The Model Practical Arithmetic

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and Bro

PREFACE.

THE present work is designed to follow the Model Elementary Arithmetic, although its arrangement and scope are such as to adapt it to schools in which but a single book can be used, while it presents every subject in arithmetic that can be thoroughly and profitably taught in the higher classes of graded and grammar schools.

Its range is sufficiently extended to qualify pupils for the solution of problems likely to arise in business; and, at the same time, to subserve the higher and more important purposes of mental discipline. It embodies every form of illustration, analysis, and exercise necessary to make it a complete manual for the class-room, and gives in a compact form a thoroughly practical and logical exposition of such subjects as are usually taught in schools of the grade for which it has been prepared.

The general plan of this and every other book of the series is to give,

1. Inductive questions and exercises unfolding the general principles of each topic as it is introduced, without the use of technical terms.

2. Definitions of such terms as are necessary to be used in a formal treatment of the subject, or in the statement of problems involving the application of the principles presented. Special care has been taken to state all definitions and rules in the same language throughout the series, so that, as pupils advance from grade to grade, they shall not be required to lose time or to renew efforts to memorize words.

3. Oral or mental exercises, placed where they properly and logically belong, as introductory to the further enunciation and formal statement of principles. The ability to perceive principles in problems orally stated, to carry them through a course of reasoning, and to apply them in the solution of problems, is of the greatest importance in disciplining and strengthening the mind. Hence, no system of arithmetic can be complete which does not combine oral with written exercises.

4. Analytical solutions of examples, containing clear and logical expositions of underlying principles, and showing their practical applications in succeeding problems. Rules, as such, are of but secondary importance; few, therefore, are given, and these, based upon principles and processes already established, are made as comprehensive as possible. In the application of general principles in advanced subjects, formulas are given as summaries of processes previously analyzed and demonstrated. They can be easily understood and quickly memorized; at the same time, they may serve as bases of rules which pupils should be required to fill out

and state in their own words. The intellect is constantly appealed to, and pupils are trained to *think*, and not merely to become expert in the solution of problems. Every step which they are required to take tends to a mental development which must ensure success in this subject, and the better prepare them to pursue every other branch of education.

5. Frequent reviews, combining principles previously learned, with new methods and processes, showing the dependence of every advanced subject upon preceding fundamental truths. These test and drill exercises are less monotonous and mechanical than would be a review of any single class of problems; at the same time they serve to present each subject, not as a detached fragment, but as a link in the chain running through the whole science.

The problems have been prepared with the design not only to afford sufficient drill to secure accuracy in calculation and to exemplify principles, but also to convey useful information respecting terms and forms stated in the phraseology of business circles, usages in commercial transactions, and values of commodities and merchandise actually found at the present day in our markets, stores, and factories. All are systematically graded and arranged so as to lead from the easy and simple to those more difficult and complicated.

The practical value of all knowledge of arithmetic must depend upon the ability of pupils to grasp the conditions of a given problem, to understand the relations existing between those conditions, to know what new relations are to be determined from them, and to apply known principles in processes and operations necessary to obtain required results. To ensure such knowledge the present work has been prepared with the view of leading pupils to rely upon principles and reason rather than upon rules, and to think, compare, and investigate for themselves.

Answers to problems are not given, because the author thinks that they always work injury to pupils by inducing them to rely upon false and unpractical tests, rather than upon their own judgment and knowledge, to determine the correctness or falsity of their calculations. The school should serve as a training and preparation for the stern duties of real life, and in it should be formed habits of attention, patient investigation, and self-reliance. Pupils should from the first be required to give indisputable evidence of their ability to perceive and apply principles, and should be held strictly responsible for the accuracy of their reasoning and computations as well as for the mechanical processes by which they obtain required results. Teachers who prefer that pupils shall have answers to aid and guide them in their work, can furnish results from the Key; in which are given also the full solutions of the more difficult problems, and some useful suggestions as to the methods of teaching this important branch of education.

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THE MODEL PRACTICAL ARITHMETIC.

CHAPTER I.

NOTATION AND NUMERATION.

Section I.

1. Express by figures the numbers—

Three thousand. Eight thousand, one hundred. One thousand, nine hundred twenty. Seven thousand, fifty-five. Nine thousand, eighty. Five thousand, one.

Twenty thousand. Ninety-one thousand. Thirty-three thousand, five hundred. Eighty-one thousand, seven hundred twenty. Seventy thousand, seventy. Forty thousand, six.

Three hundred thousand. Nine hundred ten thousand. Eight hundred thousand, one hundred. One hundred thousand, ninety. Two hundred thousand, fourteen. Seven hundred six thousand, eight hundred five. Eight hundred thousand, seven.

2. Express in words, or numerate and read, the numbers: —

3000	*4050*	*10000*	*70600*	*900000*	*300600*	*100090*
5100	*6067*	*91000*	*45060*	*180000*	*610250*	*900108*
7320	*8098*	*23400*	*60076*	*812000*	*420000*	*200020*
9103	*9005*	*87650*	*50080*	*207000*	*509070*	*800075*
2040	*1007*	*34567*	*70008*	*702300*	*100980*	*300009*

3. Express by figures, or notate, the numbers: —

Two million. Four million, sixty-five thousand. Three million, seven thousand, four hundred. Five million, twenty thousand, six hundred. Seven million, eight thousand, forty. Six million, fifty thousand, sixty. Eight million, seven hundred one. Nine million, two.

Twenty million. Forty million, one hundred thousand. Ninety million,

ten. Ten million, four thousand, seven. Eighty million, eighty thousand, eight. Thirty million, four thousand, ninety. Fifty million, two hundred one. Seventy million, forty thousand, six.

One hundred million. Nine hundred million, one hundred thousand. Two hundred million, eighty thousand, four hundred. Eight hundred million, thirty thousand, seventy. Two billion. Nine billion, forty million, eighty thousand, ten. Thirty billion. Eighty trillion, five billion, four million, twenty. Four hundred quadrillion. Seven quadrillion, forty million, four thousand. Fifty quadrillion, forty billion, three million, two hundred thousand, eleven.

4. Express in words, or numerate and read, the numbers: —

4065000	*10090000*	*70800909*	*123000000*	*400500060*
2007500	*20103000*	*60070070*	*908700000*	*650004003*
1020300	*90040060*	*90700005*	*200340050*	*500050050*
7009001	*80700500*	*50505005*	*807006500*	*900800700*
8090080	*30098001*	*80500061*	*300400050*	*100020000*
7000050	*40005006*	*77000077*	*760005004*	*800800008*

3007050030; 700065004 40; 400050005005; 900000700014; 1003-000400500; 200400050004 05; 900080007600050; 803000704000 3060; 80004000050004003; 700050005400030020.

Section II.

ADDITION.

Problems.

Find the sum of the numbers to be added in each of the following problems; read each part and each result: —

(1)	(2)	(3)	(4)	(5)	(6)
324	*23*	*2332*	*6203*	*72342*	*34*
40	*432*	*23*	*1250*	*3403*	*3421*
3413	*4302*	*70430*	*34*	*2021*	*5203*
202	*230*	*5102*	*81302*	*32*	*90220*

(7)	(8)	(9)	(10)
320 gallons.	221 quarts.	81203 pints.	3022 gills.
35 "	5330 "	530 "	63200 "
7202 "	22 "	4022 "	321 "
82030 "	202 "	43 "	402 "
401 "	93013 "	201 "	43 "

Find the sum of—

11. 1323 + 32 + 40431 + 5002 + 93211.

12. 3040 pounds + 70325 pounds + 3102 pounds + 500412 pounds + 2000 pounds.

13. What is the amount of 5000 dollars, 70025 dollars, 902512 dollars, 20220 dollars, and 2132 dollars?

14. How many ones, tens, hundreds, and thousands are in the sum of 11, 2322, 321, 6032, and 213?

15. Add together one hundred twenty-one; thirty; two hundred two; three thousand, thirteen; two hundred thousand, four hundred thirty-two.

16. Find the amount of fifty thousand, four hundred; two hundred three thousand, thirty-two; four thousand, two hundred three; four million, three hundred thousand, two hundred fifty; fifty million, four hundred thousand, four.

(17)	(18)	(19)	(20)	(21)	(22)
78	89	78870	78	987	8
7890	8	89	98076	68	89
809	9876	7896	9	7896	876
87678	897	8	9584	8	90847
9	87967	768	789	36879	9630

(23)	(24)	(25)	(26)
60789 acres.	76854 tons.	57368 cords.	30576 feet.
9086 "	8076 "	769854 "	87 "
789 "	983586 "	75 "	789 "
98765 "	76 "	9864 "	8697 "
89 "	789 "	789 "	48593 "
8697 "	9876 "	8978 "	978 "

Find the sum of —

27. 975 + 984576 + 8765 + 75978 + 596 + 869753.

28. 78 + 9807654 + 8 + 767548 + 987 + 86579 + 9087.

29. 3786 + 9876 + 47 + 98765 + 8 + 78965 + 3894576 + 385 + 98765 + 83840 + 987 + 91876.

30. 3750 bushels + 73510 bushels + 329 bushels + 8465 bushels + 75860 bushels + 937 bushels + 78756 bushels.

31. 965 tons + 7854 tons + 31416 tons + 88620 tons + 433013 tons + 2821 tons + 5236000 tons + 7525 tons + 80000 tons + 9080706 tons + 462068 tons.

32. Find the sum of 864, 9753, 13579, 24, 8642, 204060, 817263, 9283746, 3928, 192874.

33. What is the amount of 20907 yards, 3040 yards, 9182 yards, 28374 yards, 827364 yards, 82 yards, 1760 yards?

34. How many bales are 246 bales, 852 bales, 13579 bales, 86420 bales, 345 bales, 98 bales, 50000 bales, and 75 bales?

35. Add together fifty thousand, nineteen; four hundred ten; seven million, three thousand, eleven; two hundred thousand, one hundred forty-eight.

36. What is the sum of ninety thousand, ninety-six; eight thousand, sixty-nine; nineteen; seven hundred nine; four hundred thousand, eight; and five million?

37. Find the amount of ten million, ten thousand, ten; forty-seven million, nine thousand, five; six hundred thousand, six hundred six; eight thousand, four hundred; six hundred ninety-seven; eighty-six.

38. Of the Middle States, New York contains 47156 square miles, Pennsylvania 46000 square miles, New Jersey 8320 square miles, and Delaware 2120 square miles. How many square miles do all contain?

39. I bought a suit of clothes for $37.50, a pair of boots for $8.75, a hat for $4.25, and a cane for $.75. What did all cost?

40. Find the sum of 4762 dollars 25 cents, 500 dollars 80 cents, 3257 dollars 46 cents, 10 dollars 9 cents, 48 cents, and 25 dollars 75 cents.

41. What is the amount of one thousand dollars fifty cents, nine hundred dollars twenty cents, 10 dollars 10 cents, 1525 dollars 8 cents, 9 cents, and 2300 dollars?

42. What is the entire weight of four loads of hay that weigh respectively 1934, 2354, 2017, and 1875 pounds?

43. How many years elapsed from the battle of Marathon, 490 B. C., to the battle of Yorktown, A. D. 1781? The battle of Waterloo, A. D. 1815?

44. An agent bought a house for $6250. He paid $250.75 for repairs, and $150.50 for painting it. For how much must he sell it to gain $750? To gain $800.25?

45. What is the entire area of the British Isles, if England contains 51000 square miles, Wales 7000, Scotland 30000, and Ireland 33000?

46. What is the area of the Eastern Continent, if Europe contains 3764745 square miles, Asia 11196000 square miles, and Africa 10936000 square miles?

47. Find the combined length of the following South American rivers: Amazon 3600 miles, La Plata 2250 miles, Parana 2000 miles, Orinoco 1500 miles, and Colorado 1000 miles.

48. One year a merchant's sales were as follows: silk department, $341275.48; cloth, $250175.55; cotton, $92246.50; trimmings, $75127.34. What was the amount of his sales?

Section III.

SUBTRACTION.

Problems.

Find the difference between the numbers in the following problems; read each term and each result: —

(1)	(2)	(3)	(4)	(5)	(6)
17392	*92837*	*35247*	*84957*	*437969*	*364938*
4030	*71620*	*2035*	*42035*	*135042*	*243705*

(7)	(8)	(9)	(10)
43732 miles.	93847 yards.	29385 feet.	73645 inches.
2012 "	3605 "	18360 "	50343 "

Find the remainder of —

11. 7364 — 234; 19285 — 6203; 398347 — 7325.

12. 57685 bales — 1623 bales; 95847 tons — 1804 tons.

13. Find the remainder of 736579 bushels less 32046 bushels; and of 3293761 feet less 201340 feet.

14. How many ones, tens, hundreds, and thousands are in the difference between 9245 and 6203?

15. What is the difference between seven hundred ninety-six million, six hundred fifty thousand, forty-nine; and four hundred seventy-five million, three hundred twenty thousand, fifteen?

16. How many are left after taking thirty-four million, six thousand, forty-two; from seven billion, seventy-five million, six hundred eight thousand, ninety-four?

Find the difference between the numbers in the following problems; read each term and each result: —

(17)	(18)	(19)	(20)	(21)
23194	307060	5006070	70097565	60900017
7087	96374	797091	98067	900089

(22)	(23)	(24)	(25)
708005 bales.	9300036 cords.	7000604 tons.	$300750.25
60076 "	100048 "	30605 "	8760.50

Find the remainder of —

26. 305007 — 8098; 405030 — 6739; 3007001 — 78002.

27. 323434 — 9536; 100076 — 908; 4300209 — 9200.

28. 230075 gallons — 786 gallons; $500750 — $3904.

29. 30010050 tons — 501075 tons; $37050.25 — $850.50.

30. Subtract 703007050 feet from 802077075 feet.

31. Find the difference between 13050706 and 70807.

32. How many yards are 50280 yards — 789 yards?

33. How many remain after taking three billion, seventy million, two hundred four thousand, fourteen; from five billion, nine million, fifty-one thousand, seven?

34. Find the difference between one trillion, two hundred million, fifty thousand, seven; and nine hundred six million, nine thousand, eight.

35. What is the remainder of ten trillion, fifty billion, five hundred thousand, two hundred; less ninety billion, eighty million, six hundred thousand, forty?

36. What number increased by 200708 will be 30000000? What number increased by 500005?

37. What number must be taken from $700050.50 to leave a remainder of $50705.75? A remainder of $9050.81?

38. What number must be added to 70085 feet to equal 50070060 feet? To equal 50000000 feet?

39. Two sums of money amount to $300750.25. If one of them is $70825.50, what is the other?

40. The sum of two numbers is 30706800 pounds, and the less number is 790809 pounds. What is the greater?

41. If the greater of two numbers is 290007006, and the difference between them is 97007, what is the less?

42. If you subtract $70500.25 ten times from $1000000, what will each remainder be? If $90807.06 ten times?

43. A merchant bought a quantity of goods for $1320.56, and sold them for $1520. How much did he gain?

44. I sold a house for $10250, which was $325.25 more than it cost. How much did it cost?

45. At an election one man received 50125 votes, and the other received 41376. What majority had the first?

46. America was discovered in 1492. How many years passed until the Revolution, in 1775? Until the Three Years' War, in 1812? Until the Centennial Exposition, in 1876?

47. One year the earnings of a railroad were $735240.36, and the expenses were $607157.49. How much was cleared?

48. A man bought a house for $12507.50, and paid for it by giving $1250.75 a year until the house was clear. How long did it take to pay off the debt?

Problems Combining Addition and Subtraction.

NOTE.—Numbers connected by the sign + are to be united in one sum before subtracting the numbers connected or preceded by the sign —.

1. What is the result of 50262 — 9275 added to 40092 + 9078? Of 100405 — 1008 + 60302 — 907?

2. How many gallons of water must be drawn from a cistern of 100000 gallons to leave 10050 more than 30905 gallons?

3. How many dollars must be added to $25050 to make a sum of money equal to $3095 less than $30000?

4. What is the sum of 32598 feet increased by the difference between 10007 and 908 feet? Increased by 2005 feet — 109 feet?

5. How much is 50000 less 5004 diminished by the sum of 4697 and 25317? Diminished by 10008 + 9007?

6. What number increased by the difference between 20000 yards and 9005 yards equals 12002 yards?

7. What two numbers equal 15000, if one of the numbers equals 14005 less 5006? If one of the numbers is 9007 + 3009?

8. A man's salary is $1250; he receives from other sources $350, and his expenses are $1125.50. What does he save?

9. A railroad company owned 25000 acres of land, and sold 5068 acres at one time, and 10939 at another. How many acres did it still own? How many less than at first?

10. A man left $5750 to his son, $4725 to his daughter, and the balance to his wife. How much did the wife receive, the estate being worth $18000? How much more than the son?

11. The sum of four numbers is 150079, and three of them are 16327, 92837, and 9073. What is the fourth?

12. If a man's income is $5000 a year, and he pays $1200 rent, $325 for taxes, and $1525 general expenses, how much does he save? If his income were $6050.75?

13. An agent bought a farm for $11325, and paid $725.75 for improvements. How must he sell it to gain $500?

14. Some Western land cost $11075, the stock $1750, and the improvements $725.25. How much was gained or lost by selling it for $14000? For $13100.50?

15. I bought a farm for $10250, giving in exchange a house worth $6500, a note for $2500, and the balance in cash. How much money did I pay? How much more than $900.25?

16. A had $575, B $725 more than A, and C as much as A and B together, lacking $175.75. How much had C?

17. A and B had each $1000. A paid B $250, and then B loaned to A $725.50. How much had each?

18. A store valued at $25500, and goods worth $30250, were destroyed by fire. The building was insured for $16500, and the goods for $20025. What was the entire loss?

19. A grain-dealer received an order for 13500 bushels of wheat. He had in store 7265 bushels, and expected the next day 3000 bushels. How many bushels had he to buy to fill the order?

20. A merchant commenced business with a capital of $10000. The first year he gained $1750, the second he lost $2000, and the third he gained $2275.50. What was his capital then, and how much had he gained or lost in all?

Section IV.

MULTIPLICATION.

Problems.

Find the product of—

1. 50908×5; 54098×7; 567898×6; 987654×8.

2. 708098 × 6; 708908 × 7; 908706 × 8; 830745 × 9.

3. 9283746 gallons × 3; 4; 5; 6; 7; 8; 9.

4. Find the product of 465798 × 7. Of 2839475 × 9.

5. Multiply 4855964 cents by 6; 6070089 feet by 8.

6. If the multiplicand is 93847564, and the multiplier is 7, what is the product? If the multiplier is 9? 8?

7. Multiply eighty-two million, fifty-eight thousand, five hundred sixty-nine by six; by seven; by eight; by nine.

8. Find the product of four hundred eight million, fifty-nine thousand, forty-eight by seven; by eight; by nine.

9. If in one mile there are 63360 inches, how many inches are there in 5 miles? In 7 miles? 6 miles? 8 miles?

10. If $7546.58 were paid for grading and making a mile of railroad, what would 7 such miles cost? 9 miles?

(11)	(12)	(13)	(14)	(15)	(16)
1234	98765	23456	76543	34567	65764
900	230	870	345	987	405

(17)	(18)	(19)	(20)
24680 cords.	425364 gallons.	748596 bales.	968570 feet.
3456	80600	90750	80070

Multiply —

21. 70365 by 500; 600; 700.
22. 86050 by 670; 780; 890.
23. 60708 by 708; 809; 907.
24. 90075 by 923; 384; 756.
25. $78560 by 7000; 5000.
26. $85964 by 4600; 5300.
27. $975.25 by 7080; 9060.
28. $850.50 by 8006; 7009.

29. What is the product of 759780 multiplied by 70800?

30. What is the product of $7380.75 multiplied by 235?

31. Multiply 14400 minutes by 365; 1095; 7050; 90060.

32. 7050 yards × 6089 = how many? $7065 × 7065?

33. If the multiplicand is $9376.04, and the multiplier is 7054, what is the product? If the multiplier is 80090?

34. What is the product if the multiplicand is seventy thousand, fourteen, and the multiplier is nine thousand? If the multipler is eight thousand, eighty?

35. Find the product of 67000 multiplied by 8900.

36. 708900 by 700; 70800.

37. 900800 by 6800; 8090.

38. 609080 by 7060; 9800.

39. 890070 by 8006; 7066.

40. 90750 gallons by 7089.

41. $7327.50 by 9087; 70600.

42. 5076 times 40070 bushels.

43. 5000 times 2240 pounds.

44. Two factors are 4006 and 700800. Find the product.

45. If a quantity of provisions will supply 1080 soldiers 90 days, how many soldiers would it supply for one day?

46. A farmer sold 175 bushels of corn to a miller at 56 cents a bushel. How much did he receive for it?

47. At 8 cents a pound for rice, how much must a grocer pay for 700 pounds? At 9 cents? At 7 cents?

48. A farm of 234 acres was sold at an average of $75.08 an acre. What was the selling price?

49. What would be the cost of building a line of telegraph 355 miles long, at an average cost of $875.75 per mile?

50. A railroad company contracted for 74 palace cars at $11475.50 each. To how much did the contract amount?

51. How many yards of cotton goods are turned out in a year of 309 working days by a factory in which a daily average of 236 yards is made? How many in 365 days?

52. Find the product of the three factors 76, 87, and 250.

53. What is the product of the three factors 85, 100, and 250?

54. What is the continued product of 204, 980, and 678?

55. A farmer had 57 acres of corn which yielded 38 bushels per acre. What was the crop worth at 87 cents a bushel?

56. A pork-packer shipped 236 barrels of pork, each 200 pounds. What was the cargo worth at 9 cents a pound?

57. If a barrel of flour weighs 196 pounds, what is the weight of the cargo of flour on a freight-train of 23 cars, each containing

75 barrels? 4 trains, each 24 cars containing 78 barrels each?

58. What is the product if the multiplicand is 3459870, and the multiplier is 3007900? If the multiplier is 3900870?

Problems in Addition, Subtraction and Multiplication.

NOTE.—When numbers are enclosed in parenthesis marks, the operation indicated is to be performed and the result used instead of the original figures, thus in Ex. 1 $(8 \times 5) = 40$, and the example will then read $9 + 40 - 12$. Numbers connected by the sign $\times$ are to be multiplied first; then numbers or results connected by $+$ are to be added; and, lastly, numbers or results preceded by $-$ are to be subtracted.

Find the result of—

1. $9 + (8 \times 5) - 12$.
2. $(97 + 9 - 81) \times 9$.
3. $(43 - 10) + 11 \times 5$.
4. $75 - (12 \times 5 - 41)$.
5. $(72 + 35 - 46) \times 25 - 98$.
6. $98 - 49 \times (20 - 18) + 50$.
7. $(12 \times 20 - 50 + 75) \times 23$.
8. $(905 + 4561) \times 607 + 600$.

9. Find the value of $\$453 \times 58 - \$345 \times 73$.

10. What is the result of $(86 + 27 \times 27 - 50) \times 235$?

11. What is the difference between 97 times 543 and 98 times 468? The sum of 195 times 864 and 286 times 579?

12. Find the product obtained by multiplying the difference between \$9075 and \$908.25 by the sum of 846 and 454.

13. Multiply \$76 by 94, to the product add \$876.25, and from the sum subtract \$1062.50 — \$.87.

14. The sum of three numbers is 875, the least being 225, and the greatest 375. Find the continued product of the three.

15. What number must be added to 736×423 to make the amount 7654 less than 728394? 8297 less than 908006?

16. A clerk receives a salary of \$1200 a year; his board is \$365, his clothing \$156.25, and other expenses are \$212.75. How much can he save in three years?

17. A merchant bought 10 hogsheads of molasses for \$480, and retailed it at \$55.75 a hogshead. How much did he gain in all?

18. If a flour-dealer buys 1000 barrels of flour at $6.25, and sells it all for $7500, how much does he gain?

19. A grocer bought 25 hogsheads of sugar for $525. 6 hogsheads were spoiled, and he sold the remaining hogsheads at $27 each. What did he gain or lose?

20. A dealer exchanged 500 bushels of wheat, at $2 a bushel, for 150 barrels of flour, at $6.25 a barrel, receiving the difference in money. How much did he receive?

21. If an agent buys two farms, one 75 acres at $45 an acre, and the other 105 acres at $50 an acre, and pays cash $6000, how much purchase-money remains unpaid?

22. My farm-house is worth $5000; my farm is worth three times as much, lacking $4000; and my stock is worth twice as much, less $3500. How much are all worth?

23. A farmer raised 450 bushels of corn. He retained enough to feed 6 horses, allowing each 35 bushels, and sold the balance at 90 cents a bushel. What did he get for it?

24. A train by its time-table must run 355 miles in 12 hours. If it runs 29 miles an hour for 11 hours, how far must it run the last hour to arrive on time?

25. Two vessels are 3000 miles apart, and are sailing towards each other, one at the rate of 150 miles in a day, and the other 115 miles. How far apart are they at the end of 10 days?

Section V.

DIVISION.

Problems in Short Division.

(1) *4)91827(*
(2) *5)82736(*
(3) *6)73645(*
(4) *7)64536(*
(5) *8)546570(*

(6) *6)1070 feet.*
(7) *8)30045 tons.*
(8) *7)370045 cords.*
(9) *9)$37061.45*

Find the quotient of —

10. 394857 ÷ 7; 3059705 ÷ 8; 3498720 ÷ 9.

11. 2090070 ÷ 8; 40070065 ÷ 7; 300407001 ÷ 9.

12. 25000 chests ÷ 6; 80407 cases ÷ 7; 420870 sacks ÷ 8.

13. Divide 10209080 cords by 9, by 8, by 7, by 6, by 5.

14. 3607844 is 7 times what number? 8 times what?

15. The product of two numbers is 135040 bales, and the multiplier is 5. What is the multiplicand?

What is —

16. One eighth of 236835?
17. One seventh of 48376?
18. One ninth of 817540?
19. One sixth of 817261 feet?
20. One fifth of 406030 tons?
21. One ninth of $34750.50?

22. If the dividend is 2038600 gallons, and the divisor is 7, what is the quotient? If the divisor is 8?

23. Find the divisor if the dividend is $38902.50, and the quotient is 5. If the quotient is 9. If 7. If 8.

24. What is the quotient if sixty million, forty thousand, fourteen is divided by seven? by eight? by nine?

25. Divide fourteen million, four hundred thousand, fifty-five by five; by six; by seven; by nine; by four; by eight.

Find one of the —

26. 4 equal parts of 126040.
27. 6 equal parts of 456078.
28. 5 equal parts of 246800.
29. 7 equal parts of 357080.
30. 9 equal parts of 500708.
31. 6 equal parts of $56789.
32. 8 equal parts of $90807.
33. 7 equal parts of $864.20.
34. 9 equal parts of $407.50.
35. 5 equal parts of $300.10.

36. The sum of the eight equal parts of a number is 458760 bushels. What is one of the parts?

37. Mr. James is one ninth owner of a vessel valued at $17500. What is his share worth?

38. A grain-dealer shipped 125000 bushels of wheat in 4-bushel sacks. How many sacks did he fill?

Problems in Long Division.

(39) *50)987654(*

(40) *64)864234(*

(41) *71)234567(*

(42) *89)654321(*

(43) *560)192837 feet.*

(44) *607)283746 tons.*

(45) *789)$75040.50*

Divide —

46. 63047 by 420; 560; 608.
47. 70365 by 560; 607; 789.
48. 90807 by 980; 807; 765.
49. 87065 by 230; 345; 567.
50. 70654 by 753; 246; 864.
51. 307408 by 2300; 3040.
52. 135780 by 7050; 2045.
53. 203708 by 6025; 3304.
54. 900832 by 5403; 4056.
55. 807060 by 4005; 7654.

56. Find the quotient of 8476500 divided by 8705.

57. What is the quotient of 1070900 tons divided by 9086?

58. Divide 864000 yards by 596; 7098; 8908; 9007.

59. 528000 feet ÷ by 6089 = how many? $9087.50 ÷ 895?

60. If the dividend is $107040.75, and the divisor is 7908, what is the quotient?

61. What is the quotient if the dividend is twelve million, three hundred four thousand, twenty; and the divisor is nine thousand, eighty-six?

Find the quotient of —

62. 3421789 ÷ 300; 500.
63. 719203 ÷ 400; 6040.
64. 908175 ÷ 750; 8600.
65. 802090 ÷ 3040; 890.
66. 7010628 tons ÷ 4300.
67. 5100620 feet ÷ 3900 feet.
68. $7010050 by 3300; 6700.
69. $90100.75 by 2700; 9800.

70. The stock of a banking company is $250000, and it is divided into $500 shares. How many shares are there?

71. How many regiments, averaging 750 men each, will make an army of 52500 men? Of 60000 men?

72. A builder contracted to put up a block of small houses for $48000, averaging $3000 each. How many houses were there?

73. The highest mountain in the world is about 29000 feet high. How many miles high is it, each mile 5280 feet?

74. The product of two factors is 90807065, and one of the factors is 98760. What is the other factor?

75. If the dividend is 207902090, and the divisor is 24980, what is the quotient? If the divisor is 90820?

Find the quotient of—

76. \$451.72 ÷ 46; 69; 78.
77. \$673.38 ÷ 54; 81; 97.
78. \$8436.00 ÷ \$57; \$76.
79. \$96012.00 ÷ \$63; \$84.
80. \$641.25 ÷ 45 cents.
81. \$668.52 ÷ 54 cents.
82. \$15355.50 ÷ \$.29; \$.87.
83. \$20641.35 ÷ \$.31; \$.93.

84. At 8 cents a pound, how many pounds of rice cost \$90.16?

85. A farmer received \$392.40 for his crop of corn, at 72 cents a bushel. How many bushels had he?

Review Problems.

When numbers are enclosed in parenthesis marks, as $(4 \times 12 + 6)$ Ex. 1, or marked with a vinculum, as $\overline{72 + 6 + 24}$, Ex. 2, the operation indicated is to be performed and the result used instead of the original figures, thus in Ex. 1 $(4 \times 12 + 6) = 54$, and the example will then stand $54 + 9$. Operations of multiplication and division must be performed before those of addition and subtraction.

Find the result of—

1. $(4 \times 12 + 6) \div 9$.
2. $60 - \overline{72 \div 6 + 24}$.
3. $(79 - 16) \div \overline{75 - 66}$.
4. $\overline{100 + 50 \times 12} \div 35$.
5. $\overline{94 - 22} \div (50 - \overline{8 + 6})$.
6. $(216 \div 36) \times 16 + 18 - 71$.
7. $\overline{42 + (36 \div 18) + 10} \div 9$.
8. $(176 \div \overline{11 \times 4}) + \overline{60 \div 12}$.

9. Find the sum of 125×76 and $12152 \div 98$.

10. From 375 times 198 take $8765 - \overline{4376 + 2937}$.

11. Multiply $9876 + 7384$ by $\overline{65772 \div 203} + 102$.

12. Divide 8009×543 by $(79 + 987 - 60) \times 10$.

13. Find the result of $(540 + (\overline{36 - 24) \times 10}) \div (99 \div 9) + 75 - 50$. Of $4200 \div \overline{253 - 148}$ divided by $\overline{3594 - 76 \times 47}$.

14. From 7654 take 4567, multiply the remainder by 76, and divide the product by 7 times 76.

15. Divide 94068 by 804, subtract the quotient from 89 times 89, and to the remainder add one-eighteenth of 1152.

16. The factors of a number are 75 + 198 and 7365 — 6807. What is the number? What is 205 times the number?

17. What number must be added to the sum of \$75.25 and \$123.46 to amount to \$1234.65 — \$45.87?

18. The multiplier is 48, and the product is 4656. What is the multiplicand? If the multiplier were one-sixth as great, what would be the product? If the multiplicand were 5 times as great?

19. The dividend is 9725, the quotient is 106, and the remainder is 45. What is the divisor?

20. The quotient is 123, the divisor is 987, and the remainder is 901. What is the dividend? One-sixth of the dividend?

21. The product of four factors is 69392232, and three of them are 78, 89, and 98. What is the fourth?

22. Two candidates at an election received in all 113576 votes. If the first received 1356 votes more than the second, how many did each receive? If 3560 votes more?

23. A farmer had hay enough to winter 18 horses for four months. If he had sold 6 of them, how long would the hay have lasted the remaining horses?

24. A teacher receives a salary of \$1200, and his yearly expenses are \$850. If he is now worth \$1500, in how many years will he be worth \$5000? \$7500? \$10000?

25. A coal-dealer bought 150 tons of coal by the long ton of 2240 pounds, and sold it by the short ton of 2000 pounds. How many short tons did he gain? Long tons?

26. Four contractors built a railroad for \$1250000. The first built 15 miles, the second 9 miles, the third 15 miles, and the fourth 11 miles. What was the average cost per mile? What did each receive? What was the least amount?

27. A man bought 75 acres of land at \$50, and 60 acres at \$75.50. He paid \$5000 cash, and the balance in three equal yearly payments. What did he pay each year?

28. I bought 125 acres of land for \$6250, and sold 80 acres

at $55.25 an acre, and the remainder at cost. How much did I gain or lose? How much per acre?

29. A real-estate agent bought four city lots for $850, $1150, $1200, and $1500. What is the average price?

30. If a farmer buys 45 acres of land at $62.50 an acre, and 38 acres at $70.50, what was the average price per acre?

CHAPTER II.

PROPERTIES OF NUMBERS.

Section I.

Factors or Divisors.

Separate into their prime factors —

1. 108, 112, 132, 135, 140.
2. 288, 320, 352, 384, 392.
3. 420, 430, 550, 720, 755.
4. 672, 725, 824, 896, 984.
5. 1365, 2025, 2800, 3094.
6. 2916, 3584, 4536, 5670.
7. 5184, 6864, 7644, 8064.
8. 6624, 7644, 8008, 9984.

9. Resolve into their prime factors 11385 and 43197.

10. Find the prime divisors of 20020, 35343, and 50692.

11. What are the prime factors of 24035, 32725, and 41943?

12. Find every prime and every composite factor of 48 and 90.

13. What are all the numbers that will exactly divide 72 and 108? 60 and 96? 84 and 144?

14. Which of the numbers 419, 561, 847, 983, and 1024 are prime, and which are composite?

15. How many of the different factors of 96, 132, and 216 are prime, and how many are composite?

16. Find all the composite factors of 84, 112, and 144. Find the prime factors of the composite factors.

Section II.

Problems.

Find the common prime divisors of—

1. 12 and 15; 14 and 21.
2. 18 and 24; 21 and 28.
3. 35 and 49; 42 and 54.
4. 63 and 81; 72 and 96.

5. Find the prime divisors common to 26, 39, and 52.

6. What number of feet will exactly divide 34 feet, 51 feet, and 85 feet? 48 feet, 60 feet, and 72 feet?

Find all the common divisors of—

7. 36 and 42; 48 and 60.
8. 56 and 72; 60 and 84.
9. 108 and 120; 105 and 135.
10. 132 and 156; 144 and 216.

11. Find every common divisor of 112, 128, 160, and 224.

12. Find every number of bushels that will exactly divide 240 bushels, 270 bushels, and 360 bushels.

13. Find the size of every vessel that can be filled an exact number of times by 216 gallons of water and 324 gallons.

Find by the second method the greatest common divisor of—

14. 104 and 130.
15. 153 and 204.
16. 324 and 432.
17. 450 and 570.
18. 575 and 690.
19. 648 and 756.
20. 764 and 822.
21. 816 and 978.
22. 2058 and 2163.
23. 5148 and 6035.
24. 7640 and 9042.
25. 9792 and 9900.

26. 12104 and 12274; 16021 and 17205; 37303 and 44603.

27. 408, 500, and 636; 697, 861, and 1066.

28. 1368, 1766, and 1848; 3402, 5703, and 9072.

29. 10560, 13332, and 1582; 156, 192, 216, and 456.

30. 624, 1092, 1352, and 1638; 2016, 4410, 8904, and 35658.

31. What is the greatest common divisor of 3008 yards, 639 yards, and 9212 yards? $4136, $7623, and $4438?

32. What is the greatest number of feet that can be a common factor of 6006 feet, 9009 feet, and 10388 feet?

33. Find the greatest number of tons that will exactly divide 4480 tons, 5376 tons, and 9408 tons.

34. What is the length of the longest boards that can be used without cutting, to fence a garden 96 feet wide and 936 feet long?

35. A grain-dealer put 312 bushels of corn, 376 bushels of oats, and 268 bushels of wheat into the largest possible sacks of equal size. What did each sack contain?

36. From a cotton-mill were turned out three kinds of sheetings measuring 306 yards, 442 yards, and 374 yards. Into how many equal pieces of the greatest length each could they be cut?

37. Three men bought cows at the highest price that enabled each man to invest all his money. How many cows did each buy — the first man having $224, the second $352, and the third $192? What was the price of each cow?

38. A man having three farms of 98 acres, 140 acres, and 112 acres, divided them into the largest possible fields of equal size. How many acres were in each field? How many fields were there in all?

Section III.

MULTIPLES, OR DIVIDENDS.

Problems.

Find a common multiple of —

1. 3, 5, and 7.	3. 5, 7, and 9.	5. 7, 9, 10, and 11.
2. 4, 6, and 8.	4. 6, 8, and 10.	6. 8, 10, 11, and 12.

7. $5, $6, $10; 6 tons, 7 tons, 8 tons; 8 feet, 10 feet, 12 feet.

8. What number of miles is a common multiple of 5 miles, 9 miles, 10 miles, 11 miles? 8 miles, 10 miles, and 12 miles?

9. Find a number of bales that can be exactly divided by 5 bales, 7 bales, 10 bales, and 14 bales.

Find by the second method the least common multiple of—

10. 10, 12, and 15.
11. 10, 14, and 35.
12. 12, 18, and 24.
13. 13, 26, and 39.
14. 14, 21, and 28.
15. 16, 32, and 48.
16. 18, 36, and 54.
17. 21, 18, and 63.
18. 24, 8, 36, and 72.
19. 35, 14, 21, and 105.
20. 13, 104, 52, and 78.
21. 96, 8, 36, and 24.

22. 168, 14, 41, 63, 6; 70, 7, 210, 5, 280, 70.

23. 16, 4800, 80, 192, and 9; 1050, 210, 7, 40, and 210.

24. $5, $100, $50, $75; 13 feet, 200 feet, 52 feet.

25. 17 tons, 102 tons, 153 tons, and 34 tons; 38 cords, 19 cords, 114 cords, 171 cords, and 57 cords.

26. Find the least common multiple of 9702 and 12474; of 3080 and 36960; and of 2304 and 8432.

27. What number is the least common multiple of 81 acres, 2388 acres, and 597 acres? Of 15625 cords and 875 cords?

28. What is the least number that can be divided by each of the nine digits without a remainder?

29. What is the shortest piece of kite-string that could be cut into lengths of 225 feet, or 600 feet?

30. What is the narrowest box in which ribbons 2 inches wide, 3 inches, or 4 inches could be packed?

31. Find the smallest sum of money that can be made up of 2-cent, 3-cent, 5-cent, 10-cent, 20-cent, 25-cent, or 50-cent pieces.

32. What is the least number of bushels of oats that could be put in bins holding 48 bushels, 72 bushels, and 80 bushels?

33. Find the smallest sum of money for which I could hire laborers at $25, $36, $40, or $50 per month.

34. What is the least number of yards of muslin that could be cut into whole pieces containing 36 yards, 33 yards, or 40 yards?

35. Four persons traveling in the same direction go respectively 36 miles, 24 miles, 45 miles, and 48 miles per day. How far will they travel before coming together again?

Section IV.

CANCELLATION.

Problems.

Divide by cancellation —

1. 513 by 324; 1452 by 1089; 22464 by 5804.

2. $32 \times 10 \times 8$ by 40×16; $42 \times 22 \times 52$ by 28×26.

3. $225 \times 6 \times 30$ by $6 \times 33 \times 15$; $247 \times 7 \times 8$ by $81 \times 7 \times 12$.

4. $135 \times 7 \times 3$ by $21 \times 27 \times 9$; $5 \times 15 \times 375$ by 25×75.

5. Divide $21 \times 56 \times 16 \times 8$ by $50 \times 16 \times 7 \times 9$.

6. How many times is 81×35 contained in $90 \times 21 \times 3$?

7. Find the quotient of $(7 \times 153 \times 19) \div (7 \times 204 \times 14)$.

8. The dividend is $5 \times 15 \times 25 \times 35$, and the divisor is $25 \times 5 \times 3 \times 7$. What is the quotient?

9. The factors of a dividend are 7, 49, and 35, and of a divisor are 25, 7, 21. What is the quotient?

10. What is the value of $(102 \times 90 \times 61) \div (400 \times 30 \times 91)$? Of $(26 \times 28 \times 7 \times 8) \div 91 \times 4 \times 5$?

11. How many pounds of tea at 80 cents a pound are worth as much as 10 pounds of coffee at 30 cents a pound?

12. At \$.75 a day, how long will it take a boy to earn as much as a man earns in 70 days at \$2.25 a day?

13. How many tons of coal at \$7 a ton will cost as much as 28 cords of wood at \$5 a cord?

14. A farmer sold 135 pounds of butter at \$.28 a pound, which paid exactly for some sugar at \$.14 a pound. How many pounds of sugar did he buy?

15. When 84 men can mow 72 acres of grass in 18 days, how many acres can 96 men mow in 12 days?

16. A drover sold 78 horses at \$100 each, and with the proceeds he bought cattle at \$39 each. How many head of cattle did he buy?

Review Problems.

1. Which numbers between 200 and 250 are prime and which are composite?

2. Find all the prime divisors of 68616 and 94308.

3. Find the largest prime factor and the largest composite factor of 8290 and of 39858.

4. How many times does 7 occur as a factor of 12348?

5. Which of the numbers 887, 1155, 993, and 7623 are prime? Which are composite, and what are their prime factors?

6. Find all the different divisors of 264 and 384; and every common divisor. Find the greatest common divisor.

7. Find the prime divisors of 576 and 864; their composite divisors; their common divisor; their greatest common divisor.

8. Find the greatest common divisor of 36036 and 45740; of 42368 and 47012; of 41769, 51051, and 54264.

9. What is the least common multiple of 9, 117, 13, 234, 39, and 585? Of 12, 8, 144, 72, 432, and 720?

10. Divide 33075 by 1890, using their factors.

11. How many times is 54×35 contained in $9 \times 7 \times 10 \times 18$? 8 times 256 in 80 times 48?

12. Find the least common multiple of the first six odd numbers. Of the first six even numbers.

13. Divide the least common multiple of 4095, 6825, 7735, and 2730 by their greatest common divisor.

14. What is the largest number that will exactly divide 575 and 765? The smallest that can be exactly divided by each of them?

15. How many days' work at \$1.75 a day will pay for 15 bushels of potatoes at \$.75 a bushel?

16. Find the least number of tons that can exactly contain 87 tons, 290 tons, and 406 tons; and the greatest that can exactly divide each. 93 tons, 361 tons, and 496 tons.

17. A and B bought some horses at the same rate per head. A's horses cost \$623, and B's \$1068. How many did each buy?

18. What is the least number of bushels of wheat that would make an exact number of loads for drays that can haul respectively 30, 36, and 45 bushels at a load?

19. A has \$28, B \$42, C \$63, and D as many as the least common multiple of the sums that the other three have. How many dollars has D?

20. A owns 140 acres of land, and B owns 175 acres. If C owns as many as the greatest common divisor of A's and B's, and D has as many as the least common multiple of A's and B's, how many acres have C and D?

CHAPTER III.

FRACTIONS.

Section I.

REDUCTION OF FRACTIONS.

Problems.

Change or reduce to their lowest terms: —

1. $\frac{150}{200}$; $\frac{168}{216}$; $\frac{192}{288}$; $\frac{198}{330}$.
2. $\frac{224}{368}$; $\frac{288}{544}$; $\frac{396}{726}$; $\frac{450}{980}$.
3. $\frac{208}{130}$; $\frac{285}{228}$; $\frac{336}{504}$; $\frac{675}{532}$.
4. $\frac{736}{1166}$; $\frac{798}{1218}$; $\frac{952}{630}$; $\frac{1062}{1834}$.
5. $\frac{1210}{1694}$; $\frac{1320}{1848}$; $\frac{369}{1968}$; $\frac{540}{2018}$.
6. $\frac{525}{1785}$; $\frac{819}{2483}$; $\frac{2132}{12669}$; $\frac{3591}{22743}$.

7. \$$\frac{1377}{306}$; $\frac{192}{1280}$ of an acre; \$$150\frac{75}{100}$; $225\frac{189}{1260}$ gallons.

8. What are the lowest terms in which $\frac{3675}{13905}$, $\frac{4032}{19404}$, $\frac{11211}{224559}$, $\frac{174427}{406863}$, and $\frac{20448}{624771}$ can be expressed?

9. Which is the greater fraction and why, $\frac{57}{513}$, or $\frac{134}{1206}$?

10. One village lot contained $\frac{12}{16}$ of an acre, and another contained $\frac{480}{640}$ of an acre. Which was the larger of the two lots?

Problems.

Reduce or change to higher terms—

1. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to twelfths.
2. $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{9}{10}$ to twentieths.
3. $\frac{7}{6}$, $\frac{7}{10}$, $\frac{7}{12}$, $\frac{7}{15}$ to sixtieths.
4. $\frac{3}{4}$, $\frac{7}{8}$, $1\frac{1}{2}$ to 24ths; 48ths.
5. $\frac{5}{6}$, $\frac{7}{21}$, $1\frac{3}{7}$ to 42ds; 84ths.
6. $\frac{8}{9}$, $\frac{17}{18}$, $\frac{33}{34}$ to 72ds; 144ths.

7. $\frac{15}{16}$, $\frac{17}{24}$, and $\frac{47}{48}$ to fractions having 48 as the denominator.

8. Change $\frac{9}{10}$, 9, $\frac{98}{840}$, $\frac{14}{15}$, $\frac{19}{20}$, 30, $\frac{59}{60}$ each to sixtieths.

9. Express 8, $7\frac{1}{8}$, $\frac{10}{12}$, 15, $\frac{11}{16}$, $\frac{33}{34}$, 30 and $\frac{1}{48}$, and $\frac{1}{96}$ as fractions having 96 as their denominator; having 384.

10. Change $\frac{7}{12}$ of a foot and $\frac{35}{36}$ of a yard to fractions having terms 4 times, 6 times, and 10 times as great.

Problems.

Change or reduce to improper fractions—

1. 5, 7, and 9 to fifths.
2. 8, 10, and 12 to sixths.
3. 10, 15, and 25 to tenths.
4. 8, $8\frac{1}{4}$, $10\frac{3}{8}$, $27\frac{5}{8}$ to eighths.
5. $9\frac{1}{12}$, $25\frac{5}{12}$, $58\frac{7}{12}$ to twelfths.
6. $10\frac{1}{10}$, $30\frac{2}{40}$, $75\frac{19}{20}$ to twentieths.

7. 20 yards to 8ths of a yard; \$30$\frac{1}{8}$ to quarter-dollars.

8. $135\frac{1}{70}$, $200\frac{1}{20}$, $300\frac{1}{300}$, $4000\frac{1}{4}\frac{1}{20}$ to improper fractions.

9. How many hundredths of a dollar are \$20? \$50$\frac{35}{100}$? \$75$\frac{50}{100}$? \$1000$\frac{99}{100}$? \$1050$\frac{1}{4}$? \$2000$\frac{1}{10}$? \$3000$\frac{1}{50}$?

10. How many quarter-dollars will pay for a hat that costs \$3? For a coat that costs \$10$\frac{1}{2}$?

11. How many half-pound packages of tea can be made from a chest of tea weighing $56\frac{1}{2}$ pounds? Quarter-pound packages?

12. Into how many lots, each $\frac{1}{8}$ of an acre, can a field containing $7\frac{1}{8}$ acres be divided?

Problems.

Change or reduce to whole or mixed numbers—

1. $\frac{24}{8}$, $\frac{40}{10}$ of a yard to yards.
2. $\frac{33}{9}$ and $\frac{22}{12}$ of a foot to feet.
3. $\frac{48}{16}$ and $\frac{96}{24}$ of a ton to tons.
4. $\frac{40}{20}$, $\frac{66}{22}$, $\frac{80}{30}$, $\frac{94}{40}$ to ones.
5. $\frac{101}{10}$, $\frac{201}{21}$, $\frac{331}{33}$, $\frac{495}{45}$, $\frac{505}{56}$.
6. $\frac{601}{30}$, $\frac{725}{36}$, $\frac{809}{48}$, $\frac{1003}{50}$, $\frac{2010}{101}$.

7. $\frac{4500}{44}$, $\frac{6050}{80}$, $\frac{7006}{140}$, $\frac{8070}{201}$, $\frac{10008}{450}$, $\frac{125025}{5050}$, $\frac{20304005}{84682}$.

8. Express the value of $\frac{62720}{2240}$ of a ton in whole tons.

9. How many pounds of tea in 275 packages, each containing $\frac{1}{4}$ of a pound? In 600 packages? In 1730?

10. What number of acres is in a piece of land composed of 175 lots, each $\frac{1}{8}$ of an acre? In 500 lots? 875 lots?

Problems.

Change to least similar fractions —

1. $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$.
2. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{7}{12}$.
3. $\frac{5}{6}$, $\frac{5}{8}$, $\frac{7}{9}$, $\frac{9}{16}$.
4. $\frac{3}{10}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{9}{20}$.
5. $\frac{7}{12}$, $\frac{7}{18}$, $\frac{7}{24}$, $\frac{7}{36}$.
6. $\frac{9}{12}$, $\frac{9}{18}$, $\frac{9}{21}$, $\frac{9}{42}$.
7. $\frac{11}{16}$, $\frac{11}{24}$, $\frac{11}{32}$, $\frac{11}{48}$.
8. $\frac{1}{10}$, $\frac{3}{20}$, $\frac{4}{25}$, $\frac{7}{50}$.
9. $\frac{11}{24}$, $\frac{13}{48}$, $\frac{17}{72}$, $\frac{23}{96}$.

10. $\frac{9}{10}$ of an acre, $\frac{11}{16}$ of an acre, and $\frac{25}{100}$ of an acre.

11. Find the least similar fractions equal to $\frac{7}{11}$, $\frac{16}{5}$, $\frac{8}{55}$, $3\frac{1}{121}$.

12. To what least similar fractions can $\frac{3}{5}$, $2\frac{1}{16}$, $\frac{31}{50}$, and $\frac{53}{75}$ be changed? $\$1\frac{1}{4}$, $\$\frac{9}{5}$, $\$10\frac{1}{10}$, $\$\frac{40}{50}$, and $\$\frac{28}{100}$?

13. Change to equivalent fractions having the least common denominator $\frac{7}{19}$, $1\frac{1}{78}$, $\frac{3}{57}$, $\frac{5}{38}$, and $\frac{11}{95}$.

14. Express $7\frac{1}{10}$, $10\frac{9}{25}$, $\frac{21}{50}$, and $20\frac{1}{20}$ as similar fractions having the least common denominator.

Review Problems.

1. What are the lowest terms to which $\frac{91}{546}$, $\frac{1850}{2775}$, $\frac{2016}{9072}$, $\frac{2457}{10101}$, $\frac{2502}{19055}$, $\frac{7548}{26478}$, $\frac{2508}{35948}$, and $\frac{4032}{102144}$ can be changed?

2. Change 37 to the form of a fraction having 37 as its denominator. Change it to 100ths; to ones; to 1000ths; to 5ths.

3. Change $\frac{1000}{99}$, $\frac{9999}{100}$, $\frac{6666}{222}$, $\frac{80008}{999}$, $\frac{555555}{777}$, $\frac{90000}{888}$, and $\frac{200000}{90009}$ to integers or mixed numbers.

4. Name four fractions that can be changed to 36ths; 48ths.

5. What improper fraction equals $600\frac{1}{60}$? $707\frac{7}{101}$? $1000\frac{5}{102}$? $3030\frac{1}{404}$? $4004\frac{1}{999}$? $6066\frac{11}{666}$? $10008\frac{101}{334}$?

6. Change 11 twelfths to one hundred and eighths. To two hundred and sixteenths. To 1728ths. 3000ths.

7. Express $357 \div 765$ as a fraction in its lowest terms.

8. Express each of the integers 3, 7, 9, 13, 50, and 100 as a fraction having as its denominators 3, 7, 9, 13, 50, and 100.

9. If 6 cords of wood are divided among 4 persons, what fraction of a cord does each obtain?

10. In $100\frac{45}{63}$ hogsheads are how many 63ds of a hogshead? How many 7ths? How many 504ths? 21sts?

11. Which is the largest of four village lots containing respectively $\frac{30}{40}$, $\frac{7}{8}$, $\frac{75}{80}$, and $\frac{125}{160}$ of an acre?

12. Among how many boys can $\$7\frac{3}{4}$ be divided so that each boy shall receive $\frac{1}{4}$ of a dollar? $\$8\frac{2}{8}$? $\$10\frac{1}{2}$? $\$12\frac{3}{4}$?

13. One day a fruit-dealer bought 100 baskets of peaches, each containing $\frac{3}{8}$ of a bushel. How many bushels did he buy?

14. How many quarter-dollars are in $\$12\frac{1}{4}$? $\$20\frac{2}{8}$? $\$25\frac{1}{2}$?

Section II.

ADDITION OF FRACTIONS.

Find the sum of—

1. $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.
2. $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$.
3. $\frac{4}{9}$, $\frac{7}{9}$, and $\frac{8}{9}$.
4. $\frac{3}{10}$, $\frac{5}{12}$, $\frac{8}{15}$; $\frac{7}{12}$, $\frac{14}{15}$, $\frac{17}{18}$.
5. $\frac{9}{14}$, $\frac{9}{16}$, $\frac{9}{21}$; $\frac{8}{15}$, $\frac{9}{20}$, $\frac{11}{30}$.
6. $\frac{11}{18}$, $\frac{13}{12}$, $\frac{35}{36}$; $\frac{13}{20}$, $\frac{13}{25}$, $\frac{13}{50}$.
7. $\frac{7}{13}$, $\frac{17}{39}$, $\frac{27}{52}$.
8. $\frac{19}{38}$, $\frac{17}{76}$, $\frac{21}{57}$.
9. $\frac{11}{21}$, $\frac{31}{63}$, $\frac{1}{105}$.
10. $\$\frac{3}{5}$, $\$\frac{7}{20}$, $\$\frac{17}{100}$; $\frac{7}{9}$ of a yard, $\frac{25}{36}$ of a yard, $\frac{11}{12}$ of a yard.
11. Find the sum of $\frac{5}{133}$, $\frac{7}{19}$, and $\frac{9}{95}$; $\frac{71}{108}$, $\frac{87}{12}$, and $\frac{11}{84}$.
12. What is the amount of $\frac{9}{143}$, $\frac{120}{117}$, $\frac{7}{65}$? Of $\frac{121}{33}$, $\frac{1}{121}$, $\frac{9}{55}$?
13. Add $\frac{17}{20}$ of a ton, $\frac{87}{100}$ of a ton, and $\frac{7}{2000}$ of a ton.
14. What part of a dollar is the sum of $\$\frac{3}{10} + \$\frac{25}{100} + \$\frac{7}{1000} + \$\frac{18}{1000}$? Express the result in its lowest terms.
15. How much land in three village lots containing respectively $\frac{7}{8}$ of an acre, $\frac{71}{80}$ of an acre, and $\frac{159}{160}$ of an acre.

Find the sum of—

16. $7\frac{3}{5}$, 10, $9\frac{5}{8}$.
17. $\frac{6}{7}$, $9\frac{7}{9}$, 12.
18. 15, $\frac{47}{25}$, $8\frac{3}{35}$.
19. $10\frac{2}{9}$, $\frac{14}{15}$, $16\frac{7}{30}$.
20. 30, $40\frac{3}{40}$, $50\frac{5}{12}$.
21. $\frac{35}{38}$, $60\frac{1}{57}$, $25\frac{75}{76}$.
22. 100, $7\frac{11}{50}$, $\frac{27}{125}$.
23. $\frac{7}{88}$, 75, $76\frac{8}{77}$.
24. $30\frac{1}{50}$, 10, $11\frac{1}{10}$.

Section III.

SUBTRACTION OF FRACTIONS.

Problems.

Find the remainder of—

1. $6\frac{5}{8}$ less $5\frac{1}{8}$.
2. $7\frac{3}{7}$ less $4\frac{5}{8}$.
3. $9\frac{5}{9}$ less $8\frac{6}{7}$.
4. $19\frac{13}{30} - 11\frac{17}{18}$.
5. $33\frac{5}{42} - 30\frac{5}{63}$.
6. $75\frac{15}{57} - 69\frac{15}{18}$.
7. $100 - \frac{1}{1000}$.
8. $300 - \frac{299}{300}$.
9. $500\frac{1}{10} - \frac{99}{100}$.
10. From $100\frac{1}{10}$ take $99\frac{9}{10}$. Take $50\frac{5}{42}$ from $60\frac{5}{63}$.
11. How much less is $75\frac{3}{25}$ than $100\frac{2}{75}$? $99\frac{10}{11}$ than $200\frac{10}{99}$?
12. What number added to $\$37.43\frac{1}{2}$ equals \$45?
13. If $199\frac{3}{11}$ be taken from $300\frac{1}{10}$, what will be left? $20\frac{1}{22}$ from $22\frac{1}{20}$? $200\frac{1}{100}$ from 301? $500\frac{1}{50}$ from $5000\frac{3}{100}$?
14. The sum of two numbers is $600\frac{7}{100}$, and one of them is $111\frac{7}{600}$. What is the other? Subtract the less from it.
15. The greater of two numbers is $205\frac{111}{1000}$, and the less is $204\frac{10}{99}$. What is their difference? Take it from the less.
16. The minuend is $250\frac{5}{22}$, and the subtrahend is $205\frac{5}{33}$. What is the remainder? Take it from the subtrahend.
17. An agent bought a property for $\$3925.33\frac{1}{3}$, and sold it for $\$4000.62\frac{1}{2}$. How much did he gain?
18. If I pay $\$9075\frac{25}{100}$ for an estate, and sell it at a loss of $\$900\frac{2}{3}$, what do I receive for it? If at a gain of $\$100\frac{1}{8}$?

Find the value of—

19. $7\frac{3}{5} + 6\frac{2}{3} - 5\frac{6}{7}$.
20. $9\frac{7}{8} - (3\frac{4}{7} - 2\frac{8}{9})$.
21. $11 - (5\frac{5}{8} + 4\frac{4}{9})$.
22. $31\frac{1}{5} + \overline{75 - 69\frac{1}{30}} - 5\frac{11}{15}$.
23. $100 - (25\frac{3}{17} + 9 - \frac{4}{5})$.
24. $\overline{150 - 50\frac{6}{7}} + \overline{80\frac{10}{27} - 70\frac{7}{18}}$.
25. $\$2000 - (\$600 - \$550\frac{75}{100}) + \$901\frac{9}{10} - \$5\frac{3}{4}$.
26. To what must $7\frac{2}{7}$ be added that the sum may be $8\frac{1}{8}$?
27. From the sum of $10\frac{3}{20}$ and $20\frac{3}{10}$ take their difference.

28. To the difference between $30\frac{1}{18}$ and $20\frac{5}{12}$ add their sum.

29. From the sum of $27\frac{7}{9} + 15\frac{5}{12}$ take $50 - 30\frac{7}{30}$.

30. What number added to $30\frac{7}{11} - 11\frac{7}{30}$ makes $330\frac{7}{10}$?

31. The difference between two numbers is $17\frac{3}{26}$ tons, and the less is $40\frac{2}{39}$ tons. What is the greater number?

32. The sum of three numbers is $350\frac{11}{32}$ cords. One of them is $200\frac{7}{8}$ cords, and another is $100\frac{4}{18}$ cords. What is the third?

33. A man had in bank $\$500\frac{50}{100}$, then earned enough to pay $1000 on his house, lacking $\$125\frac{7}{8}$. What did he earn?

34. The sum of two numbers is $\$75.12\frac{1}{2}$, and the greater is $\$50.33\frac{1}{3}$. What is the less? Take the less from the greater. Add their sum to their difference.

35. A merchant bought some cloth for $\$750.87\frac{1}{2}$, and some silk for $1200.25. He sold the cloth for $801.90, and the silk for $\$1190.16\frac{2}{3}$. What did he gain or lose?

Section IV.

MULTIPLICATION OF FRACTIONS.

Problems.

Find the value of—

1. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$.
2. $\frac{5}{8}$ of $\frac{7}{8}$ of $\frac{8}{9}$.
3. $\frac{10}{20}$ of $\frac{50}{57}$ of $\frac{38}{75}$.
4. $\frac{31}{32}$ of $\frac{48}{49}$ of $\frac{77}{93}$.
5. $\frac{19}{33}$ of $\frac{48}{95}$ of $12\frac{1}{10}$.
6. $\frac{58}{63}$ of $\frac{84}{87}$ of $32\frac{9}{20}$.

7. Change the compound fraction $\frac{1}{4}\frac{3}{2}$ of $\frac{4}{3}\frac{9}{5}$ of $8\frac{5}{6}\frac{5}{5}$ to a simple fraction; $\frac{9}{34}$ of $\frac{85}{9}$ times 33; $66\frac{2}{3}$ times $\frac{1}{2}\frac{9}{0}$ of $1\frac{5}{76}$.

8. If a bushel of wheat is worth $\$1\frac{7}{10}$, what is the value of $\frac{4}{5}$ of 7 bushels? $\frac{5}{6}$ of $8\frac{2}{3}$ bushels? $\frac{3}{4}$ of $9\frac{3}{5}$ times 13 bushels?

Find the product of—

9. $\frac{5}{6}$ of $9 \times \frac{7}{8}$ of $\frac{1}{11}\frac{0}{} \times 8\frac{1}{4}$.
10. $\frac{7}{8}$ of $14\frac{2}{5} \times 14\frac{2}{7} \times \frac{9}{50}$.
11. $\frac{7}{11} \times \frac{5}{13}$ of $17\frac{3}{5} \times 13\frac{3}{4}$.
12. $\frac{3}{5}$ of $17\frac{1}{2}$ of $\frac{7}{26} \times 7\frac{4}{5}$ of $\frac{1}{2}\frac{0}{1}$.
13. $9\frac{3}{5} \times 9\frac{1}{6}$ times $22 \times \frac{9}{12}\frac{}{1}$.
14. $8\frac{4}{5}$ times $\frac{7}{8}$ of $5\frac{9}{11} \times 100$.

15. Multiply together $15\frac{3}{4}$, $5\frac{1}{7}$, $\frac{1}{9}$ of $8\frac{3}{4}$, and $30\frac{1}{10}$.

16. If the multiplicand is 8 times $9\frac{3}{4}$ feet, and the multiplier is $\frac{7}{8}$ of $10\frac{2}{3}$ times $\frac{2}{7}$, what is the product?

17. Three factors are $\frac{8}{9}$ of $16\frac{2}{3}$, $2\frac{7}{10}$ times $\overline{9\frac{2}{7} \times 1\frac{5}{18}}$, and $8\frac{4}{7}$ times $11\frac{1}{50}$. What is the product?

18. Find the cost of $\frac{7}{8}$ of $166\frac{2}{3}$ acres at $\frac{5}{6}$ of $\$50\frac{4}{10}$ an acre?

19. If I pay $\$275.87\frac{1}{2}$ for a horse, and sell him for $\frac{3}{4}$ of the cost, what do I get for him? If for $\frac{6}{5}$ of the cost?

20. If $2\frac{3}{8}$ tons of hay are cut from an acre, how much hay will $10\frac{2}{7}$ acres yield? $60\frac{4}{9}$ acres? $8\frac{1}{10}$ times $10\frac{5}{7}$ acres?

Find the value of —

21. $(7\frac{3}{4} + 8\frac{2}{8}) \times \frac{108}{121} - 10\frac{7}{9}$.

22. $8\frac{4}{5} \times \overline{(7\frac{5}{9} - 5\frac{7}{12}) + 12\frac{7}{20}}$.

23. $15\frac{7}{8} + 9\frac{9}{12} \times (8\frac{4}{9} - 7\frac{5}{6})$.

24. $\overline{25\frac{3}{10} + 10\frac{4}{5} \times 1\frac{7}{18}} - 35\frac{5}{8}$.

25. $60\frac{5}{8} - 3\frac{7}{15} + (10\frac{2}{7} \times 7\frac{7}{8})$.

26. $85\frac{9}{10} - (30\frac{4}{5} + 25\frac{8}{18} - 7\frac{5}{12})$.

27. What is the result of $7\frac{7}{10}$ times the sum of $8\frac{3}{11}$ and $10\frac{6}{7}$?

28. What number added to $\frac{8}{20}$ of $7\frac{7}{9}$ times $14\frac{2}{7}$ equals 50?

29. Find the sum of $10\frac{5}{16} \times 18\frac{2}{5}$ and $10\frac{3}{20} - 9\frac{99}{100}$. The difference. The product of the sum and the difference.

30. $9\frac{3}{4}$ times $\frac{7}{26}$ of $30\frac{2}{9}$ tons are how many less than $80\frac{1}{20}$ tons? How many more than $60\frac{99}{100}$ tons?

31. How much more than 3 ten-dollar bills is needed to pay for $3\frac{1}{3}$ times 4 barrels of apples at $\frac{2}{3}$ of $\$4\frac{1}{8}$ a barrel?

32. Henry earns in a month $\$8\frac{7}{8}$ less than $4\frac{9}{10}$ times $\$10\frac{2}{7}$. How much less than $4\frac{2}{7}$ times $\$10\frac{1}{2}$ does he receive?

33. A clerk's salary is \$62.50 a month, and his expenses are $\$50.87\frac{1}{2}$. What can he save in a year and 3 months?

34. How much must be paid for $13\frac{3}{4}$ tons of coal at $\$6\frac{7}{10}$ a ton, and $14\frac{3}{8}$ cords of wood at $\$5\frac{3}{4}$ a cord? What change would be due from nine 20-dollar bills offered in payment?

35. The sum of four fractions is $100\frac{3}{100}$. The first is $30\frac{8}{25}$, the second is $\frac{5}{7}$ of $42\frac{7}{10}$, and the third is $3\frac{1}{5}$ times $10\frac{5}{8}$. What is the fourth? What is their continued product?

Section V.

DIVISION OF FRACTIONS.

Fractions divided by integers.

Problems

Divide —

1. $\frac{9}{12}$ by 6.	4. $\frac{24}{65}$ by 18.	7. $45\frac{9}{11}$ by 27.	10. $200\frac{15}{28}$ by 50.
2. $\frac{10}{11}$ by 8.	5. $\frac{42}{75}$ by 56.	8. $60\frac{15}{44}$ by 51.	11. $405\frac{38}{41}$ by 76.
3. $\frac{12}{21}$ by 9.	6. $\frac{65}{96}$ by 78.	9. $80\frac{40}{58}$ by 65.	12. $650\frac{51}{77}$ by 85.

13. Divide $\$\frac{36}{49}$ by 6, by 8, by 9, by 12, by 18, by 24, by 36.

14. Find the quotient of $\frac{6}{25} \div 9$; $10\frac{3}{7} \div 12$; $50\frac{14}{25} \div 21$.

15. If a merchant's weekly sales amount to $\$2170\frac{50}{100}$, what are his average daily sales?

16. How much does a barrel of flour cost at the rate of $\frac{3}{8}$ of $500 for $\frac{8}{9}$ of 36 barrels? For $\frac{7}{15}$ of 75 barrels?

Integers divided by fractions.

Problems.

Divide —

1. 15 by $\frac{10}{11}$.	7. 72 by $14\frac{3}{8}$.
2. 27 by $\frac{18}{19}$.	8. 84 by $12\frac{3}{5}$.
3. 39 by $\frac{26}{33}$.	9. 92 by $13\frac{4}{5}$.
4. 49 by $\frac{35}{42}$.	10. 200 by $25\frac{5}{8}$.
5. 68 by $\frac{51}{75}$.	11. 505 by $30\frac{3}{10}$.
6. 87 by $\frac{58}{99}$.	12. 750 by $80\frac{7}{20}$.

13. 75 tons by $\frac{10}{11}$, by $\frac{12}{13}$, by $\frac{15}{16}$, by $\frac{20}{21}$, by $\frac{25}{27}$, by $\frac{30}{37}$.

14. Find the quotient of $250 \div 4\frac{4}{9}$; $300 \div 10\frac{5}{12}$; $450 \div 20\frac{10}{17}$; $600 \div 50\frac{15}{18}$; $2050 \div 6\frac{3}{20}$; $5000 \div 5\frac{50}{100}$.

15. How many plows costing $\$10\frac{25}{100}$ can be bought for $\$20\frac{1}{2}$?

16. 640 acres of land can be divided into how many fields each containing $24\frac{3}{5}$ acres? Each $36\frac{9}{10}$ acres?

Fractions divided by fractions.

Problems.

Divide —

1. $\frac{9}{10}$ by $\frac{75}{16}$.
2. $\frac{11}{26}$ by $\frac{17}{39}$.
3. $\frac{38}{51}$ by $\frac{57}{85}$.
4. $10\frac{2}{7}$ by $2\frac{4}{7}$.
5. $25\frac{2}{3}$ by $9\frac{1}{6}$.
6. $70\frac{5}{9}$ by $8\frac{7}{15}$.
7. $81\frac{3}{5}$ by $\frac{3}{4}$ of $9\frac{5}{7}$.
8. $90\frac{4}{11}$ by $\frac{6}{7}$ of $55\frac{3}{8}$.
9. $\frac{7}{10}$ of $100\frac{5}{9}$ by $50\frac{10}{36}$.
10. $25\frac{3}{5}$ by 80; 200 by $46\frac{7}{8}$; $\frac{7}{9}$ of $13\frac{1}{2}$ by 24 times $4\frac{7}{18}$.
11. $4\frac{2}{7}$ times $8\frac{3}{4}$ by $\frac{7}{10}$ of $\frac{15}{22}$ of $19\frac{4}{5}$; by $7\frac{3}{7}$ times $4\frac{11}{13}$.
12. What fraction multiplied by $13\frac{1}{2}$ times $6\frac{6}{9} = 16\frac{1}{4} \times 19\frac{1}{2}$?
13. If the dividend is $3\frac{9}{10}$ times $4\frac{8}{13}$, and the divisor is $\frac{7}{15}$ of $56\frac{1}{4}$, what is the quotient? If the dividend is $9\frac{3}{7} \times 8\frac{10}{11}$?
14. The dividend is $6\frac{6}{13}$ times $18\frac{4}{7}$, and the quotient is $14\frac{2}{9}$ times $1\frac{3}{2}$. What is the divisor?
15. If $22\frac{1}{2}$ bushels of wheat weigh $1404\frac{3}{10}$ pounds, what is the average weight per bushel?

Problems.

What part of —

1. 84 is 60?
2. 25 is $\frac{2}{5}$? $\frac{7}{10}$?
3. 75 is $18\frac{3}{4}$?
4. $\frac{15}{16}$ is $\frac{5}{6}$? $\frac{9}{15}$? $\frac{12}{25}$?
5. $5\frac{1}{4}$ is $\frac{7}{8}$? $\frac{9}{10}$? $\frac{14}{50}$?
6. $37\frac{1}{2}$ is $6\frac{1}{4}$? $56\frac{1}{4}$?
7. $112\frac{1}{2}$ is $\frac{5}{8}$ of $\frac{5}{9}$?
8. $\frac{6}{7}$ of $12\frac{1}{2}$ is $37\frac{1}{2}$?
9. $\frac{8}{9} \times 22$ is $10\frac{1}{12}$?
10. What is the relation of 80 to 60? Of 25 to $\frac{7}{10}$? Of $18\frac{3}{4}$ to 75?
11. Find the relation of $4\frac{2}{7}$ to 20; 50 to $4\frac{4}{9}$; $66\frac{2}{3}$ to $\frac{3}{4}$ of $62\frac{1}{2}$.
12. If from \$250 a man spends \$50, what part of his money does he spend? What part of his money remains?
13. From a piece of muslin containing $33\frac{1}{3}$ yards there were sold $18\frac{3}{4}$ yards. What part of the piece was sold?
14. If 7 yards of broadcloth cost $\$38\frac{1}{2}$, what part of a yard can be bought for $\$2\frac{3}{4}$? For $\$3\frac{7}{10}$? For $\$4\frac{5}{8}$? For $\$5\frac{11}{20}$?
15. A lady having \$50 gave \$10 for a bonnet, and $\frac{3}{10}$ of the remainder for a dress. What part of \$50 had she left? What part had she spent? What part for the bonnet? What part for the dress?

16. If a lot of ground containing $3507\frac{1}{2}$ square feet is worth \$2500, what is the price per square foot?

17. If 10 barrels of apples cost 8 times $\frac{5}{11}$ of $\$8\frac{37}{80}$, what is the price per bushel, if each barrel contains $2\frac{1}{2}$ bushels? If each barrel contains $\frac{1}{8}$ of $10\frac{18}{27}$ times $2\frac{3}{16}$ bushels?

18. Find the value of $\frac{10\frac{7}{8}}{9\frac{2}{3}}$.

19. $\frac{14\frac{2}{3}}{32}$, or $14\frac{2}{3} \div 32$.

20. $\frac{54}{6\frac{3}{10}}$, or $54 \div 6\frac{3}{10}$.

21. $\frac{24\frac{3}{8}}{10\frac{3}{4}}$, or $24\frac{3}{8} \div 10\frac{3}{4}$.

22. $\frac{10\frac{5}{9}}{\frac{19}{27}}$; $\frac{2\frac{3}{5}}{18\frac{2}{5}}$; $\frac{11\frac{4}{7}}{9\frac{1}{19}}$; $\frac{20\frac{5}{7}}{\frac{3}{10} \text{ of } 11\frac{3}{5}}$.

23. $\frac{14\frac{3}{8}}{57\frac{1}{2}}$; $\frac{3\frac{9}{10}}{8\frac{2}{3} \times 5}$; $\frac{\frac{6}{7} \text{ of } 19\frac{1}{4}}{7\frac{3}{5}}$.

24. $\frac{25\frac{3}{5}}{36\frac{4}{7}}$; $\frac{9\frac{1}{5}}{\frac{6}{7} \text{ of } 16\frac{4}{5}}$; $\frac{7\frac{5}{7} \times \frac{8}{9}}{\frac{9}{10} \text{ of } 28\frac{1}{8}}$.

25. Change to simple fractions $\frac{10\frac{2}{10}}{20\frac{3}{17}}$; $\frac{4\frac{2}{5} \text{ times } 8\frac{1}{4}}{\frac{4}{9} \text{ of } 90\frac{5}{9}}$.

Section VI.

RELATION OF NUMBERS.

To find a fractional part of a number.

Problems.

What is the result of

1. $\frac{3}{8}$ of 7? $8\frac{1}{8}$?
2. $\frac{5}{7}$ of 9? $9\frac{1}{10}$?
3. $\frac{6}{7}$ of 9 times $4\frac{2}{3}$?
4. $\frac{9}{10}$ of $16\frac{2}{3} \times 5\frac{5}{8}$?
5. $\frac{5}{6}$ of $\frac{8}{9}$ of 8 times $9\frac{3}{4}$?
6. $4\frac{2}{5}$ times $\frac{9}{11}$ of $7\frac{1}{3} \times 9$?

7. $2\frac{3}{11}$ is how many times $\frac{3}{8}$ of $9\frac{1}{7}$? $\frac{10}{11}$ of $9\frac{9}{10}$?

8. $\frac{2}{10}$ of $\$9\frac{1}{8}$ are what part of $\frac{7}{8}$ of 9 times $\$12\frac{4}{7}$?

9. If an acre of land is worth $\$75\frac{50}{100}$, what is the value of $\frac{3}{4}$ of an acre? Of $\frac{3}{10}$ of an acre? Of $\frac{2}{5}$?

10. If I buy a load of hay for $\$18\frac{7}{9}$, and sell it for $\frac{12}{13}$ as much, how much do I get for it? If for $\frac{15}{13}$?

11. What is the value of $\frac{7}{8}$ of a yard of silk, if 5 yards cost $\frac{3}{7}$ of $\$4\frac{2}{3}$? The value of $\frac{7}{12}$ of a yard? Of $\frac{25}{36}$ of a yard?

12. If a jockey buys a horse for $\$375\frac{3}{4}$, and sells it for $\frac{10}{9}$ of the cost, how much does he gain? If for $\frac{2}{3}$ of $\frac{9}{10}$ of the cost? If for $2\frac{1}{8}$ times $\frac{4}{5}$ of the cost? $\frac{8}{21}$ of $15\frac{3}{4}$ times the cost?

To find a number from its fractional parts.

Problems.

1. $55\frac{1}{2}$ is $\frac{7}{15}$ of what?
2. $62\frac{9}{80}$ is $\frac{17}{40}$ times what?
3. $105\frac{5}{9}$ is $\frac{39}{37}$ of what?
4. $\frac{3}{10}$ of $40\frac{5}{8}$ is $\frac{9}{73}$ of what?
5. $\frac{19}{27}$ of $43\frac{1}{5}$ is $\frac{75}{28}$ of what?
6. $3\frac{2}{11}$ times $50\frac{5}{6}$ are $\frac{11}{20}$ of what?
7. $\frac{8}{9}$ of 162 is $\frac{4}{5}$ of how many times 18? $21\frac{3}{5}$? $40\frac{7}{10}$?
8. $\frac{5}{9}$ of 225 is $\frac{4}{7}$ of $\frac{21}{40}$ of how many times $100\frac{2}{5}$?
9. A drover sold 72 sheep, which was $\frac{8}{15}$ of his flock. How many sheep had he at first? How many had he left?
10. A furniture-dealer sold 7 dozen arm-chairs for $\frac{8}{9}$ of \$224. What was the price of each? If he had sold $10\frac{1}{2}$ dozen?
11. $\frac{9}{13}$ of A's farm of 260 acres was equal to $\frac{9}{11}$ of B's farm. How many acres were in B's farm? B's was what part of A's?
12. A pair of horses cost \$750, which was $\frac{5}{9}$ of 10 times the cost of a carriage. What did the carriage cost?
13. If a man earns $\$18\frac{3}{4}$ a week, and spends $\frac{1}{2}$ of this amount, how long will it take him to save \$100?
14. The product of two numbers is $\frac{3}{4}$ of 14. and one of them is $\frac{1}{8}$ of 10. What is the other?

Review Problems.

1. $\frac{17}{20} - \overline{\frac{15}{38} \times \frac{54}{80} \div \frac{27}{76}}$.
2. $\overline{(\frac{7}{45} \div \frac{49}{30}) \times \frac{35}{48}} - \frac{1}{84}$.
3. $(\frac{14}{51} \div \frac{21}{34}) - \frac{1}{70} \times 8\frac{3}{4}$.
4. $(14\frac{2}{7} \times \frac{8}{25} - 31\frac{9}{21}) \div \frac{35}{38}$ of $4\frac{1}{14}$.
5. $(9\frac{9}{10} \div 2\frac{1}{16}) + \overline{10\frac{1}{17} \times \frac{11}{28}}$ of $2\frac{1}{19}$.
6. $\overline{10\frac{3}{4} - (\frac{8}{9} \div \frac{4}{9}$ of $10\frac{2}{7})} + 45$.

7. Find the sum of $8\frac{1}{6}$ times $\frac{3}{14}$ and $14\frac{4}{7} \div 9\frac{5}{7}$. The difference.

8. From $21\frac{3}{5} \div 25\frac{5}{7}$ take $\frac{1}{8}\frac{1}{5}$ of $12\frac{3}{4}$. Find their sum.

9. Multiply the sum of $26\frac{4}{2}\frac{}{1}$ and $34\frac{1}{1}\frac{1}{4}$ by $20\frac{5}{1}\frac{}{3} \div 18\frac{9}{2}\frac{}{8}$.

10. Find the quotient of $20\frac{8}{1}\frac{}{1}$ times $\frac{3}{2}\frac{}{6}$ divided by $60\frac{7}{3}\frac{}{4} - 50\frac{1}{5}\frac{1}{1}$.

11. What number divided by $\frac{8}{1}\frac{}{3}$ of $9\frac{1}{1}\frac{}{0}$ gives $55\frac{3}{1}\frac{}{4}$? Gives $207\frac{1}{2}\frac{9}{8}$?

12. Find what number multiplied by $100\frac{1}{3}\frac{}{0}$ less $40\frac{1}{2}$ gives $70\frac{1}{1}\frac{1}{5}$.

13. To what must you add the product of $105\frac{1}{9}$ multiplied by $80\frac{2}{1}\frac{}{1}$ that the sum may be $10000\frac{1}{1}\frac{}{0}\frac{}{0}$?

14. What is the smallest fraction which, added to the sum of $\frac{1}{1}\frac{0}{1}$ and $\frac{1}{6}$, will give an integer as the result?

15. The divisor is $10\frac{9}{1}\frac{}{1}$, the quotient is $72\frac{2}{7}$, and the remainder is $5\frac{1}{3}\frac{}{4}$. What is the dividend?

16. The product of three factors is $3004\frac{1}{2}\frac{3}{3}$, and two of them are $12\frac{3}{4}$ and $20\frac{1}{2}\frac{}{3}$. What is the third factor?

17. The product of two numbers increased by $110\frac{2}{1}\frac{}{5}$ is $120\frac{1}{3}\frac{9}{0}$, and one of them is 30. What is $\frac{1}{1}\frac{1}{2}$ of the other?

18. Find the sum and the difference of 30 and $\frac{1}{3}\frac{}{0}$. Divide the sum by the difference; the difference by the sum; and find the product of the two quotients.

19. If each term of $\frac{7}{9}$ is increased by 10, how much greater or less is the value of the fraction? If each is diminished by 10? Multiplied by 10? Divided by 10?

20. If $7\frac{9}{2}\frac{}{0}$ tons of hay are worth $\$140\frac{9}{1}\frac{}{0}$, how much are $22\frac{7}{2}\frac{}{0}$ tons worth? $111\frac{3}{4}$ tons? $130\frac{3}{8}$ tons? $7\frac{1}{8}$ times $\frac{1}{1}\frac{8}{9}$ of a ton?

21. A captain owned $\frac{7}{9}$ of a vessel, and sold $\frac{4}{7}$ of his share for $\$9050\frac{4}{5}$. What was the vessel worth?

22. What is the average weight of three quarters of beef weighing $135\frac{7}{8}$ pounds, $150\frac{1}{1}\frac{1}{6}$ pounds, and $110\frac{3}{4}$ pounds?

23. A man bought a horse, and paid $\$26\frac{2}{5}$ down, which was $\frac{6}{5}\frac{}{5}$ of the cost. What was the cost of the horse?

24. A horse and carriage cost \$600, and the horse cost $3\frac{1}{2}$ times as much as the carriage. Find the cost of the carriage.

25. Two men earned $\$88\frac{1}{8}$; one received $\$3\frac{7}{8}$ a day, and the other $\$2\frac{5}{1}\frac{}{0}$. How many days did they work?

26. In a piece of muslin there were $33\frac{2}{3}$ yards, and it cost $\$5\frac{1}{18}$. How must it be sold per yard to gain $\$\frac{101}{144}$?

27. A house and farm of $55\frac{7}{8}$ acres was sold for \$7500. Allowing \$4000 for the house, what was the price of the land per acre?

28. From $50\frac{7}{8}$ tons of iron were sold $20\frac{3}{4}$ tons at $\$43\frac{1}{2}$ a ton, and the remainder at $\$40\frac{2}{5}$. What was received for all?

29. A grocer bought 75 pounds of soap at 7 cents a pound. While on hand it dried away $\frac{1}{3}$ in weight. What did he gain or lose by selling it at $8\frac{2}{3}$ cents a pound?

30. A merchant one year lost $\frac{2}{7}$ of his capital, and the next year gained \$700. If he then had \$4200, what was his capital? What was his loss the first year?

31. If a cask whose capacity is 42 gallons is $\frac{5}{8}$ full, what part of it would be filled if $8\frac{3}{4}$ gallons were added?

32. Into a cistern whose capacity is $500\frac{9}{10}$ gallons there run $50\frac{3}{4}$ gallons per hour, and there run out in the same time $45\frac{7}{8}$ gallons. In what time will the cistern be filled?

33. A merchant put $\frac{1}{5}$ of his money in one bank, $\frac{2}{5}$ of it in another, $\frac{3}{10}$ in a third, and the remainder, \$7000, in his safe. How much money had he?

34. Change to its simplest form $\dfrac{\frac{2}{3}\text{ of }3\frac{2}{3}}{\frac{3}{5}\text{ of }6\frac{2}{3}+3\frac{2}{7}}\times\dfrac{1}{4\frac{2}{7}}\div\dfrac{9\frac{3}{4}}{8\frac{2}{3}}$.

35. Find the value of $\dfrac{4\frac{1}{3}+7\frac{1}{2}}{5\frac{2}{3}-4\frac{3}{5}}\div\dfrac{7\frac{1}{2}\times4\frac{1}{3}}{4\frac{3}{5}\div5\frac{2}{3}}$; of $\dfrac{\dfrac{1+\frac{1}{4}}{1-\frac{1}{4}}}{1\times\frac{1}{4}\div4}$.

Section VII.

DECIMAL FRACTIONS.

1. Express by figures, or notate, in decimal form: —

1. $\frac{3}{10}$; $\frac{4}{10}$; $\frac{6}{10}$.
2. $\frac{5}{100}$; $\frac{9}{100}$.
3. $\frac{15}{100}$; $\frac{89}{100}$.
4. $\frac{3}{1000}$; $\frac{8}{1000}$.
5. $\frac{834}{1000}$; $\frac{579}{1000}$.
6. $10\frac{8}{10}$; $97\frac{7}{100}$.
7. $100\frac{5}{100}$.
8. $50\frac{25}{100000}$.

9. $200\frac{2}{100}$; $\frac{202}{10000}$; $200\frac{2}{1000}$; $\frac{3008}{100000}$; $3000\frac{3}{10000}$; $3003\frac{3}{10000}$.

10. $\frac{40004}{1000000}$; $40000\frac{4}{100000}$; $50500\frac{505}{1000000}$; $5005000\frac{6006}{1000000}$.

11. 7 tenths; 7 hundredths; 7 thousandths; 7 ten-thousandths.

12. Eight hundred thousandths; eight hundred-thousandths.

13. 909 thousandths; 900 and 9 thousandths; 9000 and 9 millionths.

14. 1020 ten-millionths; 1020 and 10 ten-millionths; 5006 hundred-millionths; five thousand and six hundred-millionths.

15. Two hundred thirty-four thousand five hundred and nine millionths.

16. Three million fifty thousand and one hundred nine ten-millionths.

17. Four hundred three thousand and ten thousand seventy hundred-thousandths; five million fifty hundred-millionths.

2. Express in words, or numerate and read: —

1. .1; .7; .9.	3. .003; .035.	5. .0204; .0357.	7. .00005; .00607.
2. .02; .06; .08.	4. 9.1; 8.24.	6. 8.096; 70.05.	8. 20.003; 4.0005.

9. 300.03; 404; 400.004; .00004; .00404; 4.00404.

10. .00035; .00305; .03005; .30507; .000006; .006007.

11. .070089; .708090; .700009; .0070809; .0700809; .7008009.

12. .1090208; .0800207; .3006004; .05007003; .07000809.

13. 700.0007; .07007; 7000.07007; 8080.080008; 90009.90009.

14. .0005; 5005; 5005005; 50000.00005; .505005; 505000.000005.

15. 1001.101001; 2022.022022; 30303.303033; 404004.4040044.

16. 500.5005005; 6.0006006; 7.00700077; 8.08080088; .090090009.

Section VIII.

REDUCTION OF DECIMALS.

Problems.

Change to fractions in their lowest terms —

1. .8; .9; .25.	3. .625; .01875.	5. \$.75; \$.125.
2. .45; .75; .125.	4. .0875; .00625.	6. \$.008; \$.0075.

7. Reduce .00105 and .00008 to common fractions.

8. Change .00875 to a common fraction in its lowest terms.

Change to fractions in their lowest terms —

9. $.12\frac{1}{2}$; $.37\frac{1}{2}$.
10. $.16\frac{2}{3}$; $.33\frac{1}{3}$.
11. $.31\frac{1}{4}$; $.43\frac{3}{4}$.
12. $.56\frac{1}{4}$; $\$.06\frac{1}{4}$.
13. $.62\frac{1}{2}$; $\$.18\frac{3}{4}$.
14. $.66\frac{2}{3}$; $\$.41\frac{2}{3}$.
15. $.222\frac{2}{9}$; $\$.312\frac{1}{2}$.
16. $.007\frac{7}{9}$; $\$.833\frac{1}{3}$.
17. $.000\frac{1}{11}$; $\$.007\frac{3}{4}$.

Express as mixed numbers —

18. 5.755; 7.04; 9.95.
19. $8.7\frac{3}{4}$; $10.06\frac{1}{4}$; $12.562\frac{1}{2}$.
20. $9.09\frac{3}{8}$; $25.08\frac{1}{3}$; $50.000\frac{9}{11}$.
21. \$4.25; \$66.37; \$8.50.
22. $\$395.37\frac{1}{2}$; $\$724.087\frac{3}{4}$.
23. $\$246.007\frac{3}{4}$; $\$678.000\frac{4}{5}$.

Change to equivalent decimals —

1. $\frac{3}{5}$; $\frac{7}{8}$; $\frac{9}{16}$.
2. $\frac{13}{20}$; $\frac{19}{25}$; $\frac{27}{50}$.
3. $\frac{31}{32}$; $\frac{3}{5}$ of $\frac{9}{16}$.
4. $\frac{41}{48}$; $\frac{7}{80}$ of $\frac{4}{5}$.
5. $\frac{5}{8}$; $\frac{11}{16}$; $\frac{5}{20}$.
6. $\frac{95}{100}$; $\frac{7}{100}$ of $10\frac{4}{5}$.
7. Reduce $\frac{7}{9}$, $\frac{8}{9}$, and $\frac{9}{11}$ to decimals of four orders each.
8. Change to complex decimals of five places $\frac{5}{27}$ and $\frac{9}{58}$.
9. Express as decimals of five orders $\frac{41}{85}$, $\frac{37}{99}$, $\frac{113}{174}$, $\frac{105}{211}$, $\frac{403}{9009}$.

Change to the form of decimals —

10. $12\frac{7}{8}$; $25\frac{7}{10}$.
11. $8.2\frac{1}{2}$; $9.6\frac{2}{5}$.
12. $\$.7\frac{4}{5}$; $\$.00\frac{5}{8}$.
13. $.225\frac{9}{16}$; $.87\frac{3}{20}$.
14. $18\frac{24}{25}$; $20.08\frac{3}{50}$.
15. $\$25\frac{9}{20}$; $\$9.06\frac{1}{4}$.
16. $9\frac{25}{100} \times \frac{7}{37}$.
17. $\dfrac{16\frac{2}{3}}{8} \times \dfrac{18\frac{3}{4}}{2\frac{5}{10}}$.

18. $\frac{3}{7}$ of $10\frac{1}{2}$; $4\frac{1}{5}$ times $\$4\frac{1}{5}$; $\$8\frac{5}{8} \div 6\frac{4}{7}$; $\$8 + \$\frac{9}{32} - \$\frac{15}{16}$.
19. Find the decimal value of $\overline{(3\frac{4}{5} + 10\frac{5}{8}) - 12\frac{11}{20}} \times 15\frac{1}{15}$.

Section IX.

ADDITION OF DECIMALS.

Problems.

Find the sum of —

1. .7 + .85 + .0975 + .246 + .00075 + .78.

2. 324.5 + 900.102 + .0075 + 7.005 + .2317 + 100.001.

3. \$37.25 + \$90.625 + \$.125 + \$187.5 + \$.50 + \$5000.

4. $7\frac{3}{4}$, $25040.7\frac{1}{2}$, $2.06\frac{4}{5}$, 18, $.062\frac{3}{4}$, $.066\frac{7}{10}$, and $400.006\frac{1}{4}\frac{1}{0}$.

5. $\$10\frac{1}{16}$, $\$1.04\frac{1}{8}$, $\$50\frac{7}{10}$, $\$4.007\frac{3}{4}$, \$500, $\$.00\frac{3}{100}$, and $\$1000\frac{7}{32}$.

6. $24\frac{3}{8}$ rods, $8.4\frac{2}{5}$ rods, $.007\frac{1}{2}$ rods, $5.07\frac{1}{10}$ rods, $.0005\frac{7}{8}$ rods.

7. Find the sum of 25 ones, 9 tenths, 15 thousandths, 7 ten-millionths, and 4005 hundred-thousandths.

8. Add five, three and three tenths, five hundred and five hundredths, four hundred four hundred-thousandths, and six millionths.

9. Find the sum of $\frac{4}{5}$, $8\frac{3}{7}$, $\frac{9}{11}$, $\frac{8}{15}$, $4\frac{2}{3}$ to five decimal places.

10. How many cords of wood are in four piles containing 217.125 cords, $10\frac{1}{2}\frac{1}{0}$ cords, 400.0875 cords, and 175.75 cords?

11. I paid \$5.25 for a hat, \$7.50 for a pair of boots, $\$\frac{3}{4}$ for a cane, and \$37.75 for a suit of clothes. What did they all cost?

12. How many acres in four farms containing respectively 40.7 acres, $52\frac{7}{160}$ acres, $60\frac{7}{8}$ acres, and $75.33\frac{1}{3}$ acres?

13. A carter bought a horse for $\$75.62\frac{1}{2}$, and sold him at a gain of $\$25\frac{55}{100}$. What was the selling price?

14. A grocer sold a barrel of flour for $\$7\frac{7}{8}$, a chest of tea for \$37.25, a ham for $\$2\frac{5}{10}$, a bushel of potatoes for $\$\frac{9}{10}$, and a box of soap for $\$1.16\frac{2}{3}$. What did he get for all?

15. A farm cost \$2540.50, the stock $\$835\frac{3}{4}$, the farming implements $\$176\frac{9}{10}$, and the seed $\$40\frac{35}{100}$. Find the entire cost.

Section X.

SUBTRACTION OF DECIMALS.

Problems.

Find the result of—

1. 3000.2 — .0875; 3.003 — 2.9003; 7 — .9075.

2. 20.0808 — 2.00808; 7 — .001002; 900000 — .00009.

3. \$7.25 — \$.90; \$20 — \$15.75; \$500 — \$.0625.

4. 60.8 tons — 49.125 tons; 640 acres — 40.0075 acres.

5. $8\frac{5}{8}$ — $.37\frac{1}{2}$; $40.7\frac{3}{10}$ — $.00\frac{4}{5}$; $500.05\frac{1}{8}$ — $40.8\frac{9}{20}$.

6. $\$9\frac{7}{10}$ — $\$5.12\frac{1}{2}$; \$25.00$\frac{1}{4}$ — $\$7\frac{7}{8}$; \$91.75 — $\$7\frac{7}{8}$.

7. $10\frac{7}{12}$ years — 8.75 years; $100.3\frac{3}{4}$ pounds — $10\frac{9}{16}$ pounds.

8. From 1 tenth subtract 10001 ten-millionths. From 5 and $7\frac{3}{5}$ hundredths take 1 and $3\frac{1}{5}$ thousandths.

9. From five hundred seventy-five thousandths take five hundred seventy-five millionths.

10. From a farm of 160 acres there were sold three fields containing in all 40.0625 acres. How many acres were left?

11. The minuend is $18\frac{7}{12}$, and the remainder $15.2\frac{3}{5}$. What is the subtrahend? Take the subtrahend from the minuend.

12. The sum of two numbers is $\$300\frac{9}{10}$, and one of them is $\$25.18\frac{3}{4}$. What is the other? Find the difference between the two.

Find the value of—

13. $500 + .005 - .000005$.

14. $60.06 - (.0007 + 7.007)$.

15. $(8\frac{3}{16} - 1.4\frac{4}{5}) + .001\frac{7}{8}$.

16. $(3\frac{7}{8} - 1.8) - (.9 + \frac{7}{25})$.

17. $.87\frac{1}{2} - \frac{1}{8} + .75 + .93\frac{3}{4}$.

18. $\$500 - (\$250\frac{17}{100} - \$50.06\frac{1}{4})$

19. $(400 - .004) - (.006 + .09)$; $5000 + .005 - .0005$.

20. From the sum of 400 and .0002 take 300.00003.

21. From the sum of 3 thousand and 3 thousandths take their difference. To their difference add their sum.

22. If a shoemaker makes shoes worth $\$30.62\frac{1}{2}$ from leather worth \$20.50, and other material worth $\$3.12\frac{1}{2}$, what is he paid for his labor?

23. Divide the sum of $3.5 + \frac{3}{4}$ by the sum of $8.125 + \frac{5}{8} + .250$.

24. What will $16\frac{2}{3}$ bales of cotton cost, each bale weighing 510.75 lbs., at $8\frac{3}{4}$ cents per lb.?

Section XI.

MULTIPLICATION OF DECIMALS.

Problems.

Find the result of —

1. 15 times 9.5; 28 times 8.64; 124 times 8.075.

2. .5 of 8.125; .06 of 50.0075; .008 of 100.008.

3. \$93.025 × 18; 1.0625 × 7.5; 80.75 × .5; 94 × 8.21.

4. \$9.375 × 100; \$10.0625 × 10.01; \$50.005 × .00015.

5. $7.6\frac{3}{5} \times .95$; $20.07 \times .8\frac{2}{3}$; $40.075 \times .08\frac{1}{8}$; $500\frac{1}{5} \times .007\frac{3}{4}$.

6. $8\frac{3}{4} \times .07\frac{1}{2} \times 100$; $8.075 \times \frac{3}{4}$ of $.16 \times 10 \times 3\frac{1}{5}$.

7. $\$22\frac{1}{2} \times 8\frac{4}{5} \times 20$; $\$.87\frac{1}{2} \times \frac{9}{10} \times .001$; $\$\frac{3}{8} \times \frac{8}{9}$ of $.8\frac{3}{9}$.

8. 800.75 cords × $13\frac{3}{4}$; $1000\frac{7}{10}$ bales × $100.6\frac{2}{3} \times .01$.

9. Multiply .000075 by 1000; by .001; by 500000; by .01.

10. Multiply 5 hundred-thousandths by one hundred thousand; one hundred and $5\frac{3}{16}$ tenths by 10 and $4\frac{1}{2}$ hundredths.

11. How many are forty and four thousandths multiplied by four tenths? By four hundredths? Four ten-thousandths?

12. If one dollar in gold is worth $\$1.14\frac{5}{8}$ in paper-money, how much are \$1000 in gold worth?

13. What must I pay, at $\$1\frac{3}{4}$ a foot, to fence my lot, which is 3 rods by 8 rods, each rod being 16.5 feet?

14. Find the value of 12 dozen pocket-knives, each $\$.87\frac{1}{2}$.

15. Find the cost of 50.75 acres of land at $\$112.12\frac{1}{2}$ an acre.

Find the value of —

16. $7.45 + \overline{10.01 \times .008}$.

17. $\overline{25 - 2.08\frac{1}{4}} \times 7.5\frac{7}{10}$.

18. $\$3.06\frac{1}{4} \times 10.01 + \$.003\frac{1}{3}$.

19. $\$50.7\frac{3}{8} \times .06\frac{1}{5} + \$10 \times 7.04\frac{3}{4}$.

20. $\overline{500 + .005} - \frac{3}{8} + (.3\frac{1}{2} - .07\frac{3}{4} \times \frac{2}{9}$ of $9\frac{3}{8})$.

21. What is the result of $200 \times .2 - 3000$ times .0002?

22. From the product of 25 thousandths multiplied by 30 take 5 hundred-thousandths; and to the remainder add 50000.

23. From two tenths take two millionths, and multiply the remainder by one hundred thousand.

24. Multiply 325 ten-millionths by fifty thousand; and to the product add five hundred-thousandths.

25. A butcher bought a cow weighing 450 pounds at $\$.08\frac{3}{4}$ a pound. He sold the meat, tallow, and hide for \$45. What did he gain?

Section XII.

DIVISION OF DECIMALS.

Problems.

Find the quotient of —

1. .112 ÷ 700; .006 ÷ 75; .075 ÷ 150; .0036 ÷ 600.

2. .075 ÷ 2.5; .0625 ÷ .75; .01875 ÷ .375; 7.2 ÷ .018.

3. 72 ÷ .024; 7.2 ÷ .0024; 720 ÷ .00024; .72 ÷ 24000.

4. \$40.25 ÷ 100; \$2.0025 ÷ 20.05; \$400.0625 ÷ .00125.

5. 100 ÷ .001; .002 by 2000; 3000 ÷ .03; 4 by 40000.

6. $8.3\frac{4}{5}$ ÷ 72; 20 ÷ $.04\frac{3}{8}$; $.01\frac{7}{20}$ ÷ $\frac{7}{8}$; $500\frac{1}{2}$ ÷ $.0025\frac{1}{20}$.

7. \$500 ÷ \$.04; $\$37\frac{1}{4}$ ÷ 750; $\$24\frac{1}{8}$ ÷ $\$\frac{2}{5}$; \$84 ÷ .007.

8. $800\frac{3}{10}$ tons ÷ 4.5; $7\frac{3}{8}$ cords ÷ $.00\frac{1}{2}$; $125\frac{3}{4}$ bales ÷ $\frac{7}{10}$.

9. Divide five hundred and five hundredths by five hundred; two hundredths by two hundred and two hundredths.

10. Divide 400 and 5 ten-thousandths by 24 hundredths; and 5 hundred-thousandths by five hundred thousand.

11. If the dividend is .0234, and the divisor is 7800, what is the quotient? What is .00005 of the quotient?

12. 10 acres of land were divided into village lots containing .625 of an acre each. How many lots were there?

13. If a teamster hauls 1.375 cords at a load, in how many loads can he haul 34.375 cords of wood?

14. At \$5.75 per barrel, how many barrels of flour can be bought for \$1035? How many barrels for $\$1552\frac{50}{100}$?

Find the value of —

15. $50 + (.2 \times .3\frac{4}{5}) - .0005$.

16. $\overline{8\frac{7}{8} \div .016} \times \overline{6\frac{3}{20} + .07\frac{1}{2}}$.

17. $7.15\frac{3}{4} \times 500 \div .8\frac{4}{5} \times 50$.

18. $.00\frac{7}{8} \times 80 \div \frac{3}{8}$ of $.007\frac{3}{16}$.

19. $\$.005 \times 8.4 + \$3\frac{3}{8} \div .03\frac{3}{8}$.

20. $\$.27 + \$.7\frac{9}{10} \times .05$ of $8\frac{5}{8}$.

21. $\overline{7 \div .007} + (.007 \times 73)$; $8000 + (.0008 \div 8) \times 800$.

22. From 8 thousand take 8 thousandths, divide the remainder by 5 millionths, and multiply the quotient by 50000.

23. If $7\frac{3}{4}$ tons of coal cost \$43.40, what will 9.8 tons cost?

24. How much hay can be bought for \$21.875, if $\frac{1}{8}$ of a hundred pounds cost $\$.87\frac{1}{2}$? For $\$32.81\frac{1}{4}$? For $\$16.40\frac{5}{8}$?

25. If 150 bushels of apples cost $\$168\frac{3}{4}$, how many barrels, each containing $2\frac{1}{2}$ bushels, can be bought for \$297?

26. A miller gave 9.75 barrels of flour, worth $\$6\frac{4}{5}$ a barrel, for $10\frac{3}{8}$ cords of wood. What was the wood worth a cord?

Problems by Analysis. 1.

1. What is the cost of 525 bushels of oats at 25 cents?

2. At $12\frac{1}{2}$ cents each, what will 850 melons cost?

3. What would 750 penknives cost at $18\frac{3}{4}$ cents each? At 25 cents each? At $\$.33\frac{1}{3}$? At $\$.37\frac{1}{2}$? At 50 cents?

4. At $33\frac{1}{3}$ cents a pound, how much will 175 pounds of coffee cost? 8 sacks, each containing 50 pounds?

5. What is the cost of 25 yards of cassimere at $\$1.12\frac{1}{2}$ a yard? Of 12 pieces, each containing 25 yards?

6. At $\$1.06\frac{1}{4}$ per basket, what will 80 baskets of peaches cost? 400 baskets? 800 baskets? 1000 baskets?

7. How much are 200 bushels of wheat worth at $\$1.87\frac{1}{2}$ a bushel? 18 acres yielding 30 bushels per acre?

Problems. 2.

1. How many quarts of berries at $6\frac{1}{4}$ cents a quart can be bought for \$4.80? At $8\frac{1}{3}$ cents? At $12\frac{1}{2}$ cents? At $16\frac{2}{3}$ cents?

2. At $\$.16\frac{2}{3}$ a yard, how many yards of muslin can be bought for \$3? For \$4.20? For \$5? For \$6.75?

3. If a fish-woman received \$4.50 for shad at \$.25 each, how many shad did she sell? If she received \$6.50?

4. How many pounds are in a quarter of beef for which a butcher receives \$30, at $18\frac{3}{4}$ cents a pound?

5. How many pounds of butter can be bought for \$60, at 20 cents a pound? At 25 cents? At $\$.33\frac{1}{3}$? At \$.50?

6. At \$1.25 each, how many caps can be bought for \$10? For \$25? For \$75? For \$125? For \$100?

7. How many silk handkerchiefs can be bought for \$11.25, at $\$1.12\frac{1}{2}$ each? At \$1.25? At $\$1.56\frac{1}{4}$? At $\$2.08\frac{1}{3}$?

Problems. 3.

1. Find the cost of 875 pounds of beef at $\$16\frac{2}{3}$ a hundred.

2. What will 970 lath cost at \$.95 per C?

3. What is the cost of 25250 bricks at \$8.75 per M?

4. At $\$14\frac{1}{2}$ a hundred, what is the cost of 436 shad?

5. What must be paid for 2736 pounds of sugar at $\$9\frac{1}{2}$ a hundred? For 1324 pounds? For 3142 pounds?

6. At \$3.25 a thousand, what will 15475 envelopes cost?

7. If 100 cedar posts are worth \$22.50, what is the value of 869 posts? Of 1025 posts? Of 1750? Of 2345?

8. If it take 1250 shingles for the roof of my house, how much will they cost at \$4.75 per M?

9. How many hundred feet of boards at \$3.25 per hundred can be bought for $\$15.43\frac{3}{4}$? For $\$46\frac{5}{16}$? For $\$61\frac{3}{4}$?

10. At \$9.50 per M. a builder paid $\$127.06\frac{1}{4}$ for bricks. How many thousand bricks did he buy? How many bricks?

11. If 7225 pounds of rice cost \$60.69, what is the cost per thousand? The cost of 2250 pounds?

12. If 1187 pounds of merchandise can be carried for \$77.155, what is the freight per hundred? Per thousand?

Problems. 4.

1. What is the value of 2840 pounds of hay at \$22.50 a ton? At \$1.25 a hundred? At $\$1.12\frac{1}{2}$ a hundred?

2. At \$4.25 a ton, what will 3425 pounds of plaster cost? 4628 pounds? 7236 pounds? 5675 pounds?

3. What is the freight on 11426 pounds of iron at \$1.64 a ton? On 17456 pounds at \$2.12½ a ton?

4. Find the value of 1725 pounds of coal at \$7½ a ton.

5. What must be paid for 14 loads of coal, each 2240 pounds, at \$6.16⅔ a ton? For 18 loads at \$7.37½ per ton?

6. What is the cost of 75 sacks of bone-dust, each containing 137½ pounds, at \$31¼ per ton? At \$33⅓ per ton?

7. What is the value of 21540 pounds of egg-coal at \$6¼ a ton, and 12250 pounds of stove-coal at \$6½ a ton?

8. If 27000 pounds of plaster cost \$111.75, what is the price per ton? The price per hundred, and the cost of 750 pounds?

9. I paid \$6.90 for a load of coal which was found to weigh but 1725 pounds. What price per ton did I pay?

10. A man paid \$58.12½ for a car-load of plaster at \$7½ a ton. How many pounds should there have been?

Section XIII.

BILLS AND ACCOUNTS.

1. Henry Brown bought of Eli K. Brous, Cincinnati, March 5, 1875, 25 pair misses' kid boots, @ \$2.12½; 2 cases men's calf boots, @ \$65.25; 10 pair women's gaiters, @ \$1.37½; 10 gro. buttons, @ \$.87½. Find the footing.

2. Mrs. N. Hilles bought of Gilmore & Bro., Lancaster, Penna., June 10, 1901, 10 lb. sugar, @ 12½ c.; 12½ lb. dried beef, @ 18¾ c.; 5 gal. N. O. molasses, @ 62½ c.; 8¼ lb. mackerel, @ 8½ c.; 1 box starch, 20 lb., @ 11¼ c. Make out and receipt the bill.

3. Trenton, N. J., Oct. 7, 1901, James Sheddon to Richard Carpenter, debtor, for repairing house as per contract, \$25; 500

ft. pine boards, @ \$4.25 per C; 6 days' work, @ \$3.50; nails and sundries, \$2.17. Make out and receipt the bill per clerk.

4. Williamsport, Penna., Felton & Bro. sold to William Kees & Son, Sept. 10, 1901, 15000 ft. pine boards, @ \$22.50 per M; 7240 ft. plank, @ \$11.75; 5450 ft. lath, @ \$.87½ per C; 6750 ft. scantling, @ \$1.62½ per M. Receipt the bill.

5. Nov. 3, 1901, James Paton bought of Henry Dewees, Wilmington, Del., 4 bbl. flour, @ \$7.50; 70 bu. wheat, @ \$2.12½; 60 bu. oats, @ \$.33⅓; 25 bu. corn, @ \$.87½.

November 6, 1901, he paid \$50; and on Dec. 10, 1901, he rendered a bill for 5 days' work, @ \$2.25 a day.

Put in the form of an account, and find the balance due Henry Dewees.

Review Problems.

Find the value of—

1. $(.04 \times 12 + .6) \div 900.$
2. $60 - \overline{.72 \div .6 + .024}.$
3. $\overline{(70 - .07) \div .63} \times .05.$
4. $\overline{200 + .5} \times \overline{.12 \div 600}.$
5. $(500 \div \overline{.005 \times .4}) + .0025.$
6. $2.16 \div \overline{(360 + 180) \times .009}.$
7. $\overline{.007 \div 7000} + (600 \times .006).$
8. $(256 \div \overline{.32 \times .4}) + \overline{.6 \div 120}.$

9. Find the sum of $.125 \times .75$ and $.0125 \times 75000$.

10. From $6000 - \overline{5000 - .005}$ take 375 times .625.

11. Multiply $4000 \div .0005$ by $\overline{.0005 \times 4000} \div 8000$.

12. Divide $.875 \times 900$ into $(.75 + 300 - .125) \times 1000$.

13. To 75 thousand add 75 thousandths, and take the sum from the difference between 25 million and 25 millionths.

14. From the sum of 300 thousand and 300 thousandths take their difference. To their sum add their difference.

15. From the product of 25 millionths multiplied by 5 hundred thousand, take 25 hundred-thousandths; and add 50000.

16. Divide 8 hundred thousand by 8 hundred-thousandths, and multiply the quotient by 125 times 4 millionths.

17. To the quotient of 27 ten-thousandths divided by 45 thousand, add their product plus their sum.

18. Divide 625 millionths by 625 thousand, multiply the quotient by 625 hundred-thousandths, and take the product from .625.

19. Express $\frac{5}{14}$ as a decimal having four decimal places.

20. Express .00625 as a common fraction in its lowest terms.

21. Change $\frac{18}{1000}$ and $\$112\frac{7}{8}$ to the decimal form.

22. Express, decimally, the remainder of $3\frac{1}{4}$ times $\$\frac{3}{8}$ less $\frac{1}{12}$ of $\$6.31\frac{1}{4}$. Express their sum as a common fraction.

23. From the sum of $\frac{5}{8}$, 10, $\frac{4}{5}$, $13\frac{1}{2}$, and .375, take the sum of $1\frac{3}{4}$, $\frac{7}{8}$, 12, and $.2\frac{1}{4}$. Find the sum of the two amounts.

24. Change $\left(\frac{1\frac{2}{3}}{3\frac{3}{4}} \div \frac{4\frac{2}{5}}{5\frac{3}{8}}\right) \times 3\frac{3}{5}$ times $\frac{3}{8}$ to a decimal.

25. How much less than a thousand is the sum of 225.75, $236\frac{7}{8}$, $175.2\frac{1}{2}$, and 244.625? How much more than 500.05?

26. Find the value of $\left(\frac{16\frac{2}{3}}{8} - \frac{3\frac{3}{4}}{8\frac{7}{16}}\right) \div \frac{3}{5}$ of 8.25, decimally.

27. How many rods of fence will enclose a field whose sides are $44\frac{3}{8}$ rods, 95.75 rods, $46\frac{9}{10}$ rods, and $87.2\frac{3}{4}$ rods?

28. A man bought a pair of pants for $\$9.62\frac{1}{2}$, a vest for $\$4\frac{7}{8}$, and a coat for $\$24\frac{7}{10}$, and gave in payment a twenty-dollar and two ten-dollar bills. What change did he receive?

29. A coal-dealer received twenty car-loads of coal, each 5 tons, and sold at different times $20\frac{1}{4}$ tons, 32.25 tons, and 43.375 tons. How much had he left?

30. 75 acres of land were bought at $\$137\frac{1}{2}$ an acre, and sold for \$9968.75. What was the loss per acre?

31. If 32 sacks of guano, each $127\frac{1}{2}$ pounds, cost $\$133\frac{1}{20}$, what is the price per ton? The cost of 10 sacks, each $112\frac{1}{2}$ pounds?

32. A merchant sold 3 pieces of muslin, each $33\frac{1}{3}$ yards, at $18\frac{3}{4}$ c. a yard, and took in payment coal at \$6.25 a ton. How many tons did he receive?

33. A grocer bought 3 barrels of eggs, each containing 108 dozen, for \$72.90. What was the price per dozen?

34. A grocer bought a sack of Rio coffee, 112 pounds, for $\$25\frac{1}{5}$, and a caddy of tea, 22 pounds, for \$20.90. He sold the

coffee at $\$.33\frac{1}{3}$ per pound, and the tea at $\$1.12\frac{1}{2}$. How much did he gain?

35. A butcher drew \$205.75 from bank, and expended all but \$25 in the purchase of 5 cows. Find the average price.

36. A miller sold 3 barrels of flour @ $\$7\frac{1}{8}$ a barrel, and took in payment 25 pounds of coffee at $\$.37\frac{1}{2}$ a pound, and the balance in tea at $\$\frac{9}{10}$. How much tea did he receive?

CHAPTER IV.

Section I.

MEASURES.

1. A ***Measure*** is a fixed unit used in comparing or determining quantity.

Thus, *1 foot* is a *measure of length;* 1 pound is a measure of weight; 1 dollar is a measure of value.

2. ***Quantity,*** in general, is that which can be increased, decreased, or measured.

Thus, *distance, weight, time, money, numbers* are *quantities.*

Measures may be divided into *six classes:* —

1. *Measures of Extension.*
2. *Measures of Capacity.*
3. *Measures of Weight.*
4. *Measures of Money.*
5. *Measures of Circles.*
6. *Measures of Time.*

MEASURES OF EXTENSION.

3. ***Extension*** is that which has one or more of the dimensions, *length, breadth,* and *thickness.*

Extension may be a *line,* a *surface,* or a *solid.*

A ***Line*** is that which has *length* only.

A ***Surface*** is that which has *length* and *breadth.*

A ***Solid,*** or ***Body,*** is that which has *length, breadth,* and *thickness.*

4. The ***Standard Unit*** of measures of extension is the *yard.*

5. The measures of extension are of three kinds: ***measures of lines, measures of surfaces,*** and ***measures of solids.***

I. Linear Measures.

A *line* has but one dimension, — *length.*

6. ***Linear Measures*** are used in measuring lines and distances.

The units or denominations of linear measures are *inch, foot, yard, rod,* and *mile.*

Table.

12 inches (in.)	are	*1 foot, ft.*	1 *inch.*
3 feet	"	*1 yard, yd.*	
5½ yards, or *16½ feet*	"	*1 rod, rd.*	
320 rods	"	*1 mile, mi.*	

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

In measuring *cloth,* or goods sold by the yard, the linear yard is divided into *halves, quarters, eighths,* and *sixteenths.*

Note 1.—Surveyors' Linear Measures are used in measuring *roads, boundaries of land,* etc. The unit of measure is the Gunter's chain, which is composed of 100 links, each 7.92 inches long.

Table.

7.92 in.	are	*1 link, l.*	*100 l., or 4 rds.,*	are	*1 chain, ch.*
25 l.		*1 rod, rd.*	*80 ch.*	"	*1 mile, mi.*

2. ***Civil Engineers*** generally use a chain or measuring tape 100 feet long, each foot being divided into *tenths* and *hundredths.*

3. ***Mariners' Measures*** are used to measure distances at sea: —

6 feet	are	*1 fathom, in measuring depths at sea.*
120 fathoms	"	*1 cable's length.*
1.16 common mi.	"	*1 nautical mile.*
3.48 common mi.	"	*1 nautical league.*

In measuring the length of the foot, 3 barleycorns, or sizes, are *1 inch.*

In measuring the height of horses at the shoulder, 4 inches are *1 hand.*

II. Surface Measures.

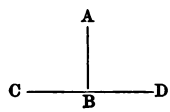
A *surface* has two dimensions, — *length* and *breadth*.

7. A ***Right Line,*** or ***Straight Line,*** is the shortest distance between two points.

Thus, the line A B is a *right* or *straight* line. A————B.

8. An ***Angle*** is the opening between two right lines that meet at the same point.

Thus, the opening between the two lines, A B and B C, is the *angle* A B C.



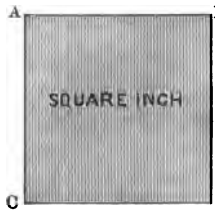
9. A ***Perpendicular Line*** is a right line that meets another so as to lean no more towards one side than towards the other.

Thus, the right line A B is a *perpendicular* line.

10. A ***Right Angle*** is an angle formed by two right lines that are perpendicular to each other.

Thus, the angle A B C, or A B D, is a *right angle.*

11. A ***Square*** is a figure or surface which has four equal straight sides and four right angles.



Thus, the *surface* A B C D is a *square.*

A ***Square Inch*** is a surface 1 inch long and 1 inch wide.

A ***Square Foot*** is a square 1 foot long and 1 foot wide.

12. ***Surface Measures,*** or ***Square Measures,*** are used in ascertaining the extent of surfaces; as of *boards, floors, land,* etc.

The units or denominations of surface or square measures are *square inch, square foot, square yard, square rod, acre,* and *square mile.*

Table.

144	*square inches (sq. in.)*	are	*1 square foot, sq. ft.*
9	*square feet*	"	*1 square yard, sq. yd.*
30¼	*square yards*	"	*1 square rod, sq. rd.*
160	*square rods*	"	*1 acre, A.*
640	*acres*	"	*1 square mile, sq. mi.*

1 A. = 160 sq. rd. = 4840 sq. yd. = 43560 sq. ft.

Note 1.* — *Surveyors' Square Measures are used in measuring or computing the area of land.

Table.

625 square links	are	*1 sq. rd.*	*10 square chains*	are	*1 acre.*
16 square rods	"	*1 sq. ch.*	*640 acres*	"	*1 sq. mi*

A *Government Section* of land is 1 square mile, or 640 acres; a *quarter-section* is 160 acres.

2. The measures of land, *perch* (30¼ sq. yd.) and *rood* (40 sq. rd.), are seldom used.

III. Solid Measures.

A *solid* has three dimensions, — *length*, *breadth*, and *thickness*.

13. A ***Cube*** is a solid having six equal square sides, called *faces*. The sides of the squares are the *edges* of the cube.

A ***Cubic Inch*** is a solid 1 inch long, 1 inch wide, and 1 inch thick.

A ***Cubic Foot*** is a solid 1 foot long, 1 foot wide, and 1 foot thick.

14. ***Solid Measures,*** or ***Cubic Measures,*** are used in ascertaining the contents or volumes of solids; such as *boxes*, *timber*, *bins*, etc.

Table.

1728 cubic inches (cu. in.)	are	*1 cubic foot, cu. ft.*
27 cubic feet	"	*1 cubic yard, cu. yd.*

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

Note 1. — In measuring *wood*, the cord foot (*cd. ft.*) and the cord (*cd.*) are used.

Table.

16 cu. ft.	are	*1 cd. ft.*	*8 cd. ft.*, or *128 cu. ft.*,	are	*1 cd.*

2. Wood cut for sale is usually 4 feet long, and is put in piles 4 feet high. 1 foot of the length of such a pile is 1 cord foot; and 8 feet of the length is 1 cord, containing 128 cubic feet.

A *Perch* of stone is 16½ ft. long, 1 ft. high, and 1½ ft. wide, and contains 24¾ cu. ft.

A *Ton*, in estimating the tonnage of vessels, is 100 cubic feet of their interior space.

MEASURES OF CAPACITY.

15. *Capacity* means extent of room or space.

Thus, the *capacity* of a box is the *extent of space* within the six surfaces of the box; the capacity of a barrel is the extent of space within the bounding inner surfaces of the barrel.

The measures of capacity are of two kinds: *measures of liquids* and *measures of dry substances.*

Note. — The measures of capacity are all cubic measures; each unit of capacity representing a certain number of cubic inches.

I. Liquid Measures.

16. *Liquid Measures* are used in measuring liquids; such as *water, milk, oil, molasses,* etc.

The units or denominations of liquid measures are *gill, pint, quart,* and *gallon.*

17. The ***Standard Unit*** of liquid measures is the *gallon.*

Table.

4 gills (gi.)	are	*1 pint, pt.*
2 pints	"	*1 quart, qt.*
4 quarts	"	*1 gallon, gal.*

1 gal. = 4 qt. = 8 pt. = 32 gi.

Note. — In estimating the capacity of cisterns, etc., —

31½ gal. are *1 barrel, bbl.* | *63 gal.* are *1 hogshead, hhd.*

The gallon, liquid measure, contains *231 cubic inches.*

18. *Apothecaries' Fluid Measures* are used by physicians in prescribing, and by apothecaries in mixing liquid medicines.

Table.

60 minims, or drops (m.),	are	*1 fluidrachm,* $f\mathfrak{Z}$.
8 fluidrachms	"	*1 fluidounce,* $f\mathfrak{Z}$.
16 fluidounces	"	*1 pint, O.*
8 pints	"	*1 gallon, Cong.*

Cong. 1 = O. 8 = $f\mathfrak{Z}$ *128 =* $f\mathfrak{Z}$ *1024 = m. 61440.*

Note 1. — *Cong.* is an abbreviation of the Latin *congius, a gallon; O.* of *octarius, one-eighth,* — a pint being one-eighth of a gallon.

2. In writing prescriptions, physicians place the ***number*** **of minims,** fluidrachms, etc., *after* the symbol.

Thus, O. 1 f℥ 2 f℥ 4 is 1 pint 2 fluidounces 4 fluidrachms.

3. For ordinary purposes, the following measures are used: 4 teaspoonfuls are 1 table-spoonful; 4 table-spoonfuls are 1 wineglass; 2 wineglasses are 1 teacupful; 4 teacupfuls are 1 pint.

II. Dry Measures.

19. Dry Measures are used in measuring dry substances; such as *grain, fruit, salt, lime,* etc.

The units or denominations of dry measures are *pint, quart, peck,* and *bushel.*

20. The ***Standard Unit*** of dry measures is the *bushel.*

Table.

2 pints (pt.)	are	*1 quart, qt.*
8 quarts	"	*1 peck, pk.*
4 pecks	"	*1 bushel, bu.*

$$1 \text{ bu.} = 4 \text{ pk.} = 32 \text{ qt.} = 64 \text{ pt.}$$

Note 1. — 1 half-peck, or 1 gallon, or 4 quarts, dry measure, contain $268\frac{4}{5}$ *cubic inches;* and 1 bushel contains *2150.42 cubic inches.*

2. In measuring grain, seeds, beans, small fruits, etc., the measure should be *even full;* but in measuring larger fruits, vegetables, etc., the measure should be *heaped.*

3. 6 quarts *dry* measure are nearly equal to 7 quarts *liquid* measure. 4 *heaped* pecks are nearly equal to 5 *even* pecks.

21. COMPARISON OF MEASURES OF CAPACITY.

	Cu. in. *in 1 gallon.*	***Cu. in.*** *in 1 quart.*	***Cu. in.*** *in 1 pint.*	***Cu. in.*** *in 1 gill.*
Liquid Measures. —	*231*	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry Measures. —	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

MEASURES OF WEIGHT.

22. Weight is the quantity of heaviness in a body.

23. The measures of weight are of two kinds: *Troy weights* and *avoirdupois weights.*

24. The ***Standard Unit*** of weight in the United States is the *Troy pound of the Mint.*

I. TROY WEIGHTS.

25. ***Troy Weights*** are used in weighing gold, silver, and jewels; and in philosophical experiments.

The units or denominations of Troy weights are *grain, pennyweight, ounce,* and *pound.*

Table.

24 grains (gr.)	are	*1 pennyweight,*	*pwt.*
20 pennyweights	"	*1 ounce,*	*oz.*
12 ounces	"	*1 pound,*	*lb.*

1 lb. = 12 oz. = 240 pwt. = 5760 gr.

The *Carat* is a small weight of about $3\frac{1}{6}$ grains, and is used to weigh diamonds and precious stones. The term *carat* means, also, a *twenty-fourth part,* when used to denote the *fineness* of gold.

26. ***Apothecaries' Weights*** are used by physicians in prescribing, and by apothecaries in mixing dry medicines.

Table.

20 grains (gr.)	are	*1 scruple,*	*sc. or* ℈.
3 scruples	"	*1 dram,*	*dr. or* ʒ.
8 drams	"	*1 ounce,*	*oz. or* ℥.
12 ounces	"	*1 pound,*	*lb. or* ℔.

1 lb. = 12 oz. = 96 dr. = 288 scr. = 5760 gr.

Note 1. — The pound, the ounce, and the grain are the same in Troy and Apothecaries' Weights. The *only* difference between the two is in the *division of the ounce.*

2. In writing prescriptions, physicians use the small letters of the Roman notation, and place the number of ounces, drams, etc., *after* the symbol, using j for i at the end of a number.

Thus, ℥ iv ℈ ii gr. viij is 4 ounces 2 scruples 8 grains.

3. For ordinary purposes, 1 teaspoonful weighs 1 dram; 1 tablespoonful, $\frac{1}{2}$ ounce; 1 pint, 1 pound.

II. AVOIRDUPOIS WEIGHTS.

27. ***Avoirdupois Weights*** are used in weighing all coarse and heavy articles; such as *groceries, grain, coal, iron,* etc.

The units or denominations of avoirdupois weights are *ounce, pound, hundred-weight,* and *ton.*

Table.

16 ounces (oz.)	are	*1 pound,*	*lb.*
100 pounds	"	*1 hundred-weight,*	*cwt.*
20 hundred-weight	"	*1 ton,*	*T.*

1 T. = 20 cwt. = 2000 lb. = 32000 oz.

Note 1. — The laws of almost all the States, as well as general use, make 100 pounds equal a hundred-weight.

2. At the United States Custom Houses, and in weighing coal and a few other articles, the gross hundred-weight of 112 pounds and the gross ton of 2240 pounds are used.

3. The ton of 2000 lb. is commonly called the *short ton,* or *net ton;* and the ton of 2240 lb. is called the *long ton,* or *gross ton.*

28. The following units or denominations are in common use:

56	*pounds of*	*butter*	are	*1 firkin.*
100	"	*grain or flour*	"	*1 cental.*
100	"	*nails*	"	*1 keg.*
100	"	*dry fish*	"	*1 quintal.*
196	"	*flour*	"	*1 barrel.*
200	"	*beef or pork*	"	*1 barrel.*
280	"	*(5 bu.) salt at N. Y. salt works*	"	*1 barrel.*

4. In many States, the weight of the bushel of the principal kinds of grain and seed is fixed by law, which must govern in buying and selling, unless special agreements in the contract are made.

29. AVOIRDUPOIS POUNDS IN A BUSHEL.

Barley,	*48 pounds.*	*Oats,*	*32 pounds.*
Buckwheat,	*42 pounds.*	*Potatoes,*	*60 pounds.*
Clover seed,	*60 pounds.*	*Rye,*	*50 pounds.*
Indian corn,	*50 pounds.*	*Wheat,*	*60 pounds.*

30. COMPARISON OF MEASURES OF WEIGHT.

	Troy.		Avoirdupois.		Apothecaries.
1 POUND. —	*5760 grains*	=	*7000 grains*	=	*5760 grains.*
1 OUNCE. —	*480* "	=	*437 $\frac{1}{2}$* "	=	*480* "
	175 pounds	=	*144 pounds*	=	*175 pounds.*

MEASURES OF VALUE.

31. ***Money*** is the standard of price or value used in trade.

32. Money is of two kinds: *coin* and *paper money*.

33. ***Coin,*** or ***Specie,*** is metal stamped and authorized by government to be used as money.

34. ***Paper Money*** is a substitute for coin, and consists of notes and bills issued by government.

35. ***Currency*** is whatever circulates in trade and commerce as money, whether coin or paper money.

36. Measures of money are coins of different values fixed by law.

I. United States Money.

37. ***United States Money*** is the legal currency of the United States.

The units or denominations of United States money are *mill, cent, dime, dollar,* and *eagle.*

38. The ***Standard Unit*** of United States money is the *gold dollar.*

Table.

10 mills (*m.*)	are	*1 cent,*	*ct. or c.*
10 cents	"	*1 dime,*	*d.*
10 dimes	"	*1 dollar,*	*$.*
10 dollars	"	*1 eagle,*	*E.*

$$1\ E. = \$10 = 100\ d. = 1000\ c. = 10000\ m.$$

Eagles are regarded as a number of *dollars*, and *dimes* as *cents* (339).

39. The ***Coins*** of the United States are of gold, silver, nickel, and bronze.

The ***Gold Coins*** are the double-eagle, eagle, half-eagle, and quarter-eagle.

The ***Silver Coins*** are the dollar, half-dollar, quarter-dollar, and dime.

The ***Nickel Coin*** is the five-cent piece.

The ***Bronze Coin*** is the cent.

Note 1.* — *Alloy is a baser metal mixed with a finer metal.

The alloy for *gold coins* is copper, or copper and silver; the silver not to exceed $\frac{1}{10}$ of the whole alloy in weight.

The alloy for *silver coins* and for *nickel coins* is copper.

The alloy for the *bronze cent* is zinc and tin.

2. The ***Standard Purity*** of the gold and silver coins is 9 parts, or .900, of metal of absolute fineness, and 1 part, or .100, of alloy.

The nickel coin consists of .25 of nickel and .75 of copper.

The bronze cent consists of 95 parts of copper, 3 of zinc, and 2 of tin.

3. The ***Legal Weight*** of the United States coins is —

Double eagle, 516 Troy grains; eagle, 258 gr.; half-eagle, 129 gr.; quarter-eagle, 64.5 gr.

Dollar, 420 gr.; half-dollar, 192.9 gr.; quarter-dollar, 96.45 gr.; dime, 38.58 gr.

Five-cent piece, 77.16 gr.; cent, 48 gr.

II. Canada Money.

40. Canada Money is the legal currency of the Dominion of Canada.

The units or denominations are *mill*, *cent*, and *dollar*.

The coins of Canada money have the same nominal value as the corresponding coins of the United States; although the real value of the *50-cent piece* is about *46* $\frac{1}{5}$ *cents*.

III. English Money.

41. English or ***Sterling Money*** is the legal currency of Great Britain.

The units or denominations of English money are *farthing*, *penny*, *shilling*, and *pound*.

42. The ***Standard Unit*** of English money is the *pound sterling*, or *gold sovereign*.

Table.

4 farthings (*far. or qr.*)	are	*1 penny*,	*d.*
12 pence	"	*1 shilling*,	*s.*
20 shillings	"	*1 pound*,	£.

1 £ = 20 s. = 240 d. = 960 far.

The *pound sterling*, or gold sovereign, is worth in U. S. money $\$4.866\frac{1}{2}$.

IV. French Money.

43. ***French Money*** is the legal currency of France.

The units or denominations are *millime, centime, decime,* and *franc.*

44. The ***Standard Unit*** of French money is the *silver franc.*

Table.

10 millimes (*m.*) *are 1 centime, ct.*	*10 decimes are 1 franc, fr.*
10 centimes are 1 decime, dc.	*20 francs are 1 Napoleon. Nap.*

The *franc* is worth in United States money $\$.193$.

V. German Money.

45. ***German Money*** is the legal currency of the Empire of Germany.

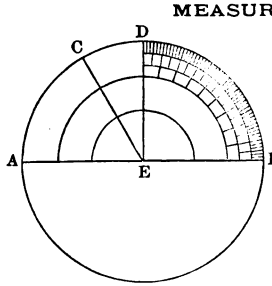
The units or denominations are *pfennig* and *mark.*

46. The ***Standard Unit*** of German money is the *silver mark.*

100 pfennige, or pennies, are 1 mark.

The *mark* is worth in United States money $\$.238\frac{1}{2}$.

MEASURES OF THE CIRCLE.



47. A ***Circle*** is a plane surface bounded by a curved line, which is everywhere at an equal distance from a point within called the *centre.*

48. A ***Circumference*** is the line that bounds a circle.

49. An ***Arc*** is any part of the circumference of a circle.

Thus, the part of the circumference from A to C is an *arc.*

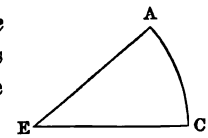
50. A ***Diameter*** is a right line drawn through the centre of a circle and terminated both ways by the circumference.

51. A ***Radius*** is a right line drawn from the centre of a circle to the circumference; and is half the diameter.

Thus, A B is the *diameter,* and A E or E B is the *radius* of the circle.

52. A ***Degree*** is one of the 360 equal parts into which the circumference of every circle is supposed to be divided.

53. The ***Measure of an Angle*** whose sides meet at the centre of a circle is that part of the circumference between the sides of the angle.



Thus, the arc A C is the measure of the angle A E C.

The *size* of an angle depends upon the *number of degrees* in the arc between the sides of the angle.

54. ***Circular*** or ***Angular Measures*** are used to measure angles and arcs of circles; to determine latitude, longitude, time, etc.

The units or denominations of circular or angular measures are *second, minute, degree,* and *circumference.*

55. The ***Standard Unit*** of angular measures is the *degree.*

Table.

60 seconds (″)	are	*1 minute,*	′
60 minutes	"	*1 degree,*	°
360 degrees	"	*1 circumference,*	*C.*

$$1\ C. = 360° = 21600' = 1296000''.$$

A *Quadrant* is 90°, or one fourth of a circumference.

A *Sextant* is 60°, or one sixth of a circumference.

A *Sign,* in astronomy, is 30°, or one twelfth of a circumference.

Note 1. — Every circle, whether great or small, has the *same number* of degrees; but the *size* or *length* of the degrees varies with the size of the circumference of the circle.

2. A *minute* of the earth's circumference is called a geographic, or nautical, mile; and is 1.16, or nearly $1\frac{1}{6}$, common miles.

3. A *degree* of latitude, or a degree of longitude at the equator, is 60 geographic miles, or nearly $69\frac{1}{6}$ common miles.

MEASURES OF TIME.

56. ***Time*** is a definite portion of duration.

The units or denominations of time are *second, minute, hour, day,* and *year.*

57. The ***Standard Unit*** of measures of time is the *day*.

Table.

60 seconds (sec.)	are	*1 minute,*	*min.*
60 minutes	"	*1 hour,*	*hr.*
24 hours	"	*1 day,*	*d.*
365 days	"	*1 common year,*	*yr.*
366 days	"	*1 leap year,*	*yr.*
100 years	"	*1 century,*	*C.*

1 yr. = 365 da. = 8760 hr. = 525600 min. = 31536000 sec.

Note 1. — 7 days are 1 week (*wk.*). For many purposes 4 weeks are called a month (*mo.*); and 52 weeks a year. In most business transactions, 30 days are considered as a month, and 12 months as a year.

58. The names of the months, and the number of days in each month, are as follows: —

		Days.			Days.
January,	*Jan.*,	31.	July,	*July*,	31.
February,	*Feb.*,	28 or 29.	August,	*Aug.*,	31.
March,	*Mar.*,	31.	September,	*Sept.*,	30.
April,	*Apr.*,	30.	October,	*Oct.*,	31.
May,	*May*,	31.	November,	*Nov.*,	30.
June,	*June*,	30.	December,	*Dec.*,	31.

2. The ***Solar Year*** is the time required for the earth to revolve around the sun, and is 365 da. 5 hr. 48 min. 49.7 sec. in length.

Hence, by counting only 365 days as a year, almost $\frac{1}{4}$ of a day is lost every year, or 1 day every four years.

This 1 day is, therefore, added every fourth year to February, giving it 29 days, and making what is called *leap year*, which contains 366 days.

59. The following rules for leap year will keep the calendar correct within 1 day for 4000 years: —

I. *Every year not a centennial year whose number can be exactly divided by* 4, *is a leap year.*

Thus, 1812, 1852, 1864, 1876, 2000 are leap years.

II. *Every centennial year whose number can be exactly divided by* 400, *is a leap year.*

Thus, 1200, 1600, 2000 are leap years; 1500, 1700, 1900 are common years.

MISCELLANEOUS MEASURES.

60. In ***counting*** certain articles, the units or denominations *dozen, gross,* and *great gross* are used.

Table.

12 things	are	*1 dozen,*	*doz.*
12 dozen	"	*1 gross,*	*gro.*
12 gross	"	*1 great gross,*	*grt. gro.*

1 grt. gro. = 12 gro. = 144 doz. = 1728 things.

Two things are often called a *pair;* six things, a *set;* twenty things, a *score;* as, a *pair* of shoes; a *set* of chairs, spoons, etc.; a *score* of years.

61. In ***buying and selling paper,*** the units or denominations *sheet, quire, ream, bundle,* and *bale* are used.

Table.

24 sheets	are	*1 quire.*	*2 reams*	are	*1 bundle.*
20 quires	"	*1 ream.*	*5 bundles*	"	*1 bale.*

Paper is bought and sold at *retail* by the quire and ream; and at *wholesale* by the bundle and bale.

62. In ***printing*** and ***binding books,*** the terms *folio, quarto, octavo,* etc., are used to denote the number of leaves into which a sheet of paper is folded.

When a sheet is folded into	The book is called	And 1 sheet of paper makes
2 leaves,	*a folio,*	*4 pp. (pages.)*
4 "	*a quarto or 4to,*	*8* "
8 "	*an octavo or 8vo,*	*16* "
12 "	*a duodecimo or 12mo,*	*24* "
16 "	*a 16mo,*	*32* "
18 "	*an 18mo,*	*36* "

Section II.

REDUCTION OF DENOMINATE NUMBERS.

Problems.

Change, or reduce —

1. 12 mi. to yd.; 25 mi. to ft.
2. 32 mi. 35 rd. 29 ft. to feet.
3. 43 rd. 7 ft. 9 in. to inches.
4. 9 rd. 2 yd. $2\frac{1}{2}$ ft. to feet.
5. 1 mi. 20 ch. to links.
6. 8 A. to sq. rd.; 18 A. to sq. ft.
7. 24 A. 100 sq. rd. to sq. ft.
8. 8 sq. mi. to acres; to sq. ft.
9. 50 A. 10 sq. yd. to sq. feet.
10. 600 sq. ch. to sq. links.
11. 18 cd. to cu. ft.; 27 cd. to cu. in.
12. $27\frac{3}{5}$ cu. yd. to cu. ft.; to cu. in.
13. 15 gal. 2 qt. $1\frac{1}{2}$ pt. to pt.
14. 2 bbl. 10 gal. $3\frac{1}{2}$ qt. to gi.
15. 500 hhd. 50.075 gal. to qt.
16. 50 bu. 3 pk. to qt.; to pt.
17. 75 bu. $3\frac{1}{2}$ pk. to pk.; half-pk.
18. 273 cwt. 25 lb. to lb.; to oz.
19. 200.05 T. to cwt.; to lb.
20. 7 long tons to oz.; a short ton.
21. 10 lb. 8 oz. $3\frac{1}{2}$ pwt. to gr.; 12 lb. 8.5 oz. to pwt.
22. How many minutes in the year 1875? In 1876?
23. How many minutes in the earth's circumference? Seconds?
24. $\frac{9}{11}$ common yr. to min.; $\frac{1}{20}$ wk. to sec.; $\frac{9}{11}$ leap yr. to da.
25. $\frac{1}{10}°$ to ″; £$8\frac{1}{3}$ to far.; $\frac{8}{9}$ long T. to lb.; $\frac{8}{9}$ short T. to oz.
26. $\frac{10}{11}$ lb. Troy to gr.; $\frac{10}{11}$ lb. apoth. to gr.; $\frac{10}{11}$ lb. avoir. to gr.
27. $8\frac{3}{5}$ bu. to pt.; $\frac{12}{13}$ hhd. to pt.; $3\frac{1}{10}$ bbl. to gi.; $10\frac{3}{100}$ hhd. to gi.
28. $\frac{3}{70}$ cu. yd. to cu. in.; $\frac{8}{11}$ cu. yd. to cu. ft.; $\frac{7}{15}$ A. to sq. yd.; $\frac{7}{20}$ A. to sq. in.; $\frac{5}{22}$ sq. ch. to sq. ft.
29. $\frac{9}{40}$ sq. mi. to sq. ft.; $\frac{11}{50}$ mi. to ch.; $\frac{13}{55}$ mi. to yd.; $\frac{11}{80}$ mi. to in.
30. .75 of a ream to sheets; .05 grt. gro. to ones; .0875° to ″.
31. .025 leap year to hr.; .0025 com. yr. to min.; £4.015 to far.
32. .007 long T. to lb.; .0007 short T. to oz.; .325 lb. Troy to gr.
33. 4.005 lb. apoth. to gr.; .75 hhd. to qt.; 9.05 bbl. to gi.
34. .007 sq. mi. to sq. rd.; .075 A. to sq. in.; .009 mi. to in.

Change to integers of lower denominations —

35. $\frac{3}{4}$ grt. gro.; $\frac{7}{8}$ doz.; $\frac{1}{2}$ score.
36. $\frac{4}{5}$ reams; $\frac{7}{8}$ da.; $\frac{4}{9}$ wk.
37. $\frac{9}{11}$ common yr.; $\frac{9}{11}$ leap yr.
38. $\frac{8}{9}°$; $\frac{5}{7}'$; $\frac{3}{5}$ lb. Troy; $\frac{4}{11}$ oz.
39. $\frac{7}{12}$ T.; $\frac{5}{16}$ cwt.; $\frac{9}{13}$ lb.
40. $\frac{7}{15}$ bu.; $\frac{8}{21}$ bu.; $\frac{7}{20}$ hhd.
41. $\frac{3}{17}$ cd.; $\frac{5}{13}$ cu. yd.; $\frac{7}{11}$ A.
42. $\frac{3}{10}$ of $7\frac{1}{2}$ mi.; $\frac{8}{11}$ of $7\frac{1}{3}$ rd.
43. .0325 mi.; .006$\frac{1}{2}$ mi.
44. .0204 A.; .5625 sq. ch.
45. .00125 cd.; .034$\frac{1}{3}$ cu. yd.
46. .563 hhd.; .0064 bbl.
47. .0005$\frac{1}{8}$ gal.; .075$\frac{3}{16}$ bu.
48. .008$\frac{3}{5}$ bu.; .0006 T.
49. 6.5$\frac{1}{2}$ cwt.; .008$\frac{1}{3}$ year.
50. $\frac{5}{6}$ of $3\frac{3}{10}$ times .0048 yr.

51. Express $\frac{9}{13}$ of a day as a compound number; $\frac{4}{9}$ yr.

52. Change $\frac{5}{7}$ of $4\frac{1}{2}$ T. to a compound number; $\frac{4}{5}$ of .225 A.

53. How many sq. yd. and sq. ft. in $2\frac{1}{3}$ times .002$\frac{1}{2}$ A.?

What is the cost of —

54. A silver cup weighing 10 oz. 1.5 pwt. at 12$\frac{1}{2}$ cents a pwt.?

55. 12 reams 6 quires 12 sheets of paper at $\frac{1}{2}$ c. per sheet?

56. 12 cwt. 45$\frac{7}{8}$ lb. rice at 6$\frac{1}{2}$ c. a lb.? At 7$\frac{3}{4}$ cents?

57. 2 lb. 5.5 oz. of gold at \$14$\frac{3}{8}$ an ounce? At 87$\frac{1}{2}$ c. a pwt.?

58. A grt. gro. of lead-pencils at 31$\frac{1}{4}$ c. a doz.? At 3$\frac{1}{2}$ c. each?

59. 3$\frac{1}{2}$ bbl. of flour at 5$\frac{1}{2}$ c. a pound? At \$.06$\frac{3}{4}$? At \$.07$\frac{1}{4}$?

60. $\frac{7}{8}$ mi. of fence at \$1.12$\frac{1}{2}$ per rd.? .075 mi.? .005 mi.?

61. 75 lb. gold at \$.91$\frac{3}{4}$ a pwt.? .75 quarter section of land at \$2$\frac{1}{4}$ an acre? 4.75 cwt. of hay at 1$\frac{1}{4}$ c. a lb.?

62. How many more times will a clock tick seconds in February in a leap-year than in February in a common year?

63. How many papers of seed, each 2.5 lb., can be filled from 17$\frac{1}{2}$ lb. of peas? From .75 cwt. of beans?

64. How many rods of fencing are needed to run a fence on both sides of a lane $\frac{1}{4}$ of a mile long?

65. A druggist put up ℔ 1 ℥ 10 ʒ 4 ℈ 2 of morphine in pills each weighing a half-grain. How many gross of pills were there?

66. How many two-ounce weights can be made from 100 lb. of brass, allowing 2 lb. for waste and loss?

67. A grocer bought $3\frac{3}{4}$ bbl. of flour at $\$6\frac{3}{4}$ a bbl., and sold it at $5\frac{1}{4}$ cents a pound. What did he gain?

68. How many gold rings, each $3\frac{1}{5}$ pwt., can be made from a bar of gold weighing .75 lb.? From $2\frac{2}{3}$ lb.? 1 lb. $5\frac{3}{10}$ oz.?

69. How many times will a wheel $15\frac{1}{3}$ ft. in circumference turn in going 10 mi.? In going 10 mi. 100 rd.? .025 mi. $\frac{7}{8}$ rd.?

70. A boy bought $3\frac{3}{4}$ bu. of chestnuts at $\$5\frac{1}{2}$ a bu., and retailed them at $12\frac{1}{2}$ cents a pt. What did he gain?

Reduction Ascending.

Written Exercises.

Change, or reduce, to integers of higher denominations —

1. 925 in. to yd.; 100 ft. to rd.
2. 2000 ft. to mi.; 2100 yd. to mi.
3. 30250 sq. in. to sq. yd.
4. 100000 sq. ft. to acres.
5. 4900 sq. yd. to sq. ch.; to A.
6. 103570 cu. in. to cu. yd.
7. 75 gi. to qt.; 5902 gi. to gal.
8. 4832 pt. to pk.; to bushels.
9. 4073200 sec. to da.; to wk.
10. 13025 to gro.; to grt. gro.

Change, or reduce, to higher denominations —

11. 2000 single things; 1025 doz.; 10000 sheets; 12960″.
12. 7800′; 7300 da.; 43800 hr.; 262801 min.; 20025 pence.
13. 10205 far.; 40302 mills; 20050 lb. avoir.; 100750 oz.
14. 10650 gr. Troy; 123456 gr.; 20304 gr. apoth.; 20005 gr.
15. 1500 qt., dry; 2520 gal.; 3200 pt. to bbl.; 10001 gi.
16. 10000 cu. in.; $1500\frac{1}{2}$ cu. ft.; 1200 sq. ft.; 8250 sq. yd.
17. 20000 sq. in.; 900005 sq. in.; 3875 ft.; 50000 ft.; 100000 in.
18. 9000 sq. in.; $10304\frac{1}{2}$ yd.; 203004.5 sq. in.; $300045\frac{2}{3}$ ft.

Change to the fraction of the higher denomination —

19. 15 doz. to gro.; 288 sheets to reams; 420 sec. to hours.
20. 480 min. to da.; 510 min. to wk.; 52560 min. to yr.

21. 1080″ to °; 1400′ to C.; 162 d. to £; 504 far. to £.

22. 800 oz. to cwt.; 2050 lb. to T.; 40000 oz. to T.; $\frac{8}{9}$ gr. to lb.

23. $38\frac{1}{2}$ rd. to mi.; $1361\frac{1}{4}$ sq. rd. to sq. mi.; $100\frac{2}{3}$ yd. to rd.

24. $\frac{7}{50}$ in. to rd.; $\frac{3}{200}$ sq. ft. to A.; $200\frac{1}{2}$ ft. to mi.; $400\frac{1}{5}$ sq. rd. to A.

Change to the decimal of the higher denomination —

26. 50 mills to $; 60 far. to £; 30 pt. to bu.; 36 qt. to bbl.

27. 128 pt. to hhd.; 288 gr. to lb.; 640 oz. to cwt.; 100 lb. to T.

28. 270 cu. in. to cu. yd.; 242 sq. yd. to A.; 1089 sq. in. to sq. ch.

29. .25 yd. to mi.; .3125 ft. to rd.; $\frac{7}{8}$ ft. to mi.; 396 in. to ch.

30. $10\frac{1}{2}$ sq. in. to sq. yd.; 8.5 ft. to mi.; 9.025 sq. yd. to A.; .5 in. to rd.; $4.37\frac{1}{2}$ sq. ch. to A.; $.075\frac{1}{2}$ yd. to mi.

Change, or reduce —

33. 3 yd. 1 ft. $3\frac{3}{4}$ in. to the fraction of a rod; of a mile.

34. 3 cd. ft. 8 cu. ft. to the fraction of a cd.; the decimal.

35. 1 gal. 3 qt. $1\frac{3}{4}$ pt. to the fraction of a bbl.; of a hhd.

36. 5 oz. 6 pwt. $11\frac{1}{2}$ gr. to the fraction of a lb. Troy; the decimal.

37. 754 yd. $10\frac{2}{7}$ in. to the fraction of a rd.; the decimal.

38. 85 rd. 1 yd. $2\frac{1}{2}$ ft. to the fraction and the decimal of a mi.

39. 8 hr. $27\frac{9}{13}$ sec. to the decimal and fraction of a day.

40. 4 A. 1 sq. rd. $18\frac{3}{20}$ sq. yd. to the mixed decimal of an A.

What is the cost of —

41. 1 T. hay, at $1.12\frac{1}{2}$ per cwt.?

42. 682 lb. flour, at $7 a bbl.?

43. 1254 ft. fence, at $\frac{7}{8}$ a rd.?

44. 2000 pencils, at $.31\frac{1}{4}$ a gro.?

45. 1764 pt. wine, at $94 a hhd.?

46. 40 sq. rd., at $500 an acre?

47. 22 in. ribbon, at $\frac{3}{8}$ a yd.?

48. $17\frac{1}{2}$ cwt. hay, at $22\frac{1}{2}$ a T.?

49. 3.75 pk. oats, at $.45 a bu.?

50. 75 pt. Cologne, at $2\frac{1}{2}$ a qt.?

51. At $62\frac{1}{2}$ c. a bushel, what is the cost of 1750 lb. of wheat?

52. A farmer divided 64 A. 5 sq. ch. in 8 equal fields. How many acres were in each field?

53. What is the value of 8 bu. 3 pk. of chestnuts, at $3\frac{1}{2}$ a bu.?

54. A mason worked 15 days, averaging 7 hr. a day. What is the amount of his wages at $3\frac{1}{2}$ a day of 10 hours?

55. How many bushels are 6250 quarts of oats? How many sacks, each holding $3\frac{1}{2}$ bu., will the oats fill?

56. If a silver-dollar weighs $412\frac{1}{2}$ grains Troy, how many pounds avoirdupois do \$1000000 weigh? How many tons?

57. A printer used 5 rm. 10 quires 10 sheets of paper for half-sheet posters. How many did he print? What did they cost at \$1.75 per C.? At \$9.75 per thousand?

58. How many days will it take a watch to tick a million times, if it ticks four times every second?

59. An iron-founder cast 2500 flat-irons, each 5 lb. If the waste was 50 lb., how many tons of iron did he use?

60. At \$.75 a rod, it cost \$465 to inclose a tract of land. What was the distance around the tract, in miles?

61. In making a plank-road 41250 planks, averaging 1 ft. wide, were used. How long was the road?

62. How many bushels of peaches are needed to fill 100 quart-cans? What are they worth at $\$.87\frac{1}{2}$ a bushel?

63. If, at 10 cents a foot, the Atlantic cable cost \$1689600, what is its length?

To Change a Denominate Number to the Fraction of Another.

Problems.

1. Change 2 pk. 3 qt. 1 pt. to the fraction of a bushel.
2. What part of 1 rod are 2 yd. 4 ft. $6\frac{1}{2}$ in.? Of 3 rd.?
3. Change 9 oz. $3\frac{3}{7}$ dr. to the fraction of $3\frac{1}{2}$ lb. apoth.
4. Reduce 10 gal. 3 qt. $1\frac{3}{4}$ pt. to the decimal of a hhd.
5. Express £3 10 s. $8\frac{1}{4}$ d. as a decimal of a £; of $£4\frac{1}{2}$.
6. What decimal part of 2 T. 10 cwt. 50 lb. are 5 T. 96 lb.?
7. What decimal of 10 cords of wood are 1000 cu. ft.?
8. Express $\frac{3}{7}$ of 25 lb. as a fraction of a ton. Of 4 T. $10\frac{1}{5}$ cwt.
9. What fractional part of $30° 40' 45\frac{1}{2}''$ are $10° 30' 40''$?
10. Change 7 rd. 3 yd. 2.25 ft. to the fraction of 4.025 mi.

11. From a hhd. of molasses 32 gal. 1 qt. were drawn. What fractional part of the hogshead remained?

12. In a farm of 150 A. 50 sq. rd. there are 60 A. 20 sq. rd. of woodland. What part of the farm is woodland?

13. At $4.25 a gallon, how much will 1 hhd. 20 gal. 2 qt. of wine cost? 10 hhd. 5 gal. $1\frac{1}{2}$ pt.? 3 qt. 1 pt. $1\frac{1}{2}$ gi.?

14. What part of a bushel are 2 pk. 6 qt. $\frac{5}{9}$ pt.?

15. What decimal of $2\frac{1}{2}$ T. of nails can be bought for $137.25, at $4\frac{1}{2}$ cents a pound? For $62.50? $100.75?

16. What part of a hogshead of wine are 10 gal. 2 qt. $\frac{4}{9}$ pt.?

17. What part of a right angle are 6° 30′? What part of a circle? What part of a quadrant?

Section III.

ADDITION OF COMPOUND NUMBERS.

Problems.

Find the sum of—

1. 10 mi. 5 rd. 2 ft. + 9 mi. 5 yd. + 20 rd. $2\frac{1}{2}$ ft. + 25 mi. 5 yd.

2. 12 A. 50 sq. rd. + 100 sq. rd. 30 sq. yd. + 50 A. 75 sq. rd. 20 sq. yd. + 10 A. 80 sq. yd. + $10\frac{1}{4}$ sq. yd.

3. 160 A. 25 sq. yd. + 200 A. 80 sq. rd. + 300 A. 5 sq. ch. + 75 A. 8 sq. ft. + 50 A. 100 sq. rd.

4. 3 mi. 120 rd. 2 yd. + 6 mi. 100 rd. 2 ft. + 1 mi. 5 yd. + 5 mi. 20 rd. $5\frac{3}{4}$ ft. + 50 mi. 5 yd. $1\frac{1}{2}$ ft.

5. 30 A. 125.25 sq. yd. + 75 A. 90 sq. rd + 132 A. 50.5 sq. yd. + 150 A. $70\frac{1}{2}$ sq. rd. + 200 A. $75.06\frac{1}{4}$ sq. yd.

6. 40 ch. 56 l. 7.5 in. + 75 ch. 50 l. 5.25 in. + 100 ch. 75 l. + 80 ch. 40 l. 3.5 in. + 70 ch. 30 l.

7. 38 bu. 3 pk. $9\frac{7}{8}$ qt., 31 bu. 2 pk. $7\frac{7}{12}$ qt., 45 bu. $\frac{5}{16}$ qt., 50 bu. 1 pk. $7\frac{3}{4}$ qt., 100 bu. $4\frac{3}{32}$ qt., and 1 pk. $3\frac{1}{4}$ qt.

8. 50 T. 10 cwt. $7\frac{3}{5}$ oz., 5 T. 50 lb. 7.25 oz., 75 T. 95 lb., 19 cwt. 75 lb. $9.06\frac{1}{4}$ oz., and 10 cwt. 15 lb. .0625 oz.

9. Find the sum of $32\frac{4}{7}$ cwt., $50\frac{7}{8}$ lb., and $15\frac{1}{2}$ oz.

10. Find the sum of $37\frac{2}{3}$ cd., $75\frac{3}{4}$ cu. ft., and 75 cd. $50\frac{1}{8}$ cu. ft.

11. Add $\frac{7}{8}$ of a day, $\frac{4}{5}$ of a week, and $\frac{7}{16}$ of an hour.

12. Add 4 yd. 2 ft. $8\frac{3}{4}$ in., $38\frac{3}{4}$ ft., $5\frac{1}{2}$ yd., and 5 yd. 8.25 in.

13. Add $3\frac{3}{4}$ hhd., 84 gal. 3 qt. $2\frac{3}{4}$ pt., $\frac{7}{16}$ gal., and 3 qt. .25 pt.

14. To .005 of a ton add $\frac{3}{64}$ cwt. + $\frac{9}{10}$ of a pound.

15. If a vessel sails from a port in latitude 40° 15′ N. to a port 22° 30″ S., through how many degrees does it sail?

16. Express in bushels the sum of .75 bu., $2\frac{16}{8}$ pk., $\frac{1}{2}$ bu., $10\frac{4}{8}$ qt., 5 bu. $2\frac{1}{2}$ pk., and 3 pk. 2 qt. .5 pt.

17. Three coal-cars contained respectively 5.125 T., $4\frac{7}{12}$ T., and 9350 lb. What was the coal worth at $\$6\frac{1}{4}$ a ton?

Section IV.

SUBTRACTION OF COMPOUND NUMBERS.

Problems.

Find the remainder of —

1. 8 mi. 75 rd. 4 yd. — 3 mi. $5\frac{3}{4}$ yd.; 10 rd. — 5 yd. 9 in.

2. 100 A. 20 sq. rd. — 90 A. 10 sq. yd.; 50 cd. — $8\frac{9}{11}$ cu. in.

3. 10 T. 14 cwt. $5\frac{2}{3}$ lb. — 5 T. 15 cwt. 20 lb. 7.25 oz.

4. 100 yr. 30 da. — 1 yr. 23.75$\frac{1}{2}$ hr.; 1 C. — 50 yr. $364\frac{5}{9}$ da.

5. 100 bu. 4 qt. $3\frac{3}{5}$ pt. — 90 bu. $4\frac{7}{8}$ pt.; 10 T. 7 cwt. — 5 lb. 20.75 oz.; 75 hhd. — 47 gal. 7 pt. 3.05 gi.

Find the difference between —

7. $\frac{3}{11}$ mi. — $\frac{3}{11}$ rd.; 10 mi. — 5.7 rd.

8. $20\frac{4}{9}$ A. — 10 sq. rd. 8.4 sq. ft.

9. 20.3 cd. — 100 cu. ft. $900\frac{7}{12}$ cu. in.

10. $\frac{3}{16}$ hhd. — $2\frac{1}{8}$ gal.

11. $7\frac{8}{20}$ lb. — 10.65 ʒ.

12. $17\frac{1}{16}$ bu. — 6.3 qt.

13. Washington is in longitude 77° 0′ 15″ W. from Greenwich, and San Francisco is 122° 26′ 45″ W. What is their difference in longitude?

14. The longer of two pieces of muslin is 34 yd. $10\frac{3}{4}$ in., and the difference in their length is $\frac{7}{8}$ yd. How long is the shorter?

15. Two masons plastered 650 sq. yd. 20.25 sq. ft. of wall. If one of them plastered 409 sq. yd. 100.5 sq. in., how much did the other plaster?

16. Tell the exact time from Jan. 25 to May 15, 1901.

17. How long from April 19, 1783, to Dec. 24, 1814?

18. A note given June 14, 1900, was paid Nov. 10, 1901. How long had it run? If it had been given Dec. 10, 1900?

19. The second war with England commenced June 18, 1812, and continued 2 yr. 6 mo. 6 da. When did it end?

20. A note was dated August 23, 1901, payable in 90 days. When was it payable, if the time was extended three days?

Find the result of —

21. $1\frac{4}{5}$ bu. + $3\frac{3}{8}$ pk. — 4.75 qt. | 23. 10.005 A. — ($\frac{8}{11}$ sq. ch. — $\frac{4}{8}$ sq. rd.)

22. .785 T. — (4 cwt. + $100\frac{4}{7}$ lb) | 24. ($8\frac{4}{9}$ mi. — 4 rd.) — $3.2\frac{1}{3}$ yd.

25. From the sum of 1.025 bu. and 2 pk. 6 qt. $3\frac{4}{5}$ pt. take their difference. To their sum add their difference.

26. A ship in latitude 40° 35′ N. sailed southward 50° 10′ 30″; then northward 15° 30′; then southward 60° 15′ 15″. What was her latitude?

27. A contractor engaged to put up 45 mi. of telegraph; after building 20 mi. 20 rd., a storm destroyed 1 mi. 200 rd. How much remained to be built?

Section V.

MULTIPLICATION OF COMPOUND NUMBERS.

Problems.

Find the product of —

1. 8 T. 50 lb. 3 oz. × 5; by 6; by 11; by 25; by 46.

2. 3 yr. 40 da. $7\frac{5}{7}$ hr. × 6; by 8; by 10; by 34; by 58.

3. £50 10 s. $5\frac{3}{4}$ d. × 7; by 12; by 17; by 22; by 60.

4. 30° 12′ 34.75″ by 8; by 14; by 20; by 32; by 71; by 86.

5. 75 A. 40 sq. rd. $8\frac{3}{4}$ sq. ft. by 23; 35; 79; 101; 123; 189; 245.

6. 105 mi. 25 rd. 1 yd. 6 in. by 46; 68; 80; 123; 234; 456.

7. If 20 cars of coal average 4 T. $500\frac{3}{16}$ lb., what is the entire weight? If they average 5 T. 3.75 cwt.?

8. If a steamboat averages a mile in 5 min. 15 sec., how long will it be in making a trip of 75 miles? Of 250 miles?

9. What is the cost of 5 pieces of cloth, each containing $33\frac{1}{3}$ yd., at 12 s. 8 d. a yard? At 10 s. $6\frac{3}{4}$ d.?

10. If the daily session of a school is 5 hr. 30 min., how many school hours in a term of 13 weeks of 5 days each?

11. How many pounds of grapes are needed to fill a half gross of boxes averaging 4 lb. $7\frac{3}{4}$ oz. each?

Find the value in all lower denominations of—

12. $8\frac{3}{5}$ bu. — 2 pk. $2\frac{1}{2}$ qt. × 3.

13. 4.35 A. × 7 — $3\frac{1}{7}$ sq. rd.

14. $9\frac{2}{3}$ mi. + ($7\frac{3}{11}$ rd. × 2) — 100 ft.

15. 75 cd. × 3.5 — 5 cd. $50\frac{1}{5}$ cu. in.

16. (10.$6\frac{1}{4}$ T. + $10\frac{1}{3}$ cwt. — 25.75 lb.) × 4.

17. $\overline{20.8\frac{1}{2}}$ hhd. × $5\frac{1}{2}$ — ($\frac{9}{25}$ of 10.$6\frac{1}{4}$ gal. + $\frac{7}{8}$ gal.).

18. From a hogshead of wine were drawn 30 gal. $3\frac{1}{2}$ qt. What was the value of the balance, at \$1.$62\frac{1}{2}$ a gal.?

19. What was the value of 4 loads of hay averaging 2 T. $2\frac{1}{4}$ cwt., at \$$21\frac{7}{8}$ a ton, after deducting the weight of the wagon, which was 1250 lb.?

20. From a farm of 45 A. 40 sq. rd. there were sold 16 building lots, each $\frac{3}{4}$ A. $30\frac{1}{8}$ sq. rd. How much remained?

21. A wood merchant has 16 piles of wood, each containing 14 cu. yds. and 16 cu. ft. How many cords and cu. ft. has he?

22. If one field contains 8 acres, 140 sq. rods, 15 sq. yds., what will 25 similar fields contain?

23. If a farmer sells 5 bus., 3 pks., 5 qts. of fruit in one week, how much will he sell in five weeks?

Section VI.

DIVISION OF COMPOUND NUMBERS.

Problems.

Find the quotient of—

1. 70 mi. 40 rd. 3 yd. ÷ 4; by 6; by 8; by 10; by 15.

2. 100 A. 80 sq. rd. 20 sq. ft. ÷ 5; by 7; by 9; by 12; by 22.

3. 200 cd. 20 cu. ft. 8 cu. in. ÷ 7; by 8; by 19; by 26; by 45.

4. 100 bbl. 30 gal. $3\frac{1}{2}$ qt. ÷ 18; by 40; by 62; by 88; by 130.

5. If in 9 casks of wine there are 540 gal. 3 qt. 2 pt., what is the average quantity in each?

6. In 12 sacks of wheat there are 36 bu. $2\frac{7}{8}$ pk. How much is in each sack? If there is the same quantity in 17 sacks?

7. A quantity of tea, consisting of 25 chests, weighs 12 cwt. 56 lb. $8\frac{5}{8}$ oz. What is the weight of each chest?

9. If a steamboat runs 15 mi. 50 rd. $5\frac{1}{2}$ yd. in an hour, in what time will it run 200 miles?

10. If 6 men mow 61 A. 65 sq. rd. of grass in 6 days, how much does one man mow in a day?

11. How many sets of spoons can be made from 10 lb. 4 oz. 16 pwt. of silver, if each spoon weighs $2\frac{2}{3}$ oz.?

12. If a township of government land be divided into farms of 135 A. $5\frac{5}{7}$ sq. ch., how many farms will it make?

13. If a railroad train runs 86 mi. 300 rd. in 6.5 hr., how far does it run per hour?

14. How many lots, each 5 rd. by 12 rd., can be made from a piece of land containing $5\frac{3}{4}$ A.?

Find the value of—

15. 1.325 T. × 7 — 5 T. $25\frac{1}{2}$ lbs.

16. ($8\frac{7}{8}$ bu. ÷ 8) + 5 pk. $\frac{4}{5}$ qt.

17. $\overline{7\frac{3}{10}}$ lb. + $4\frac{5}{6}$ pwt. ÷ 12.

18. $.002\frac{1}{2}$ yr. + $7\frac{1}{5}$ da. × 3.5.

19. (7.8 mi. ÷ $3\frac{1}{4}$) — $\frac{2}{3}$ of 10 rd.

20. 45 A. × 95 — $8\frac{3}{4}$ sq. ch.

21. How much is $\frac{3}{5}$ of 15 T. $95\frac{3}{4}$ lb.? $\frac{3}{8}$? $\frac{9}{10}$? $\frac{7}{12}$?

22. To $3\frac{3}{4}$ mi. add 30 rd. $4.02\frac{1}{2}$ yd., and divide the sum by 15.

23. From $50.72\frac{1}{2}$ A. take $\frac{2}{5}$ of $9\frac{3}{10}$ sq. ch., and divide the remainder by 25.

24. From the sum of 3 mi. $8\frac{2}{3}$ rd. and 10.005 mi. take their difference, and divide the result by 30.

Section VII.

LONGITUDE AND TIME.

Problems.

1. The difference in longitude between Washington and St. Louis is 13° 15′. What is the difference in time?

2. New York is in longitude 74° 3′ W., and Chicago in longitude 87° 37′ 48″ W. What is the difference in time?

3. What is the difference in time between Boston, 71° 3′ 30″ W., and Detroit, 80° 58′ W.? St. Paul, 95° 4′ 55″ W.?

4. How much does a watch, showing correct time in London, differ from a watch in New York, 74° 5′ W. from London?

5. The difference in time between two places being 1 hr. 20 min. 35 sec., what is their difference in longitude?

6. If the difference in time between Washington, 77° 0′ 15″ W., and St. Louis is 53 min., what is the longitude of St. Louis?

7. When it is noon at Philadelphia, it is 5 o'clock 10 min. P. M. at Paris. What is the longitude of Paris?

8. When it is noon at Washington, what is the time at Rome, in longitude 12° 27′ E.? The time at San Francisco?

9. How much earlier or later does the sun rise in Boston than in Berlin, in longitude 13° 23′ 45″ E.? Than in Mexico, 90° 5′ W.?

10. A vessel sailed from San Francisco to Yeddo, 139° 40′ W. Did the ship's chronometer gain or lose time, and how much?

Section VIII.

RECTANGULAR MEASUREMENTS.

Rules for Rectangular Surfaces.

I. When the length and the breadth are given to find the area,

Multiply the length by the breadth. The product will be the area required.

II. When the area and one dimension are given to find the other dimension,

1. *Divide the area by the length. The quotient will be the breadth required.* Or,

2. *Divide the area by the breadth. The quotient will be the length required.*

GENERAL FORMULAS.

1. *Length* × *breadth* = *area.* | 2. *Area* ÷ *length* = *breadth.*

3. *Area* ÷ *breadth* = *length.*

Note 1. — When the length and the breadth are given to find the area, two factors are given to find the product.

2. When the area and one dimension are given to find the other dimension, the product of two factors and one of the factors are given to find the other factor.

3. Glaziers and stone-cutters estimate work by the *square foot.*

4. Painters, plasterers, etc., estimate work by the *square foot,* or the *square yard.*

5. Builders generally estimate roofing, flooring, slating, etc., by the *square* of *100 square feet;* sometimes by the square foot, or square yard.

A *thousand shingles,* averaging 4 in. wide, and laid 5 in. to the weather, cover a *square;* and 100 lath cover *5 sq. yd.* of surface.

6. Bricklayers estimate by the *square* foot, the *square* yard, or by the *thousand bricks.*

Problems.

Find the area of rectangles having the following dimensions, and give the results in higher denominations: —

1. 18 ft. by 16 ft.; by 18 ft.
2. $19\frac{1}{2}$ rd. by $20\frac{1}{4}$ rd.
3. $30\frac{1}{4}$ yd. by 35.5 yd.
4. 40 yd. $2\frac{1}{2}$ ft. square.
5. 20 ft. 6 in. by 15 ft. 9 in.
6. 35 yd. 4 ft. by 25 yd. 8 in.
7. 12 ch. 10 l. by 10 ch. 25 l.
8. 50 rd. 5 yd. by 40 rd. 8 yd.
9. How many acres are in a field $11\frac{1}{4}$ ch. by $10\frac{2}{5}$ ch.?

10. How many sq. yd. of plastering in the sides of a room 20 ft. long, 15 ft. 6 in. wide, and 9 ft. 6 in. high? In the ceiling?

11. How many sq. ft. in 10 boards 12 ft. long, 6 in. wide, and 12 boards 10 ft. by 8 in.? What are all worth at $3.50 per C.?

12. What is the area of the foundation of a square building, the side of which is 46 ft. 8 in.?

13. How many acres in a rectangular field 105 ch. long and 25 ch. wide? How many A. and sq. rd.?

14. How many floor-boards 12 ft. long and 5 in. wide will be needed to floor a room 24 ft. 6 in. by 20 ft.?

15. How many yards of carpet $\frac{3}{4}$ yd. wide will cover a floor 25 ft. 6 in. by 15 ft. 8 in.? A room $30\frac{3}{4}$ ft. by $20\frac{1}{3}$ ft.?

16. How many yards of carpet 2 ft. 4 in. wide will cover a floor 22 ft. by 18 ft. 9 in.? Carpet 2 ft. 8 in. wide?

17. What will it cost to cover a floor $28\frac{1}{2}$ ft. by 18 ft. 4 in. with carpet 2 ft. wide, at $\$1\frac{1}{8}$ a yd.? 32 ft. 6 in. by 25.8 ft.?

18. At $\$2\frac{3}{8}$ a yd. for carpet $\frac{7}{8}$ yd. wide, what will it cost to carpet a room 18.75 ft. by 12 ft. 8 in.? At $\$1\frac{9}{10}$ per yd.?

19. How many tiles 10 in. square will lay a hall 18 ft. by 6 ft.

20. At $2.50 a sq. yd., what will it cost to flag a walk 250 ft. by $4\frac{1}{2}$ ft.? How many flags 2 ft. by 15 in. are needed?

21. What is the cost of glazing 8 windows, each 6 ft. 6 in. by 3 ft. 4 in., at $\$.87\frac{1}{2}$ a sq. ft.? 12 windows $7\frac{1}{2}$ ft. by $3\frac{1}{4}$ ft.?

22. If the area of a rectangle is $275\frac{1}{8}$ sq. ft., and the length is 18 ft. 6 in., what is the breadth? If the area is 50.25 sq. yds.

23. The area of a piece of land is 10 A. 60 sq. rd., and the breadth is 8 ch. 25 l. What is the length?

24. If the area of a floor is 27 sq. yd. $8\frac{1}{8}$ sq. ft., and the length is 20 ft. 6 in., what is the breadth?

25. A rectangular farm containing 60 A. 5 sq. ch. is 140 rods in length. What is the breadth? How many rods of fencing will inclose it?

26. At $75 an acre a farm costs $4181.25. How long is it, the breadth being 55 rd.?

Problems.

Find the volume, or solidity, of solids having the following dimensions, and give each result in higher denominations: —

1. $4\frac{1}{2}$ in. by $3\frac{2}{3}$ in. by $6\frac{1}{2}$ in.
2. 3.25 ft. by 4.5 ft. by 7.05 in.
3. $8\frac{1}{3}$ yd. by $6\frac{3}{4}$ ft. by $7.2\frac{1}{2}$ ft.
4. $10\frac{3}{8}$ yd. by $3\frac{1}{12}$ ft. by 4.2 in.

5. Of a solid 4 yd. 2 ft. 6 in. by 3 ft. 4 in. by 2 ft. 9 in.

6. Of a cube whose edge is 2 yd. 2 ft. 9 in. The edge $9.3\frac{1}{2}$ ft.

7. Of a solid whose end is $6\frac{2}{3}$ ft. square, and length 20.75 ft.

8. How many blocks, each one cubic foot, can be put in a box whose inside dimensions are $8\frac{2}{3}$ ft. by $7\frac{3}{4}$ ft. by 6 ft.?

9. How many loads of earth, each 1 cu. yd., are removed in digging a cellar 35 ft. 9 in. long, $20\frac{1}{2}$ ft. wide, 6 ft. deep?

10. How many cu. ft. of air in a room 18 ft. 6 in. long, 12 ft. 4 in. wide, 10 ft. 3 in. high? If the dimensions were twice as great?

11. At $4.25 a cord, what is the value of a pile of wood 21 ft. 4 in. long, 8 ft. wide, 6 ft. high? If it were 8 ft. high?

12. How many boxes of grapes 8 in. by 6 in. by 4 in. can be packed in a box 3 ft. by 2 ft. by 16 in. in the clear?

13. How many perches of masonry in the walls of a stone building 45 ft. long, 36 ft. wide, 24 ft. high, and 1 ft. thick?

14. How many perches of stone are required to inclose a lot 20 rd. long and 12 rd. wide with a wall 5 ft. high, 2 ft. thick?

15. What is the cost of leveling a street 600 ft. long and 50 ft. wide, averaging 4.5 ft. above grade, at $.37\frac{1}{2}$ per cu. yd.?

16. How many board feet in 10 joists 18 ft. long, 8 in. wide, and 3 in. thick? 20 ft. long and 10 in. by $3\frac{1}{2}$ in.?

17. What is the cost of 20 planks 18 ft. long, 10 in. wide, $2\frac{1}{2}$ in. thick, at $2.50 per hundred feet?

18. At $25 a thousand, what will it cost to floor two rooms, each $20\frac{1}{4}$ ft. by 11 ft., with boards $1\frac{1}{2}$ in. thick?

19. How many bushels will fill a bin that is 10 ft. 3 in. long, 5.5 ft. wide, $4\frac{1}{4}$ ft. deep?

20. How many bushels of grain will fill a bin $10\frac{1}{4}$ ft. long, 5 ft. wide, 4 ft. deep? How many bushels of apples?

21. At $\$1.87\frac{1}{2}$ a bushel, what is the value of the wheat that can be placed in a bin $12\frac{1}{2}$ ft. long, 6 ft. 3 in. wide, 4.5 ft. deep?

22. A bin $7\frac{1}{2}$ ft. long, 5 ft. wide, 4 ft. deep is half full of Schuylkill white-ash coal. What is it worth at $5.85 a ton?

23. How many gallons will a tank that is 6 ft. long, 4 ft. wide, 3 ft. deep contain? How many barrels?

24. How many cu. ft. are required to hold 50 hhd.?

25. There are 3 in. of water in a cellar 20 ft. front by 25 ft. long. How many gallons are there? How many hhd.?

26. If the solidity of a rectangular body is 2000 cubic feet, the length 8 ft., and the breadth 6 ft., what is its height?

27. The volume is 15 cu. ft., the length 18 ft., the breadth 15 in. What is the height? If the volume is 5 cu. yd. 2 cu. ft.?

28. If a pile of wood contains $20\frac{3}{4}$ cd., and is $41\frac{1}{2}$ ft. long, 8 ft. wide, how high is it? If $62\frac{1}{4}$ ft. long?

29. What is the length of each of 50 joists 10 in. by 3 in., whose solidity is 250 cu. ft.? Whose solidity is $291\frac{2}{3}$ cu. ft.?

30. What length of a board 8 in. wide contains 9 board feet? Of a joist 8 in. by 3 in. contains 3 cu. ft.?

31. What must be the depth of a bin to contain 240 bushels, the length being $10\frac{3}{4}$ ft., and the width $6\frac{2}{3}$ ft.?

32. What must be the width of a tank 6 ft. long, 5 ft. 6 in. deep, to contain 25 bbl. of water? To contain $15\frac{1}{2}$ hhd.?

Review Problems.

1. Change $\frac{1}{12}$ of $4\frac{4}{5}$ times $\frac{6}{7}$ mi. to the fraction of a foot.

2. What part of a sq. ch. are $\frac{7}{16}$ of $5\frac{1}{2}$ times $3\frac{4}{5}$ A.?

3. What decimal part of a ton are $2\frac{1}{2}$ times $\frac{1}{5}$ of $9\frac{3}{5}$ lb.?

4. Find the value in lower denomination of $\frac{7}{9}$ of 2.25 A.

5. If a bu. of wheat makes 49 lb. of flour, how many sacks of flour, each $24\frac{1}{2}$ lb., will 300 bu. of wheat fill?

6. How many bbl. of pork can be cut from 150 hogs averaging 225 lb. each? What is it worth at $\$7.87\frac{1}{2}$ per cwt.?

7. At $\$.12\frac{1}{2}$ per yd., what would it cost to fence a tract of land 1 mi. 25 rd. long and 275 rd. wide?

8. A farmer bought 75 A. 8 sq. rd. of land at $12.50 per sq. ch., and sold it at $\$3.87\frac{1}{2}$ per sq. rd. Find the gain.

9. Find the cost of a dozen watch-cases, each $2\frac{1}{4}$ oz., at $\$.87\frac{1}{2}$ per pwt., and $18.75 each for making.

10. If a watch loses 8 min. 24 sec. in a week, how much does it lose every hour? How much every minute?

11. If $12\frac{3}{8}$ A. of land are bought at $200 an acre, and sold in lots 6.5 rd. by $10\frac{2}{3}$ rd., at $125, what are the profits?

12. If I pay for a 3-ounce gold watch-case that weighs only 56 pwt., what am I overcharged for it, at $\$.87\frac{1}{2}$ per pwt.?

13. If I buy 5 bbl. of salt at $\$2\frac{1}{2}$ per bbl., and sell it at $\frac{3}{4}$ of a cent per lb., how much do I gain?

14. Find the number of rings, each $4\frac{1}{8}$ pwt., that can be made from a bar of gold weighing 1 lb. avoirdupois.

15. How many cannon-balls, each 66 lb. $10\frac{2}{3}$ oz., can be made from 1 T. of iron, making no allowance for waste?

16. How long is a degree of a circle whose circumference is 1200 feet? How long is a minute? A second?

17. How many gal. does a milkman dishonestly make out of 64 qt. of pure milk by putting $\frac{1}{2}$ gi. of water in every pint?

18. If a pint of water weighs a pound, what is the weight of a rain-fall of 1 inch over an acre of ground?

19. How many shingles are needed to make a roof $42\frac{3}{16}$ feet by 25 ft. 8 in., if a bundle of 500 covers $6\frac{1}{4}$ sq. yd.?

20. A dealer paid $3.75 for $18\frac{3}{4}$ yd. of ribbon, and sold it for

badges, each $\frac{1}{8}$ yd.; at \$.12$\frac{1}{2}$. Find the gain.

21. If 6 T. 10 cwt. 50 lb. of hay cost \$111, what will be the cost of 4 T. 15 cwt. 12$\frac{1}{2}$ lb., at the same rate?

22. Which is the heavier, how much, and why, — a pound of gold, or a pound of lead? An oz. of gold, or an oz. of lead?

23. How many tin quarts of peas in 3 bu. 3$\frac{1}{2}$ pk.?

24. How many pounds Troy equal 200 lb. avoirdupois? 200 gallons liquid measure are how many gallons dry measure?

25. If \$1 of standard gold weighs 24$\frac{4}{5}$ gr., how many tons do 1000000 gold-dollar coins weigh?

26. A miner sold a nugget of gold weighing 1$\frac{3}{4}$ lb. avoirdupois at \$15$\frac{1}{4}$ per ounce Troy. What did he receive for it?

27. How many bu. of wheat will fill an empty hogshead?

28. The capacity of a tank is 63 cu. ft. How many gallons of water will it contain? How many bu. of wheat?

29. A coal-dealer bought 175 T. of coal at \$5.75 per long ton, and sold it at \$6.25 per short ton. Find the gain.

30. How many more than 32 quarts of water are needed to fill a box which exactly holds a bushel of wheat?

31. How many quarts, liquid measure, are in 2 bu. 2 pk. 2 qt. of beans? How many dry quarts, even measure? Heaped?

32. From $\frac{3}{7}$ of 9$\frac{3}{4}$ mi. + 28.06 rd. take 3$\frac{1}{2}$ rd. 5 yd.

33. A grocer buys 3$\frac{3}{4}$ bu. of cranberries at \$3$\frac{1}{2}$ per bu., and sells them at 16$\frac{2}{3}$ cents a quart, tin measure. Find his gain.

34. From a quarter-section of land were sold a farm of 75 A. 150.5 sq. rd., and another 50 A. 5 sq. ch. 30$\frac{1}{4}$ sq. yd. What remained?

35. What length of time must elapse from 9 o'clock A. M., May 10, 1876, till 10 o'clock 30 min. P. M., November 10?

36. The American Revolution began April 19, 1775, and continued 7 yr. 9 mo. 1 da. When was it concluded by treaty?

37. Find the difference between 25 rd. 5 yd. 2 ft. 11 in. and 24 rd. 5 in. From the sum of 2 rd. 16 ft. 11 in. and 3 rd. 5 in. take 1 rd. 5 yd. 1 ft. 11 in.

38. In what time does the earth make one hundred revolutions around the sun? Express the time in lower denominations.

CHAPTER V.

PERCENTAGE.

Section I.

Definitions and General Cases.

1. What is $\frac{1}{100}$ of \$100? $\frac{2}{100}$? $\frac{5}{100}$? $\frac{10}{100}$? .25? .50? .75?
2. How much is $\frac{3}{100}$ of \$100? Of 200 feet? .08 of 500 tons?
3. What part of \$100 is \$1? What part are \$7? \$17?
4. How many hundredths of \$100 are \$6? \$7? \$8$\frac{1}{2}$?
5. How many hundredths of \$300 are \$15? \$30? \$50?
6. How many hundredths of any number equal that number?
7. How many hundredths of any number equal $\frac{1}{2}$ of it? $\frac{1}{4}$?
8. What part of 100 hundredths are 50 hundredths? 25 hundredths?

Definitions.

63. ***Per Cent.*** means *by the hundred.*

It is a contraction of the Latin *per centum.*

Thus, *5 per cent.* means *5* of every *100;* 12$\frac{1}{2}$ per cent. is 12$\frac{1}{2}$ hundredths, or 12$\frac{1}{2}$ of every 100; 33$\frac{1}{3}$ per cent. is .33$\frac{1}{3}$, or 33$\frac{1}{3}$ of every 100, etc.

64. Any ***Per Cent.*** of a number or quantity is so many hundredths of that number or quantity.

Thus, 5 per cent. of \$25 is .05, or $\frac{5}{100}$, of \$25; 6 per cent. of 50 bushels is .06, or $\frac{6}{100}$, of 50 bushels, etc.

65. The ***Sign of Per Cent.*** is %. It is read *per cent.*

Thus, 6% is read 6 per cent.; 8$\frac{1}{3}$% is 8$\frac{1}{3}$ per cent.; $\frac{1}{2}$%, $\frac{1}{2}$ per cent.

66. ***Percentage*** is the process of computing by the hundred.

67. Since per cent. is a number of *hundredths,* it is usually expressed in the form of a decimal; but it may be expressed either as a decimal or a common fraction. Thus,

1% is .01, or $\frac{1}{100}$.	$6\frac{1}{2}$% is .065, $\frac{6\frac{1}{2}}{100}$, or $\frac{13}{200}$.
25% is .25, $\frac{25}{100}$, or $\frac{1}{4}$.	$12\frac{1}{2}$% is .125, $\frac{12\frac{1}{2}}{100}$, or $\frac{1}{8}$.
100% is 1.00, $\frac{100}{100}$, or 1.	$\frac{1}{2}$% is .005, $\frac{\frac{1}{2}}{100}$, or $\frac{1}{200}$.
125% is 1.25, $\frac{125}{100}$, or $1\frac{1}{4}$.	$\frac{3}{4}$% is .0075, $\frac{\frac{3}{4}}{100}$, or $\frac{3}{400}$.

Exercises.

1. Express in the form of decimals: —

4%; 10%; 25%; 75%; 100%; 125%; $12\frac{1}{2}$%; $37\frac{1}{2}$%; $8\frac{1}{3}$%; $16\frac{2}{3}$%; $66\frac{2}{3}$%; $106\frac{1}{4}$%; $118\frac{3}{4}$%; $\frac{1}{2}$%; $\frac{1}{3}$%; $\frac{3}{4}$%; $\frac{3}{8}$%; $100\frac{5}{16}$%.

2. Express in the form of decimals and of fractions: —

5%; 20%; 50%; 200%; 250%; $2\frac{1}{2}$%; $62\frac{1}{2}$%; $87\frac{1}{2}$%; $31\frac{1}{4}$%; 56%; $33\frac{1}{3}$%; $66\frac{2}{3}$%; $112\frac{1}{2}$%; $\frac{1}{4}$%; $\frac{3}{8}$%; $\frac{5}{6}$%; $100\frac{6}{8}$%; .5%; $37\frac{1}{2}$%.

3. Express as rate with the sign % : —

.06; .09; .18; .25; .50; .75; 1; $.12\frac{1}{2}$; $.18\frac{3}{4}$; $.31\frac{1}{4}$; $.56\frac{1}{4}$; $.08\frac{1}{3}$; $.16\frac{2}{3}$; 1.33; $.66\frac{2}{3}$; $.01\frac{1}{2}$; $.02\frac{1}{4}$; $.04\frac{1}{8}$; $1.05\frac{3}{8}$; $2.07\frac{7}{8}$; $.00\frac{1}{2}$; $.00\frac{2}{3}$; $.00\frac{3}{4}$; $.00\frac{1}{5}$; $.00\frac{2}{5}$; $1.00\frac{5}{6}$; $1.00\frac{1}{8}$; $2.00\frac{7}{10}$; 1.25; 2; 2.50.

4. What per cent. of a number is $\frac{1}{3}$ of it?

68. ***Analysis.*** — Since the whole of any number is 100%, or $\frac{100}{100}$, of itself, $\frac{1}{3}$ of the number is $\frac{1}{3}$ of $\frac{100}{100}$, or $\frac{33\frac{1}{3}}{100}$, or $33\frac{1}{3}$% of it.

Express as per cent. $\frac{1}{2}$ of any number; $\frac{1}{4}$ of it; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{10}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{7}{8}$; $\frac{5}{12}$; $\frac{3}{16}$; $\frac{3}{20}$; $\frac{7}{20}$; $\frac{4}{25}$; $\frac{7}{25}$; $1\frac{1}{4}$; $1\frac{1}{2}$; $2\frac{1}{3}$; $2\frac{3}{4}$.

69. In the operations and applications of percentage, at least three terms or elements are considered: the *Base*, the *Rate*, and the *Percentage*.

If any two elements are given, the third may be found.

70. The ***Base*** is the number of which the per cent., or number of hundredths, is taken.

Thus, in the expression 5% of \$250, the *base* is \$250, since it is the number of which 5 hundredths are to be taken.

71. The ***Rate*** is the number of hundredths taken.

Thus, in the expression 5 per cent. of \$25, the *rate* is 5 hundredths.

The *Rate Per Cent.* is the decimal or the fraction denoting the number of hundredths taken; as 6% is .06 or $\frac{6}{100}$.

72. The ***Percentage*** is the result obtained by taking any per cent. of the base.

Thus, in the expression 5% of \$80 is \$4, the *base* is *\$80*, the *rate per cent.* is *.05*, and the *percentage* is *\$4*.

73. The ***Amount*** is the sum of the base plus the percentage.

Thus, if the base is \$80, and the percentage is \$4, the *amount* is the *sum of \$80 + \$4*, or *\$84*.

74. The ***Difference*** is the remainder of the base less the percentage.

Thus, if the base is \$80, and the percentage is \$4, the *difference* is the *remainder of \$80 — \$4*, or *\$76*.

Exercises.

1. Express in both decimal and fractional form the amount of 1 at —

4%; 15%; 50%; 75%; 100%; 125%; $6\frac{1}{4}$%; $8\frac{1}{3}$%; $12\frac{1}{2}$%; $18\frac{3}{4}$%; $66\frac{2}{3}$%; $112\frac{1}{2}$%; $156\frac{1}{4}$%; $\frac{1}{2}$%; $\frac{1}{3}$%; $\frac{3}{4}$%; $\frac{4}{5}$%; $\frac{7}{8}$%; $100\frac{3}{10}$%.

2. Express in both fractional and decimal form the difference of 1 at —

6%; 18%; 35%; 64%; 95%; $2\frac{1}{6}$%; $6\frac{1}{4}$%; $8\frac{1}{3}$%; $12\frac{1}{2}$%; $20\frac{3}{8}$%; $40\frac{4}{5}$%; $66\frac{2}{3}$%; $\frac{1}{3}$%; $\frac{1}{4}$%; $\frac{1}{2}$%; $\frac{2}{5}$%; $\frac{3}{8}$%; $\frac{7}{10}$%; $\frac{9}{10}$%; $\frac{9}{20}$%.

3. Express in both forms the amount and the difference of 1 at —

3%; 11%; 47%; 90%; 100%; $1\frac{1}{4}$%; $2\frac{1}{8}$%; $5\frac{4}{5}$%; $10\frac{7}{8}$%; $14\frac{3}{5}$%; $30\frac{3}{10}$%; $41\frac{2}{3}$%; $62\frac{1}{2}$%; $93\frac{3}{4}$%; $\frac{1}{2}$%; $\frac{3}{4}$%; $\frac{2}{3}$%; $\frac{7}{8}$%; $\frac{7}{20}$%; $\frac{5}{10}$%.

CASE I.

To Find the Percentage when the Base and the Rate are given.

1 What is 5% of \$80?

75. ***Analysis.*** — Since 5% is $\frac{5}{100}$, 5% of \$80 is $\frac{5}{100}$, or $\frac{1}{20}$, of 80, which is 4. Hence, etc.

2. What is 7% of 200? Of 300? Of \$400? Of 500 A.?
3. Find 8% of 15; 9% of \$30; $12\frac{1}{2}$% of 65 T.; $16\frac{2}{3}$% of 20 bu.
4. What is the amount of \$200 increased by 5% of itself?

76. Analysis. — Since 5% is $\frac{5}{100}$, 5% of \$200 is $\frac{5}{100}$, or $\frac{1}{20}$, of \$200, which is \$10; and \$200 + \$10 are \$210. Hence, etc.

5. What is the amount of 400 ft. increased by 5% of itself? Increased by 10%? By 20%? 50%? $12\frac{1}{2}$%? $33\frac{1}{3}$%?

6. What is the difference of 600 gal. diminished by 4% of itself? By $6\frac{1}{4}$%? By $16\frac{2}{3}$%? 20%? $37\frac{1}{2}$%? $66\frac{2}{3}$%?

7. A farmer owned 80 A. of land, and sold 20% of it. How many did he sell? How many had he left?

8. A grocer sold \$150 worth of goods on Monday, and on Tuesday 10% more. What was the amount of his sales on Tuesday?

Principle.

The percentage of any number is the product of that number multiplied by the rate per cent.

Written Exercises.

77. Example. — What is 75% of 800 yards? Find the amount and the difference.

SOLUTION.

Base × *rate %* = *percentage.*

800 yd. × *.75* = *600 yd.* Or,

800 yd. × $\frac{3}{4}$ = *600 yd.*

800 yd. + *600 yd.* = *1400 yd., the amount.*

800 yd. − *600 yd.* = *200 yd., the difference.*

EXPLANATION. — Since 75% of any number is .75 of that number, 75% of 800 yd. is the product of 800 yd. × .75, which is *600 yd.* Or,

Since 75% of any number is $\frac{75}{100}$, or $\frac{3}{4}$, of that number, 75% of 800 yards is $\frac{3}{4}$ of 800 yd., or the product of 800 yd. × $\frac{3}{4}$, which is *600 yd.*, the percentage required.

Since the amount is the sum of the base plus the percentage, 800 yd. + 300 yd., or *1400 yd.*, is the amount required. Also,

Since the difference is the remainder of the base less the percentage, 800 yd. — 600 yd., or *200 yd.*, is the difference required.

78. Rule to Find the Percentage, the Amount, and the Difference when the Base and the Rate are given.

I. *Multiply the base by the rate %. The product will be the percentage.* Or,

Take such part of the base as the given rate is of 100%.

II. *Add the percentage to the base. The sum will be the amount.*

III. *Subtract the percentage from the base. The remainder will be the difference.*

79. FORMULAS. — *Base* × *rate %* = *percentage.*
Base + *percentage* = *amount.* | *Base* − *percentage* = *difference.*

Note. — When the base and the rate % are given to find the percentage, two factors are given to find their product.

Problems.

Find —

1. 3% of 875; 4%; 5%.
2. 4% of 1250; 5%; $6\frac{1}{2}$%.
3. 5% of 137.5; $6\frac{1}{4}$%; .$7\frac{1}{2}$%.
4. $6\frac{1}{2}$% of $250\frac{1}{2}$; $8\frac{1}{3}$%; .$12\frac{1}{2}$%.
5. $12\frac{1}{2}$% of 1200 bu.; 116%.
6. $118\frac{3}{4}$% of 4800 yd.; $133\frac{1}{3}$%.
7. $\frac{1}{2}$% of \$7500; $\frac{3}{4}$%; .$1\frac{7}{8}$%.
8. $\frac{2}{3}$% of $\frac{5}{6}$; $\frac{3}{4}$% of $\frac{9}{20}$ yd.
9. $\frac{1}{4}$% of \$$8600\frac{7}{8}$; 250% of $\frac{7}{12}$ yr.; 500% of $3\frac{3}{8}$ mi.
10. A dealer bought some wheat for \$750, and sold it at a profit of 4%. What did he gain? What did he get for it?
11. From a hogshead of molasses $5\frac{7}{9}$% were drawn out. How many gallons were drawn out? How many remained?

Find the amount of —

12. \$4500 at 5%; $6\frac{1}{4}$%.
13. 500 gal. at 7%; $8\frac{1}{3}$%.
14. $625\frac{1}{4}$ ft. at 8%; $12\frac{1}{2}$.

Find the difference of —

15. \$.50 at 10%; $12\frac{1}{2}$%; $\frac{1}{2}$%.
16. $31\frac{1}{4}$ gal. at $2\frac{1}{2}$%; $16\frac{2}{3}$%; $\frac{7}{8}$%.
17. $\frac{7}{8}$ at $\frac{7}{8}$%; .75 at .75%; 1.25%.

18. If a man, who received \$1650 a year, had his salary increased 10%, to what did his salary then amount?
19. A merchant bought 3 pieces of muslin, each $33\frac{1}{2}$ yd., and $6\frac{1}{4}$% of it was damaged. How many yards were salable?
20. An agent paid \$2050 for a city lot, which in 4 years increased in value $116\frac{2}{3}$%. What was it then worth?
21. From a farm of 1000 A. $12\frac{1}{2}$% were sold to one man, and $16\frac{2}{3}$% of the remainder to another. How many acres remained?
22. A captain owned $\frac{5}{8}$ of a vessel, and sold $37\frac{1}{2}$% of his share for \$10500. What part of the vessel did he then own? What was his share worth?

CASE II.

To Find the Rate when the Base and the Percentage are given.

1. What per cent. of 75 is 25?

80. Analysis. — Since 75 is 100% of itself, 25, which is $\frac{1}{3}$ of 75, is $\frac{1}{3}$, or $\frac{1}{3}$, of 100%, which is $33\frac{1}{3}$% of 75. Hence, etc.

2. What part of 45 is 9? What per cent. of 45 is 9?

3. What part of \$50 are \$$12\frac{1}{2}$? What per cent. of \$50 are \$$12\frac{1}{2}$?

4. What per cent. of 30 lb. are 15 lb.? Of 60 da. are 45 da.?

5. What % of \$5 are \$.25? Of \$10 is \$$\frac{1}{4}$? Of \$$1\frac{1}{2}$ are \$$2\frac{1}{2}$?

6. What % of $\frac{3}{4}$ is $\frac{1}{4}$? Of $\frac{7}{8}$ is $\frac{3}{4}$? Of $\frac{8}{9}$ is $2\frac{2}{3}$? Of $\frac{8}{15}$ is $4\frac{4}{5}$?

7. John had 50 ct., and spent $37\frac{1}{2}$ ct. What % did he spend?

8. If I pay \$7 for the use of \$100 for a year, what rate % do I pay? If I pay \$8? If \$$7\frac{3}{10}$?

81. Principle.

The rate % is the quotient of the percentage divided by the base.

Written Exercises.

82. Example. — What per cent. of 600 yd. are 450 yd.?

SOLUTION.

Percentage ÷ base = rate %.

450 yd. ÷ 600 yd. = .75, or 75%.

Or, *450 yd.* $= \frac{450}{600}$, or $\frac{3}{4}$ of *100%* = *75%*.

EXPLANATION. — Since the percentage is the product of the base multiplied by the rate, the rate is the quotient of the percentage, 450 yd., divided by the base, 600 yd., which is .75, or *75%*. Or, 450 yd., the percentage, is $\frac{450}{600}$, or $\frac{3}{4}$, of the base, which is 100% of itself; therefore, $\frac{3}{4}$ of 100% is *75%*, the rate required.

83. Rule to Find the Rate when the Base and the Percentage are given.

Divide the percentage by the base. The quotient will be the rate per cent. Or,

Take such part of 100% *as the percentage is of the base.*

84. FORMULA. — *Percentage ÷ base = rate %.*

Note. — When the percentage and the base are given to find the rate %, the product of two factors and one of them are given to find the other.

Problems.

What per cent. of —

1. 300 is 20? 50? $37\frac{1}{2}$?
2. 450 is 25? 75? $87\frac{1}{2}$?
3. \$40 are \$6? $\$2\frac{1}{2}$? \$50?
4. \$5 are \$.50? 5 ct.? $\$6\frac{1}{4}$?
5. 25 lb. are $5.3\frac{3}{4}$ lb.? $10\frac{7}{8}$ lb.?
6. 50 gal. are 5 gal. 2 qt.? $6\frac{1}{4}$ gal.?
7. 12 yd. 2 ft. are 7 yd. $1\frac{4}{5}$ ft.?
8. $2\frac{1}{2}$ is $\frac{3}{4}$? $\frac{4}{5}$ is $\frac{7}{8}$? $\frac{8}{9}$ is $6\frac{2}{3}$?
9. What per cent. is 5 of 500? Is $37\frac{1}{2}$ of $337\frac{1}{2}$? \$.50 of \$4?
10. What % are 60 da. of a common year? Of 1 leap yr.?
11. What % are 750 lb. of 1 T. 5 cwt.? 19 rd. 1 yd. of 10 mi.?
12. $\frac{3}{4}$ of an acre is what per cent. of an acre? $\$\frac{1}{2}$ of $\$2\frac{1}{2}$?
13. $\frac{4}{5}$ of your money is what per cent. of your money?
14. What % of a number is $\frac{1}{4}$ of it? $\frac{2}{3}$ of it? $\frac{7}{8}$ of it? $\frac{9}{10}$ of it? .06 of it? $.12\frac{1}{2}$ of it? $.37\frac{1}{2}$ of it? $.93\frac{3}{4}$ of it?
15. The bread made from a barrel of flour weighs 264.6 lb. What per cent. more than the flour does the bread weigh?
16. If from a farm of 160 A. a man sells $\frac{1}{2}$ at one time, and $\frac{1}{2}$ the remainder at another time, what per cent. of the farm remains?

CASE III.

To Find the Base when the Rate and the Percentage are given.

1. 8 is 25% of what number?

85. Analysis. — Since 25%, or $\frac{25}{100}$, of some number is 8, 1%, or $\frac{1}{100}$, of the number is $\frac{1}{25}$ of 8, or $\frac{8}{25}$; and 100%, or $\frac{100}{100}$, of the number, or the number, is 100 times $\frac{8}{25}$, which are $\frac{800}{25}$, or 32. Hence, etc.

2. 15 is $\frac{1}{10}$ of what number? 10% of what number?
3. 20 is $\frac{3}{8}$ of what number? $37\frac{1}{2}$% of what number?
4. 35 is $6\frac{1}{4}$% of what number? $8\frac{1}{2}$%? $12\frac{1}{2}$%? 100%?
5. \$75 is 12.5% of how much? $\$\frac{7}{8}$ is 1.5% of what?
6. 25 lb. are $6\frac{1}{4}$% of how many? $8\frac{1}{3}$%? 150%? 200%?
7. Of what number are 17.5 lb. $10\frac{2}{3}$%? $\frac{1}{4}$%? $3\frac{1}{2}$%? .5%?

8. If I pay \$12 a year for the use of some money at the rate of 6%, how much money do I use?

86. Principle.

The base is the quotient of the percentage divided by the rate.

Written Exercises.

87. *Example.*—225 acres are 75% of what number?

SOLUTION.

Percentage ÷ rate % = base.

225 acres ÷ .75 = 300 A.

Or, $225 \div \frac{75}{100} = \frac{225}{1} \times \frac{4}{3} = 300$ *A.*

EXPLANATION.—Since the percentage is the product of the base multiplied by the rate, the base is the quotient of the percentage, 225 A., divided by the rate %, .75, which is 300, or *300 A.* Or,

Since the percentage is $\frac{75}{100}$, or $\frac{3}{4}$, of the base, the base must be the quotient of the percentage, 225 A., divided by $\frac{3}{4}$, which is 300, or *300 A.*, the base required.

88. Rule to Find the Base when the Rate and the Percentage are given.

Divide the percentage by the rate per cent. The quotient will be the base.

89. FORMULA. — *Percentage ÷ rate % = base.*

Note. — When the percentage and the rate % are given to find the base, the product of two factors and one of them are given to find the other.

Problems.

Of what number is —

1. 88 10%? $12\frac{1}{2}$%?
2. 1089 $8\frac{1}{3}$%? 125%?
3. .75 $6\frac{1}{4}$%? $112\frac{1}{2}$%?
4. $22\frac{2}{5}$ $\frac{1}{2}$%? $2\frac{1}{2}$%? $6\frac{1}{4}$%?
5. \$$62\frac{1}{2}$ 7%? $\frac{7}{8}$%? $\frac{7}{10}$%?
6. $6.2\frac{1}{2}$ rd. $1\frac{1}{4}$%? $18\frac{3}{4}$%?
7. 37 bu. 2 pk. $\frac{1}{2}$%? 350%?
8. 15 mi. 8 rd. $\frac{3}{4}$%? 400%?
9. $25\frac{5}{7}$ mi. are 4% of how many mi.? $\frac{1}{2}$%? $37\frac{1}{2}$%?
10. 22.4 lb. are $\frac{7}{8}$% of what? $66\frac{2}{3}$%? $112\frac{1}{2}$%? 200%?
11. $33\frac{1}{3}$% a long ton is $16\frac{2}{3}$% of how many pounds?
12. Find the base, if the rate is 5% and the percentage $2\frac{2}{3}$.

13. A clerk spends \$1250, and has 25% of his salary left. What is the amount of his salary? $\frac{3}{4}$ of $87\frac{1}{2}$% of it?

14. If a farmer sells 42 A. 130 sq. rd. of land, and has 75% of his farm left, what is the extent of his farm?

15. A man owning 65% of a mill sells 40% of his share for \$21000. What is the value of the mill?

16. A grocer sold 25% of a hhd. of sugar to one man, and $33\frac{1}{3}$% of the remainder to another, and had 250 lb. left. What did the hhd. weigh?

CASE IV.

To Find the Base when the Amount, or the Difference, and the Rate are given.

1. What number increased 25% of itself amounts to 75?

90. Analysis. — Since the number is 100% of itself, 75 is 100% plus 25%, or 125% of the number. If 75 is 125%, or $\frac{125}{100}$, of some number, 1%, or $\frac{1}{100}$, of the number is $\frac{1}{125}$ of 75, or $\frac{3}{5}$; and 100%, or $\frac{100}{100}$, of the number is 100 times $\frac{3}{5}$, or $\frac{300}{5}$, or 60. Hence, etc.

2. 125 is 25 more than what? $12\frac{1}{2}$% more? $6\frac{1}{4}$%? $2\frac{1}{2}$%?

3. $43\frac{1}{3}$ lb. are $8\frac{1}{3}$% more than how many? $66\frac{2}{3}$%?

4. What number increased by $6\frac{1}{4}$% is $87\frac{1}{2}$ ft.? Is $212\frac{1}{2}$ ft.?

5. What number diminished by 20% of itself equals 60?

91. Analysis. — Since the number is 100% of itself, 60 is 100% less 20%, or 80%, of the number. If 60 is 80%, or $\frac{80}{100}$, of some number, 1%, or $\frac{1}{100}$, of the number is $\frac{1}{80}$ of 60, or $\frac{3}{4}$; and 100%, or $\frac{100}{100}$, of the number is 100 times $\frac{3}{4}$, or $\frac{300}{4}$, or 75. Hence, etc.

6. \$250 are 50% less than what? 25%? $12\frac{1}{2}$%? $6\frac{1}{4}$%?

7. 40 mi. are $12\frac{1}{2}$% less than how many miles? $16\frac{2}{3}$%?

8. What number diminished by $2\frac{1}{2}$% equals $3\frac{1}{3}$? $\frac{2}{5}$? 9.5?

9. A horse costs \$175, which was 25% less than the cost of a carriage. What was the cost of the carriage?

10. A grocer sold tea at $\$1.12\frac{1}{2}$ a pound, which was 25% more than the cost. What did he pay for it?

Written Exercises.

92. *Example* 1.—What number increased by 35% is 2430?

SOLUTION.

Amount ÷ (*1* + *rate %*) = *base.*

2430 ÷ (*1* + *.35* = *1.35*) = *1800.*

Or, $2430 \div \frac{135}{100} = \frac{\overset{90}{\cancel{2430}}}{1} \times \frac{20}{\cancel{27}} = 1800.$

EXPLANATION. — Since any number is 100% of itself, a number increased by 35% of itself is 100% + 35%, or 1 + .35, of itself. Hence, if 2430 is 1.35 times a number, the number is the quotient of 2430 ÷ 1.35, which is *1800.* Or,

Since the amount is $\frac{135}{100}$ of the base, the base must be the quotient of 2430 divided by $\frac{27}{20}$, which equals $\frac{20}{27}$ of 2430, or *1800*, the base required.

93. *Example* 2. — What number diminished by 17% of itself is 3735?

SOLUTION.

Difference ÷ (*1* — *rate %*) = *base.*

3735 ÷ (*1* — *.17* = *.83*) = *4500.*

Or, $3735 \div \frac{83}{100} = \frac{\overset{45}{\cancel{3735}}}{1} \times \frac{100}{\cancel{83}} = 4500.$

EXPLANATION. — Since any number is 100% of itself, a number diminished by 17% of itself is 83% of itself, or .83 of itself. Hence, if 3735 is .83 of a number, the number is the quotient of 3735 ÷ .83, which is *4500.* Or,

Since the difference is $\frac{83}{100}$ of the base, the base must be the quotient of 3735 ÷ $\frac{83}{100}$, which equals $\frac{100}{83}$ of 3735, or *4500*, the base required.

94. Rule to Find the Base when the Amount, or the Difference, and the Rate are given.

I. *Divide the amount by* 1 *plus the rate %. The quotient will be the base.* Or,

II. *Divide the difference by* 1 *minus the rate %. The quotient will be the base.*

95. FORMULAS. — 1. *Amount* ÷ (1 + *rate %*) = *base.*

2. *Difference* ÷ (1 — *rate %*) = *base.*

Problems.

What number increased by	What number diminished by
1. 10% of itself is 418?	5. 20% of itself is 3400?
2. $12\frac{1}{2}$% is \$242.50? \$$\frac{3}{8}$?	6. $6\frac{1}{4}$% is \$45.30? \$$\frac{5}{8}$? .$4\frac{2}{7}$?
3. $8\frac{1}{3}$% is $28\frac{3}{5}$ bu.? $3\frac{1}{13}$?	7. $62\frac{1}{2}$% is 40 bu.? $1\frac{7}{8}$ bu.?
4. $\frac{1}{4}$% is \$20.05? $4\frac{1}{2}$? .$8\frac{1}{50}$?	8. $\frac{1}{2}$% is $447\frac{3}{4}$ ft.? $\frac{2}{5}$? .$49\frac{3}{4}$?

9. The amount is $21\frac{1}{4}$ A., and the rate $6\frac{1}{4}\%$. Find the base.

10. The difference is 31.5 gal., and the rate 5%. Find the base.

11. $855\frac{5}{9}$ is $7\frac{7}{9}\%$ more than what number? $\frac{7}{9}\%$? $11\frac{1}{9}\%$?

12. $16\frac{2}{3}\%$ of $34\frac{2}{7}$ mi. is $8\frac{1}{3}\%$ less than what? $87\frac{1}{2}\%$? $\frac{3}{4}\%$?

13. A clerk, after paying out $62\frac{1}{2}\%$ of his salary for necessary expenses, had \$656.25 remaining. What was his salary?

14. In two years the profits of a firm were \$2800, and the profits the second year were $33\frac{1}{3}\%$ more than the first. Find each year's profits.

15. A drover sold 20 horses for \$3210, which was $33\frac{1}{3}\%$ more than he paid for them. Find the cost of each horse.

Applications of Percentage.

96. The three elements, *base, rate, percentage,* enter very extensively into business transactions. They receive *different names,* according to the nature of the transaction, the *principles* remaining always the *same.*

97. The principles of percentage apply to transactions of two classes: —

1. Those in which *time* is *not* considered; as Profit and Loss, Commission, etc.

2. Those in which time *is considered;* as Interest, Discount, etc.

All questions and problems arising in such transactions may be solved according to one or more of the four rules and formulas of percentage.

Section II.

PROFIT AND LOSS.

1. At a gain of 10% on the cost of an article, what part of the cost equals the gain?

98. ***Analysis.*** — Since the gain is 10%, it is $\frac{10}{100}$, or $\frac{1}{10}$, of the cost. Hence, etc.

2. At a gain or loss of 5%, what part of the cost equals the gain or loss? 25%? 50%? $6\frac{1}{4}$%? $33\frac{1}{3}$%?

3. I bought a barrel of flour for \$8, and sold it at a gain of 20%. What was the gain? The selling price?

99. Analysis. — Since the gain was 20%, it was $\frac{20}{100}$, or $\frac{1}{5}$, of the cost; and $\frac{1}{5}$ of \$8 is \$1.60. The selling price was the sum of \$8 and \$1.60, or \$9.60. Hence, etc.

4. For how much must a grocer sell tea that costs \$.80 a pound to gain 10%? 20%? 25%? $12\frac{1}{2}$%? $37\frac{1}{2}$%?

5. At what price must an article that costs \$10 be sold to gain 20%? 100%? 150%? To lose 10%? $12\frac{1}{2}$%? 25%?

6. A dealer bought boots at \$4 a pair, and sold them at \$5 a pair. What per cent. did he gain?

100. Analysis. — Since he bought them at \$4 a pair, and sold them at \$5, he gained the difference between \$5 and \$4, which is \$1. Since on \$4 the gain is \$1, or $\frac{1}{4}$ of the cost, the gain per cent. is $\frac{1}{4}$ of 100%, or 25%. Hence, etc.

7. If a grocer buys sugar at $12\frac{1}{2}$ cents a pound, and sells it at 10 cents, what per cent. loss does he suffer?

8. What % gain or loss is made by selling an article for $\frac{4}{5}$ of its cost? For double its cost? $\frac{3}{4}$ of its cost? $\frac{1}{2}$?

9. A merchant gained 20% by selling velvet at a profit of \$2 a yard. What was the cost?

101. Analysis. — Since the gain is 20%, then $\frac{20}{100}$, or $\frac{1}{5}$, of the cost equals the gain, which is \$2. If \$2 is $\frac{1}{5}$ of the cost, $\frac{5}{5}$ of the cost, or the cost, is 5 times \$2, or \$10. Hence, etc.

10. If a grain merchant makes 25 cents a bushel on wheat by selling it at an advance of $12\frac{1}{2}$%, what is the cost?

11. A drover, by selling a cow for \$15 more than he paid for her, gained $33\frac{1}{3}$%. What was the cost of the cow?

12. By selling a watch for \$75, a jeweler made 25%. What was the cost of the watch?

102. Analysis. — Since the gain was 25%, then $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the gain, which, added to $\frac{4}{4}$ of the cost, is $\frac{5}{4}$ of the cost, or \$75. If \$75 is $\frac{5}{4}$ of the cost, $\frac{1}{4}$ of the cost is $\frac{1}{5}$ of \$75, or \$15, and $\frac{4}{4}$ of the cost, or the cost, is 4 times \$15, or \$60. Hence, etc.

13. By selling a sewing-machine for $75, an agent made 25% profit. For what should he have sold it to lose 25%?

14. A pawnbroker sold two watches at $75 each. On one he gained 25%, and on the other he lost 25%. How much did he gain or lose on both?

Definitions.

103. Profit and ***Loss*** are terms used to denote the gain and the loss in business transactions.

The profit or the loss is commonly estimated at a certain rate per cent. on the *cost*, or the sum of money *invested*.

104. The corresponding terms of Profit and Loss and Percentage are —

1. The ***Cost*** or ***Sum Invested*** is the *Base*.
2. The ***Rate*** % of ***Profit*** or ***Loss*** is the *Rate*.
3. The ***Profit*** or ***Loss*** is the *Percentage*.
4. The ***Selling Price,*** or cost plus the profit, is the *Amount*.

The ***Selling Price,*** or cost minus the loss, is the *Difference*.

Written Exercises.

105. I bought a farm for $4500, and sold it at an advance of 16⅔%. What was the gain? What was the selling price?

SOLUTION.

Cost × *rate %* = *profit*.
$4500 × .16⅔ = $750.
Cost + *gain* = *sell. price*.
$4500 + 750 = $5250.

EXPLANATION. — Since the gain is 16⅔% of the cost, it is .16⅔ of $4500, or the product of $4500 × .16⅔, which is $750, the gain required. And,

Since the cost is $4500, and the gain is $750, the sum of $4500 + $750, or $5250, is the selling price required.

106. FORMULAS. — *Cost* × *rate %* = *profit or loss*.
Cost + *profit* = *selling price*. | *Cost* — *loss* = *selling price*.

Problems.

Find the profit or loss, and selling price, of —

1. Butter bought at $.18¾, and sold at a loss of 20%.

2. Drugs costing \$125, and sold at a profit of 175%.

3. Goods bought for \$162.50, and sold at an advance of 18%.

4. An investment of \$5000 on which $16\frac{2}{3}\%$ was lost.

5. A ton of hay that cost $\$1\frac{7}{8}$ per cwt., at a profit of $16\frac{2}{3}\%$.

6. A merchant sold some remnants at a reduction of 25% on cost price. What did he get for those which cost $\$.12\frac{1}{2}$ per yd.?

7. If I buy 50 yd. of cloth for \$150, what must be my selling price per yard to realize 10% gain? $12\frac{1}{2}\%$? $16\frac{2}{3}\%$?

8. If a horse costs \$175, and his keeping costs \$12.50, for what must he be sold to gain $18\frac{3}{4}\%$? $31\frac{1}{4}\%$?

9. A provision dealer bought a barrel of pork for \$12.50, and sold it at a profit of $12\frac{1}{2}\%$. What did he get per pound?

10. A barrel of flour was bought at $3\frac{1}{2}$ cents a pound, and sold at a profit of $37\frac{1}{2}\%$. For what was the barrel sold?

11. A drover bought a horse for \$225, and sold it at a gain of 10% to a physician, who sold it at a loss of 20%. What did the physician receive for him?

12. I bought two building lots for \$750 each, and sold one at a gain of $37\frac{1}{2}\%$, and the other at a loss of $37\frac{1}{2}\%$. How much did I gain or lose on both?

107. An agent bought a sewing-machine for \$75, and sold it for $\$65.62\frac{1}{2}$. What was the loss %?

SOLUTION.

$\$75 - \$65.62\frac{1}{2} = \$9.37\frac{1}{2}$ *loss.*

Loss ÷ cost = rate %.

$\$9.37\frac{1}{2} \div \$75 = .12\frac{1}{2}$, *or* $12\frac{1}{2}\%$.

$.12\frac{1}{2}$, or $12\frac{1}{2}\%$, the rate required.

EXPLANATION. — Since the loss is the product of the cost multiplied by the rate %, the rate per cent. must be the quotient of the loss, $\$9.37\frac{1}{2}$, divided by the cost, \$75, which is

108. FORMULA. — *Profit or loss ÷ cost = rate %.*

Problems.

Find the rate per cent. of profit or loss on —

1. Tea bought at \$.75 a pound, and sold at $\$.87\frac{1}{2}$.

2. Tea bought at $\$.87\frac{1}{2}$ a pound, and sold at \$.75.

3. Oil-stock bought at \$10 a share, and sold at \$1.25.

4. Paper, cost \$2.50 a ream, selling price \$.20 a quire.

5. 8 bbl. of apples at \$3.50, that were sold for \$20.

6. An investment of \$5000, which realized but \$4500.

7. Goods costing \$.31¼ were marked \$.37½. Find the % advance.

8. If muslin bought at \$.12½ a yd. is sold at \$.15, what is the gain per cent.? If sold at 16¼ cents? At 13¾ cents? At \$.17½?

9. What % gain or loss is made by buying tea at 80 cents a pound, and selling it at \$1? At 90 c.? At 75 c.?

10. A merchant marked prints that cost him 9 cents a yard, to be sold at 12½ cents. What was the % advance on cost?

11. A dealer bought 125 tons of coal at \$5.12½, and sold it all at a profit of \$64.06¼. What was the gain %?

12. A merchant bought some silks for \$750, which was \$250 less than their real value, and sold them at 25% above their real value. What % did he make?

109. By selling corn at 15 cents above cost, a grain dealer cleared 20 per cent. What was the cost?

SOLUTION.
Profit ÷ rate % = cost.
15 c. ÷ .20 = 75 c.

EXPLANATION. — Since the profit is the product of the cost multiplied by the rate per cent., the cost must be the quotient of the profit, 15 cents, divided by the rate %, .20, which equals *75 cents*, the cost required.

110. FORMULA. — *Profit or loss ÷ rate % = cost.*

Problems.

Find the cost of —

1. Sugar sold at a profit of 1 c. a lb., at 8⅓% gain.

2. Linen at 5% below cost, at a loss of \$37.50.

3. A book sold so as to gain \$.25, or 2½% of the cost.

4. 25 lb. raisins, sold at a loss of 10%, or 2½ c. per lb.

5. Lumber sold at a profit of \$3.50 per M., or 22½% gain.

6. A broker invested in stocks and lost \$1200, which was 8⅓% of his capital. What sum did he invest?

7. If a stock farmer sells his cattle for $1750 advance, at a profit of $16\frac{2}{3}\%$, what did they cost him?

8. By putting up a block of small houses for $1500 less than cost, a builder lost $7\frac{1}{7}\%$. What was the contract price?

9. By selling 55 bbl. of apples at an advance of $48.12\frac{1}{2}$, I gained 28%. What was the cost per barrel?

10. How many tons of iron rails must be bought at $56 a ton, and sold at an advance of $18\frac{3}{4}\%$, to clear $126?

111. By selling a city lot for $750, a real-estate dealer cleared 20%. What was the cost?

SOLUTION.

Selling price ÷ (*1* + *rate %*) = *cost.*

$750 ÷ (*1* + *.20* = *1.20*) = *$625.*

EXPLANATION. — Since the cost is 100% of itself, the cost increased by 20% of itself is 120% of itself, or 1.20 times itself; and if $750 is 1.20 times the cost, the quotient of $750 divided by 1.20, or *$625*, is the cost required.

112. FORMULAS. — 1. *Selling price* ÷ (1 + *rate %*) = *cost.*

2. *Selling price* ÷ (1 — *rate %*) = *cost.*

Problems.

Find the cost of —

1. Oats sold at $.40 a bu., at a gain of 20%.

2. A carriage sold for $245, at a loss of $8\frac{1}{3}\%$.

3. A buffalo-robe sold for $18.06\frac{1}{4}$, at a gain of $6\frac{1}{4}\%$.

4. Fruit sold for $252.31\frac{1}{4}$, at a loss of $8\frac{1}{4}\%$; of $6\frac{1}{4}\%$; 10%.

5. A watch sold for $74, at a loss of $7\frac{1}{2}\%$; of $4\frac{4}{9}\%$; 20%.

6. By selling a house and lot for $7762.50, a man gained 15%. What was the cost?

7. A dealer sold 20 bbl. of flour for $7\frac{1}{2}$ per barrel, and gained $6\frac{1}{4}\%$. What did the flour cost him?

8. A grocer sold coffee at 40 cents a pound, and cleared 25%. For what should he have sold it to clear $12\frac{1}{2}\%$?

9. By selling a table for $22.50, a man gained $12\frac{1}{2}\%$. What % would he have gained or lost by selling it for $21.00?

10. A dealer sold a piece of carpeting for 15% more than cost, and made $12\frac{3}{8}$ on it. What did he get for it?

11. If by selling muslin at 16 cents a yard, a merchant gains $14\frac{2}{7}$%, how much % will he gain or lose by selling at $.12\frac{1}{2}$?

12. A grocer sold sugar for $.15 a pound, at a gain of 20%. What % would he have gained by selling it at $.16\frac{1}{4}$?

13. A Western farmer sold two farms, 100 A. each, at $3.50 an acre, and by so doing gained 10% on one farm, and lost 10% on the other. What did he gain or lose on both?

14. What must a grocer mark tea that costs 75 c. a pound, so that he may fall 10%, and still make a profit of 20%?

15. What must a builder ask for a house that costs $3200, so as to fall $12\frac{1}{2}$%, and yet realize a gain of $37\frac{1}{2}$%?

Section III.

COMMISSION.

1. An agent charged 5% for selling goods to the amount of $250. How much did he receive?

2. What rate per cent. does a real-estate agent charge, who receives $20 for selling a house at $4000?

3. An attorney received $30 for collecting a note, charging 5% of the amount. What sum did he collect?

4. How much will remain after a collector takes out a fee of $1\frac{1}{2}$% from $300? Of $2\frac{1}{4}$% from $500?

5. An agent received $206, with which he was to buy flour, after deducting 3% for his fees. What did the flour cost?

113. Analysis. — Since he received 100% of the cost of the flour plus 3% of the cost for his fee, he received 103% of the cost of the flour.

If 103% of the cost of the flour is $206, 1% of the cost is $\frac{1}{103}$ of $206, or $2, and 100% of the cost is 100 times $2, or $200. Hence, etc.

6. If a person receives $630 with which to buy goods after retaining 5% of the cost, how much can he expend for goods?

Definitions.

114. An ***Agent, Factor,*** or ***Commission Merchant,*** is a person who buys or sells merchandise, or transacts other business for another.

Note. — The person for whom an agent transacts business is called the *Principal.* The person who sends goods to be sold is called the *Consignor,* or *Shipper.* The person to whom goods are sent to be sold is called the *Consignee,* or *Correspondent.* A quantity of goods sent to a commission merchant to be sold is called a *Consignment.*

115. ***Commission*** is the sum or percentage allowed an agent or commission merchant for transacting business.

116. Commission is usually computed at some rate per cent. on the *amount of sales,* or the *sum collected* by the agent.

117. The ***Net Proceeds*** of a sale or other transaction is the sum left after the commission and other charges are deducted.

118. A ***Broker*** is a person who buys and sells gold, stocks, bonds, etc., for a commission called *Brokerage.*

119. The corresponding terms of Commission or Brokerage and Percentage are: —

1. The ***Amount of Sales*** or ***Sum Collected*** is the *Base.*
2. The ***Rate % of Commission*** is the *Rate.*
3. The ***Commission*** or ***Brokerage*** is the *Percentage.*
4. The ***Amount of Sales*** or the ***Sum Collected,*** plus the commission, is the *Amount.*

The ***Amount of Sales*** or the ***Sum Collected,*** minus the commission, is the *Difference,* or *Net Proceeds.*

Written Exercises.

120. *Example.* — Find the commission on a sale of flour for \$3725, at $3\frac{1}{2}\%$.

SOLUTION.

Amount of sale $\times$ *rate %* = *com.*

$\$3725 \times .03\frac{1}{2} = \$130.37\frac{1}{2}$.

EXPLANATION. — Since the rate of commission is $3\frac{1}{2}\%$, or $.03\frac{1}{2}$, of the amount of the sale, the product of \$3725 $\times$ $.03\frac{1}{2}$, or $\$130.37\frac{1}{2}$, is the commission required.

121. FORMULA. — *Amount of sales* × *rate %* = *commission.*

Problems.

Find the commission or brokerage on —

1. The sale of flour for \$7250, at $2\frac{1}{4}\%$; $2\frac{3}{4}\%$.

2. The purchase of \$5000 in gold, at $\frac{1}{4}\%$. At $\frac{3}{8}\%$.

3. The sale of furniture for \$625.25, at $4\frac{1}{2}\%$. At $1\frac{4}{5}\%$.

4. The sale of 325 bbl. of pork at \$11.75. Commission $2\frac{1}{2}\%$.

5. 75 shares of stock bought at $\$112\frac{1}{2}$ a share. Brokerage $\frac{1}{4}\%$.

6. A factor sold 125 bales of cotton, averaging 420 lb., at $16\frac{3}{8}$ cents a pound. What was the amount of his commission at $1\frac{1}{2}\%$?

7. An agent sold 20 sacks of coffee, averaging $56\frac{1}{2}$ lb., @ $\$.31\frac{1}{4}$, commission $1\frac{1}{4}\%$. What did he remit to his principal?

8. An attorney collected 75% of a note for \$500, and charged $7\frac{1}{2}\%$ commission. What amount did he pay over?

122. An agent received \$270 for furnishing a school-house for \$2250. What was the rate %?

SOLUTION.

Commission ÷ amt. of sales = rate %.
\$270 ÷ 2250 = .12, or 12%.

EXPLANATION.—Since the commission is the product of the amount of sales multiplied by the rate % of commission, the rate % must be the quotient of the commission, \$270, divided by the amount of sales, \$2250, or *.12*, or *12%*, the rate % required.

123. FORMULA. — *Commission ÷ amount of sales = rate %.*

Find the rate of commission or brokerage —

9. If \$31.25 are paid for collecting a debt of \$1250.

10. If \$165 are paid for buying a farm for \$7500. For \$8250.

11. If a broker charges \$8.25 for selling stock worth \$6600.

12. If a factor charges $\$40.68\frac{3}{4}$ for selling 50 bales of cotton, averaging 420 pounds, at $15\frac{1}{2}$ cents a pound.

13. If I pay a broker \$19.05 for buying 60 shares of railroad stock, at $\$95\frac{1}{4}$ a share. If I pay $\$28.57\frac{1}{2}$.

14. An agent collected \$450, retained his commission, and paid over \$425.25. What was his rate of commission?

15. A commission-merchant sold 125 bbl. of apples at \$3½ per bbl., and 200 bbl. of potatoes at \$2.87½ per bbl. If his commission amounted to \$35.43¾, what rate % did he charge?

124. What amount of sales does an agent make if I pay him \$42.90 commission, at the rate of 2¾%?

SOLUTION.

Commission ÷ rate % = amount of sales.

\$42.90 ÷ .02¾ = \$1560.

EXPLANATION.—Since the commission is the product of the amount of sales × rate %, the quotient of the commission, \$42.90, divided by the rate per cent., .02¾, is *\$1560*, the amount of sales.

125. What is the amount of sales if the net proceeds is \$3152.50, and the commission 3%?

SOLUTION.

Net proceeds ÷ (1 — rate %) = amt. of sales.

\$3152.50 ÷ (1 — .03 = .97) = \$3250.

EXPLANATION. — Since the net proceeds is the amount of sales less the commission, it is 100% of the sales — 3% of the sales, or 97%, or .97 of the sales; and the amount of the sales must be the quotient of the net proceeds, \$3152.50, divided by .97, or *\$3250*, the amount of sales required.

126. FORMULAS. — 1. *Commission ÷ rate % = amount of sales.* 2. *Net proceeds ÷ (1 — rate %) = amount of sales.*

Find the amount of sales —

16. If \$25.20 commission are paid at 2⅓%; at 4½%.

17. If \$17¼ are charged as brokerage at ⅛%; at ¼%; ⅜%.

18. If \$93.50 are received for selling muslins at 2%; 2.5%.

19. If the net proceeds are \$2653.75, and the commission 3½%.

20. If the commission is 5%, and the net proceeds are \$1125.

21. An auctioneer received \$135 for selling a house at 1½% commission. What did the owner receive?

22. A tax receiver charged 5% commission, and paid over \$2422.50. What amount had he collected?

23. At 2½%, a wool buyer received \$311.85 commission for buying wool, at \$.44 per lb. How many pounds did he buy?

24. A starch manufacturer paid a grain buyer $254.37\frac{1}{2}$, at $2\frac{1}{2}\%$, for buying wheat at $1.87\frac{1}{2}$ per bu. How many bushels did he buy? What was the amount of the agent's bill?

25. An agent received 20% for selling sewing-machines at $75, and paid over $1440. How many did he sell?

127. What is the sum invested if $5250 are sent to an agent to be invested after deducting his commission of 5%?

SOLUTION.

Sum remitted ÷ (*1* + *rate %*) = *sum invested.*

$5250 ÷ (*1* + *.05* = *1.05*) = *$5000.*

EXPLANATION. — Since the sum sent, or remitted, includes both the sum invested and the commission, it is 100% of the sum invested + 5% of the sum invested, or 105%, or 1.05 of the sum invested; and the quotient of the sum remitted, $5250, divided by 1.05, or *$5000*, is the sum invested.

128. FORMULA. — *Sum remitted* ÷ (1 + *rate %*) = *sum invested.*

Find the sum or amount invested —

26. If $4356.90 are remitted, 3% com. to be deducted.

27. If $3843.75 are remitted, $2\frac{1}{2}\%$ com. to be deducted.

28. If $7518.75 are received, $\frac{1}{4}\%$ brokerage to be retained.

29. If $\frac{1}{8}\%$ brokerage is to be deducted from $6408, and the balance is to be invested in stocks.

30. If I send an agent $2756 with which to buy books, after deducting 4%, how much can he lay out in books?

31. How many pounds of coffee at $18\frac{3}{4}$ cents a pound can be bought for $230.62\frac{1}{2}$, after deducting $2\frac{1}{2}\%$ commission?

32. How many shares of stock at $95 a share can a broker buy for $7708.80, after taking out $\frac{3}{8}\%$ brokerage?

33. I sent to my agent in Chicago $5178\frac{3}{5}$, which he is to invest in flour at $5\frac{3}{5}$ a bbl., after retaining $2\frac{3}{4}\%$ commission. How many barrels did he buy? Find his commission.

34. I sent to my agent $7687.50 to expend for cotton, and to pay his commission of $2\frac{1}{2}\%$ on the purchase price of the cotton. How many pounds at $37\frac{1}{2}$ cents could he buy? What was his commission?

Review Problems.

1. Find $37\frac{1}{2}\%$ of 37 bu. 16 qt.; $\frac{4}{5}\%$ of $\frac{1}{16}$ lb. avoir.

2. What % of $\frac{2}{3}$ is $\frac{3}{8}$? $1\frac{1}{2}$? 7.5? Of 25 is $18\frac{3}{4}$? $66\frac{2}{3}$? $87\frac{1}{2}$?

3. Find the number of which \$50 is $12\frac{1}{2}\%$; $\frac{1}{2}\%$; 5%; $100\frac{1}{2}\%$.

4. A gold ring is 21 carats fine. What % of it is gold?

5. $12\frac{1}{2}$ lb. is $\frac{2}{3}\%$ of what? $\frac{2}{5}\%$? 250%? 7.5%? 100%?

6. James is 16 years old, and $37\frac{1}{2}\%$ of his age is $33\frac{1}{3}\%$ of his brother's age. How old is his brother?

7. The standard of United States gold coins is 9 parts pure gold, and 1 part alloy. What % is gold? What % alloy?

8. If gold is at a premium of $11\frac{1}{9}\%$, what is the gold value of a dollar note? A 5-dollar note?

9. When gold is at a premium of $11\frac{1}{9}\%$, what is the currency value of an eagle? A quarter-eagle?

10. A boy spent 60% of his money for clothing, and 25% of the remainder for books, and then had \$6. What had he at first?

11. What % of a long T. is a short T.? Of a short T. is a long T.?

12. A farmer sold 80 A. from his farm, and this was $6\frac{1}{4}\%$ less than the number remaining. How many acres had he at first?

13. A flour dealer bought 480 bbl. of flour, and sold $\frac{3}{7}$ of it. What % of it did he retain?

14. A merchant received \$332.50 for a bill from which he threw off 5% for cash. What was the amount of the bill?

15. A horse and carriage are worth \$525. If the carriage is worth \$125, its value is what % of the value of the horse?

16. If a grocer buys tea at $\$.87\frac{1}{2}$ a lb., and sells it at $\$1.12\frac{1}{2}$, for what % of the cost does he sell it?

17. A man's expenses are 75% of his income, and $33\frac{1}{3}\%$ of his income is $4\frac{1}{2}\%$ of his property, valued at \$18000. What are his expenses?

18. A drover sold some cows and sheep for \$4590, and received for his sheep 70% of what he received for his cows. What did he get for his cows?

19. A man drew from bank 50% of his deposit, and paid $87\frac{1}{2}\%$ of the amount drawn out for cattle worth $218.75. What amount was deposited at first?

20. A man owned $\frac{3}{4}$ of a woolen-mill, and sold $33\frac{1}{3}\%$ of his share for $8750. What part did he still own, and what was it worth?

21. I deposited in bank $5000, and drew out 20% of it, and afterwards 20% of the remainder. If I again deposit $33\frac{1}{3}$ of what I have drawn out altogether, what will I have in bank?

22. A man sold 125 A. of land for $11718\frac{3}{4}$, at a loss of $6\frac{1}{4}\%$. What price per acre did he pay for it?

23. A grocer bought 750 lb. of coffee at $31\frac{1}{4}$ c. per lb., and paid $.50 per hundred for freight, and $1 for cartage. What % did he make by selling it @ $.37$\frac{1}{2}$ per lb.?

24. I realized a gain of $62\frac{1}{2}\%$ by selling a lot for $1462.50, which was 10% less than my asking price. What did I ask for it? What did it cost? What did I gain?

25. A commission merchant bought some produce, paid 21\frac{1}{2}$ for freight, and charged $61.25 commission. If his entire bill to his employer was $1852.75, what was the rate of commission?

26. After deducting $3\frac{1}{2}\%$ commission, and $18.90 for freight and cartage, a commission merchant remitted to the shipper $4275.35. Find the amount of the sales?

27. A grocer bought a hogshead of New Orleans molasses containing $87\frac{1}{2}$ gallons. $7\frac{1}{7}\%$ leaked out, and he sold 24% of the remainder. How much remained?

28. A grocer bought 3 hhd. of molasses, 63 gal. each, at 80 c. a gal., and sold it at 10% below cost. What did he lose?

29. I remit to my agent 941.37\frac{1}{2}$. After paying $3.50 for insurance, he retains $2\frac{1}{2}\%$ commission. How many barrels of apples at $3 can he purchase?

30. If a milkman adds 1 pint of water to every gallon of milk that he sells, what % of the price does he dishonestly make?

31. I bought a piece of cloth containing 36 yd. at $2.50 per yard. 12 yd. of it was damaged so that I had to sell it at a loss of 50%. The remainder I sold at 20% profit. What did I gain or lose on all?

32. A factor received \$4070 to invest in cotton at 16 cents a pound. How many pounds can he purchase if he charges $1\frac{3}{4}\%$ commission on the purchase price?

33. A real-estate agent bought a house, charging $2\frac{1}{2}\%$ for his services. He sent in a bill for \$4612.50. Find the cost of the house.

34. A lawyer collected 75% of a note of \$1250, and charged $6\frac{1}{4}\%$ commission. Find his commission and the amount paid over.

35. An agent bought 20 bbl. of pork at $\$10.37\frac{1}{2}$ per bbl., and paid \$6.50 for insurance and \$4.50 cartage. His commission being $2\frac{1}{4}\%$, what was the total amount of his bill to his employer?

36. An administrator of an estate paid the heir $\$9439\frac{1}{2}$, charging $1\frac{7}{8}\%$ commission and \$137.50 for other expenses. What was the value of the estate?

37. A grocer bought 3 hhd. of molasses at $\$.87\frac{1}{2}$ per gal., and sold one hhd. at 75 cents, and the other two at \$1. Find the whole profit and the rate of profit.

38. When cloth is worth \$3.30 a yard, what must be the asking price, that, if an abatement of $12\frac{1}{2}\%$ be made, there will be a profit of $16\frac{2}{3}\%$?

39. A grocer paid \$3 for 4 lb. of tea, and sold 3 lb. for \$4. What % did he make on the tea that he sold?

40. A merchant bought 6 pieces of silk, each 48 yards, at \$72 a piece, and sold it at \$2 a yard. Find the gain and the gain %.

41. A sold a pair of horses to B, and gained $16\frac{2}{3}\%$; B sold them to C for $\$367\frac{1}{2}$, and lost $12\frac{1}{2}\%$. What did A pay for them?

42. The net profits of my business in 2 years were \$3570, and the profits the second year were 10% greater than the first year. What were the profits each year?

Review Questions.

1. Of what is the term ***Per Cent.*** a contraction? What does it mean? What is any *per cent.* of a number or quantity? What is ***Percentage?*** How is the percentage of any number usually expressed? Express 5 per cent. in three different ways.

2. Name the terms or elements that enter into all the operations and applications of percentage. What is the ***Base?*** The ***Rate?*** The

rate per cent.? The ***Percentage?*** Give an example of each. Define ***Amount*** and ***Difference,*** and give an example of each.

3. State the principle of finding the percentage. Repeat the rule, and write the formula. The principle of finding the rate. Repeat the rule, and write the formula. The principle of finding the base. Repeat the rule, and write the formula. Explain how to find the base when the amount or the difference and the rate are given. Repeat the rules, and write the formulas.

4. Define ***Profit*** and ***Loss.*** Upon what is the profit or loss commonly estimated? To what principles are those involved in profit and loss similar? Name the *corresponding terms* of Profit and Loss and Percentage. Write the formula to find the profit or loss. The rate per cent. The formulas to find the cost. The selling price.

5. What is a ***Commission Merchant?*** A ***Broker?*** Who is the principal? What is commission? Brokerage? How are commission and brokerage usually computed? What are net proceeds? What principles are involved in all computation in commission and brokerage. Name the *corresponding terms* of Commission and Percentage. Explain how to find commission. To find the rate. The proceeds. The sum invested. Write each formula.

Section IV.

INTEREST.

1. At 5%, what decimal part of the money borrowed equals the sum paid for its use? At 6%? At 7%? At 9%?

2. At 7% a year, how many dollars must be paid for the use of $100 for 1 year? For 2 yr.? For $2\frac{1}{2}$ yr.? 3 yr.?

3. How much should be paid for the use of $200 for 3 years at 5% a year? At 6% a year? At 7%?

4. If $6 are paid for the use of $100 for 1 year, what rate per cent. is charged? If $7 are paid? $8?

5. If $14 are paid for the use of $200 for 1 year, what is the rate per cent.? If $24 for the use of $300?

6. If $24 are paid for the use of $100 for 4 years, what is the rate per cent. for 1 year? If $32 are paid for $100?

7. If I lend $300, and charge 6% a year for its use, how much will the borrower owe me at the end of 3 years?

8. If $200 be loaned at a yearly rate of 6% for its use, to how much will it amount in 4 years?

Definitions.

129. ***Interest*** is the sum paid for the use of money.

130. The ***Principal*** is the sum for the use of which interest is paid.

131. The ***Amount*** is the sum of the principal and the interest.

132. ***Simple Interest*** is interest on the principal only.

133. The ***Rate of Interest*** is the rate per cent. of the principal paid for its use for one year.

134 The ***Legal Rate of Interest*** is the rate allowed by law.

Note. In some States a higher rate than the legal rate is allowed, if agreed upon in contract by the parties. The legal rate of interest in the various States is as follows:

In Illinois, Louisiana, Michigan, ...	5%
In Ark., Conn., Del., Dist. of Col., Ind., Iowa, Kan., Ky., Maine, Md., Mass., Minn., Miss., Mo., N. H., N. J., N. M., N. Y., N. C., Ohio, Oregon, Pa., R. I., Tenn., Tex., Vt., Va., W. Va., Wis.,	6%
In Arizona, Cal., Georgia, Idaho, Neb., Nev., N. D., Okla., S. C., S. D., Wash., ...	7%
In Ala., Col., Fla., Utah, Wyo., ...	8%
In Montana, ...	10%
In England and France, 5%; *in Canada and Ireland,* . .	6%

In the problems following, if no particular rate is named or implied, 6% is to be understood.

135. In the operations of interest, *four* elements or quantities are considered: —

The *Principal,* the *Rate % per Annum,* the *Time,* the *Interest,* and the *Amount.*

136. The corresponding terms of Interest and Percentage are,

1. The ***Principal*** is the *Base.*

2. The ***Rate % per Annum*** is the *Rate.*

3. The ***Interest*** is the *Percentage.*

4. The ***Amount,*** or the sum of the principal and the interest, is the *Amount.*

137. The new element, ***Time,*** is introduced in interest to be considered always in connection with the rate % per annum.

If any three of the terms or elements are given, the fourth may be found.

CASE I.

To Find the Interest and the Amount.

1. At 5% for 1 year, what decimal part of the principal equals the interest at 6%? At 7%? At 8%?

2. At 5% for 3 yr., what decimal and fractional part of the principal equals the interest? At 6%? At 8%? At 10%?

3. What is the interest of $50 for 1 year at 6%?

138. Analysis. — Since the interest of any principal at 6% for one year is $\frac{6}{100}$, or $\frac{3}{50}$, of the principal, the interest of $50 for 1 year is $\frac{3}{50}$ of $50, or $3.

4. What is the interest of $60 for 1 yr. at 4%? 5%? 6%?

5. Find the interest of $80 for 1 yr. at 8%. 9%. 10%.

6. What is the interest of $50 for 2 yr. at 6%?

139. Analysis. — Since the interest of any principal is $\frac{6}{100}$, or $\frac{3}{50}$, of the principal, the interest of $50 at 6% for 1 yr. is $\frac{3}{50}$ of $50, or $3; and the interest for 2 years is 2 times $3, or $6.

7. Find the interest of $30 for 2 yr. at 5%. 3 yr. at 6%.

8. Find the interest of $40 for 4 yr. at 7%. For 5 yr. at 8%.

9. Find the interest of $40 at 5% for 2 yr. 6 mo.

140. Analysis. — Since the interest of any principal at 5% for one year is $\frac{5}{100}$, or $\frac{1}{20}$, of the principal, the interest of $40 at 5% for one year is $\frac{1}{20}$ of $40, or $2; and for 2 years 6 mo., or $2\frac{1}{2}$ yr., the interest is $2\frac{1}{2}$ times $2, or $5. Hence, etc.

10. What is the interest of $18 for 2 yr. 2 mo. at 5%? Of $25 at 6% for 3 yr. 4 mo.? Of $50 for 4 yr. 9 mo. at 8%?

618. Principle.

The interest is the product of three factors: principal, rate %, and time expressed as years.

Written Exercises.

141. *Example.*—Find the interest and the amount of $162.50 for 3 yr. 9 mo. at 7%.

SOLUTION.

Prin. × *rate %* × *time* = *interest.*

$\$162.50 \times .07 \times 3\frac{3}{4} = \$42.6562\frac{1}{2}$.

.07	
$\$11.3750$	*Int. 1 yr.*
$3\frac{3}{4}$	
$\$42.6562\frac{1}{2}$	*Int.* $3\frac{3}{4}$ *yr.*
162.50	*Prin.*
$\$205.1562\frac{1}{2}$	*Amt.*

EXPLANATION. — Since the interest of any principal at 7% for 1 yr. is .07 of the principal, the product of $162.50 multiplied by .07, or $\$11.375$, is the interest for 1 yr.; and the interest for 3 yr. 9 mo., or $3\frac{3}{4}$ yr., is $3\frac{3}{4}$ times $11.375, or $42.6562½; and the principal, $162.50, + $42.6562½, the interest, is $\$205.1562\frac{1}{2}$, the amount required.

Note 1. — In reckoning interest, partial results need not be carried to more than four places of decimals. In *final* results, the mills, if less than 5, may be rejected, and if 5 or more, they may be considered as 1 cent.

2. In computations of interest, 30 days are a month, and 12 months or 360 days are a year.

3. In finding the difference between dates for months and days, take the number of entire months between the dates, and then add the number of days left; or, count the actual number of days between the dates.

Thus, from January 15 to July 4 is 5 mo. 20 da.; for between the dates are Feb., Mar., Apr., May, and June, or 5 months; also in Jan. 16 da. are left, and in July there are 4 da., making 20 da., or 5 mo. 20 da. in all.

142. General Rule to Find Interest.

I. *Multiply the principal by the rate per cent. The product will be the interest for* 1 *year.*

II. *Multiply the interest for* 1 *year by the time expressed as years. The product will be the interest required.*

III. *Add the principal and the interest together, and the sum will be the amount.*

143. FORMULAS. — 1. *Principal* × *rate %* × *time* = *interest.*
2. *Principal* + *interest* = *amount.*

Note. — When the principal, the rate %, and the time are given to find the interest, three factors are given to find their product.

Problems.

Find the interest of —

1. \$50 for 1 yr. 3 mo. at 5%. At 6%; 7%; 8%; 9%.
2. \$250.75 for 3 yr. 4 mo. at 4%; 5%; 7½%; 8%; 8½%.
3. \$375.50 for 5 yr. 8 mo. at 6¼%; 7%; 8½%; 10%.
4. \$1250.75 for 7 yr. 10 mo. at 8%; 9%; 10%; 12½%.
5. \$2000 for 5 yr. 7 mo. 15 da. at 5%. For 6 yr. 7 mo. 25 da.
6. \$3025.10 for 7 yr. 5 mo. 20 da. at 7$\frac{3}{10}$%. For 8 yr. 20 da.
7. \$500 at 6% for 8 mo. At 7½% for 8 mo. 10 da.; at 8¼%.
8. \$450 at 7½% for 9 mo. At 8¼% for 7 mo. 6 da.; at 7$\frac{3}{10}$%.

Find the amount of —

9. \$75 for 5 mo. at 5%. At 6%; 6¼%; 6½%; 7%; 7¼%.
10. \$87.50 for 6 mo. 10 da. at 7%. At 7¼%; 7½%; 7$\frac{3}{10}$%; 8%.
11. \$125 for 7 mo. 12 da. at 7½%. At 7¾%; 8%; 8¼%; 8½%.
12. \$137.50 for 8 mo. 18 da. at 8%. At 8⅓%; 8½%; 8⅔%; 9%.
13. \$200.75 for 6 mo. 10 da. at 6%. For 2 yr. 8 mo. 15 da.
14. \$325.40 for 7 mo. 12 da. at 7%. For 3 yr. 4 mo. 12 da.
15. \$405.25 for 8 mo. 15 da. at 8%. For 4 yr. 5 mo. 18 da.
16. \$590.75 for 9 mo. 18 da. at 9%. For 5 yr. 6 mo. 20 da.
17. \$620.62½ for 10 mo. 25 da. at 10%. For 6 yr. 7 mo. 22 da.
18. \$750.37½ for 11 mo. 26 da. at 12½%. For 7 yr. 8 mo. 25 da.
19. A man borrowed \$1250 for 1 yr. 5 mo. at 6½%. How much did he owe at the end of that time?
20. If you lend \$575.50 for 2 yr. 5 mo. at 7$\frac{3}{10}$%, how much will there be due you? How much in 3 yr. 7 mo. 6 da.?
21. What is the interest of \$75.80 from Aug. 15, 1900, to Oct. 10, 1901, at 8%? What is the amount?
22. If a note for \$450, on interest at 7%, dated Dec. 26, 1898, was paid July 5, 1901, what was the amount due?

23. If I pay \$350 a year house rent, how much would I gain or lose in 5 yr. 6 mo. by borrowing \$4000 at 7% to buy the house?

24. A man bought a house for \$7250, paying cash \$3500, and giving a mortgage for the balance, due in 4 yr. 9 mo. What was the amount due on the mortgage when paid?

25. A farmer owes a \$1000-mortgage with interest for 2 yr. 4 mo. If he pays \$750 on account, how much is still due?

26. If a man borrows \$10000 in Philadelphia, and lends it immediately in Nebraska, how much will he make by the operation in $4\frac{2}{3}$ mo.? In 1 yr. $3\frac{1}{2}$ mo.? 2 yr. 4 mo. 20 da.?

27. A London merchant borrowed £500 10 s. 6d., Jan. 1, 1900. What was the total amount due Oct. 15, 1901?

28. I bought a house for \$4500, and paid \$2000 cash, \$1000 in 9 mo., and the remainder in 1 yr. 6 mo., with interest at 6%. What was the whole amount paid for it?

29. A minor at the age of 15 yr. 6 mo. 20 da. inherited \$7500, which was put at 7% simple interest until he became 21 years of age. What amount was then due him?

30. A land speculator borrowed \$8437.50, Sept. 10, 1900, at 6%, and bought a farm at \$112.50 per acre. He sold the farm, Nov. 5, 1901, at \$125 an acre cash. What did he clear?

Different Methods.

1. *By Aliquot Parts.*

144. *Example.* — Find the interest of \$500 for 2 yr. 3 mo. 21 da. at 6%.

SOLUTION.

\$500
.06

12)\$30.00 × 2 = \$60.00 *Int. for 2 yr.*

\$2.50 × 3 = 7.50 " *3 mo.*

15 da. = $\frac{1}{2}$ mo. = 1.25 " *15 da.*

6 da. = $\frac{1}{5}$ mo. = .50 " *6 da.*

\$69.25 *Int. for given time.*

EXPLANATION. — Since the interest for 1 yr. is the product of the principal multiplied by the rate per cent., or \$30, the interest for 2 yr. is 2 times \$30, or \$60.

Since the interest for 1 mo. is $\frac{1}{12}$ the interest for a year, or \$2.50, the interest for 3 mo. is 3 times \$2.50, or \$7.50.

Since the interest for 1 mo. is \$2.50, the interest for 15 da., or $\frac{1}{2}$ of a mo., is $\frac{1}{2}$ of \$2.50, or \$1.25, and for 6 da., or $\frac{1}{5}$ of a mo., is $\frac{1}{5}$ of \$2.50, or \$.50. Hence, *\$69.25* is the interest required for 2 yr. 3 mo. 21 da.

Note. — Those who prefer to find interest by aliquot parts will find the method above given shorter than those generally used. In a majority of the problems, however, the method of changing months and days to the fraction or decimal of a year is preferable.

2. *By Six Per Cent. Method.*

145. The interest on one dollar at 6% per annum,

For	*12*	*mo., or*		*1 yr., is*	*6*	*cents, or*	*.06 of the principal.*
"	*2*	"	$\frac{1}{6}$ *of*	"	*1*	*cent, or*	*.01* "
"	*1*	"	$\frac{1}{12}$ *of*	"	$\frac{1}{2}$	"	*.005* "
"	*6*	*da., or*	$\frac{1}{5}$ *of*	*1 mo., is*	$\frac{1}{10}$	"	*.001* "
"	*1*	"	$\frac{1}{30}$ *of*	"	$\frac{1}{60}$	"	$.000\frac{1}{6}$ "

Hence the following —

146. Principles.

1. *The interest of any principal at 6% is one-half as many hundredths of the principal as there are months in the given time.* And,

2. *The interest of any principal at 6% is one-sixth as many thousandths of the principal as there are days in the given time.*

Written Exercises.

147. *Example* 1. — Find the interest of \$525 for 1 yr. 6 mo. at 6%.

SOLUTION.

1 yr. 6 mo. = 18 mo.

	\$525
$\frac{1}{2}$ *of .18 = .09,*	*.09*
Int. = .09 of prin.,	*\$47.25*

EXPLANATION. — 1 yr. 6 mo. = 18 mo. Since the interest at 6% is one-half as many hundredths of the principal as there are months in the time, the interest for 18 months must be $\frac{1}{2}$ of .18, or .09 of the principal; and the product of the principal, \$525, multiplied by .09 is *\$47.25*, the interest required.

148. *Example* 2.— Find the interest of \$525 for 93 da. at 6%.

SOLUTION.

$\frac{1}{6}$ *of* .09 = .015$\frac{1}{2}$.

Int. = .015$\frac{1}{2}$ *of prin.*

$525	
.015$\frac{1}{2}$	
8.137\frac{1}{2}$	

EXPLANATION. — Since the interest at 6% is $\frac{1}{6}$ as many thousandths of the principal as there are days in the time, the interest for 93 days must be $\frac{1}{6}$ of .093, or .015$\frac{1}{2}$ of the principal; and the product of $525 multiplied by .015$\frac{1}{2}$ is 8.137\frac{1}{2}$, the interest required.

149. *Example* 3. — Find the interest of $525 for 2 yr. 3 mo. 21 da. at 6%.

SOLUTION.

2 yr. 3 mo. = *27 mo.*

$\frac{1}{2}$ *of* .27 = .135

$\frac{1}{6}$ *of* .21 = .003$\frac{1}{2}$

Int. for time, .138$\frac{1}{2}$ *of the prin.*

$525
.138$\frac{1}{2}$
72.712\frac{1}{2}$

EXPLANATION. — The time is 2 yr. 3 mo. 21 da., or 27 mo. 21 da.

Since the interest for months at 6% is one-half, etc.

Since the interest for days at 6% is one-sixth, etc.

Hence, for 27 mo. 21 da. the interest must be .135 + .003$\frac{1}{2}$, or .138$\frac{1}{2}$, of the principal; and the product of $525 × .138$\frac{1}{2}$ is 72.712\frac{1}{2}$, the interest required.

150. Rule to Find Interest by the Six Per Cent. Method.

Multiply the principal by the decimal expressing one-half as many hundredths as there are months, and one-sixth as many thousandths as there are days, in the given time. The result will be the interest at 6 per cent.

151. FORMULA. — *Prin.* × *int. of* $1 *at* 6% = *interest at* 6%.

Note 1. — To find the interest at any other rate than 6%, find the interest at 6%, and increase or decrease this interest by such part of itself as the given rate is greater or less than 6%.

Thus, if the rate is 7%, increase the interest $\frac{1}{6}$; if 5%, decrease the interest $\frac{1}{6}$; if 4%, decrease it $\frac{1}{3}$, or $\frac{1}{3}$, etc.

2. To avoid fractions in the decimal used as the multiplier, multiply the principal by the months expressed as hundredths, and divide the product by 2; or, multiply the principal by the days expressed as thousandths, and divide the product by 6.

3. Exact or ***Accurate Interest*** requires that 365 days shall be reckoned to the year. Hence, for days take such part of 1 year's interest as the number of days is of 365 for a common year, or of 366 for a leap year. Bankers and business men, however, consider 360 days as a year.

4. Teachers who prefer the method of reckoning either by aliquot parts or the six per cent. method, may require the solution of the problems under Case I by such method as they desire.

CASE II.

To Find the Principal when the Time, the Rate, and the Interest or Amount are given.

1. What sum of money will gain \$20 in 2 years at 5%?

152. Analysis. — Since the interest of \$1 at 5% for 2 years is \$.10, \$20 is the interest of as many dollars as the number of times \$.10 are contained in \$20, or \$200. Hence, etc.

2. What principal will gain \$25 in 5 years at 5%? \$36 in 3 yr. at 6%? \$42 in 3 yr. at 7%?

3. \$45 in 3 yr. at 6%? \$104 in $6\frac{1}{2}$ yr. at 8%? At 12%?

4. What sum of money amounts to \$228 in 2 yr. at 7%?

153. Analysis. — Since the interest of \$1 at 7% for 2 yr. is \$.14, the amount of \$1 is \$1 + \$.14, or \$1.14; and \$228 is the amount of as many dollars as the number of times \$1.14 is contained in \$228, or \$200. Hence, \$200 in 2 yr. at 7% amount to \$228.

5. What principal will amount to \$533 at 5% in 6 yr.? \$630 at $6\frac{1}{2}$% in 4 years? \$828 at 8% in $4\frac{3}{4}$ yr.?

6. \$$797\frac{1}{3}$ at 4% in 5 yr. 8 mo.? \$1352 at 8% in 6 yr. 3 mo. 10 da.?

7. What principal will give an annual income of \$60 at 6%?

8. At $7\frac{3}{10}$%, what sum of money will amount to \$819 in 5 yr.?

Written Exercises.

154. Example 1. — What principal will gain \$406.25 in 5 yr. at $6\frac{1}{2}$%?

SOLUTION.

$\$1 \times .06\frac{1}{2} \times 5 = \$.32\frac{1}{2}$ *Int. on* \$1.

$\$406.25 \div \$.32\frac{1}{2} = \$1250$, *Prin.*

Or, *Int.* $\div$ (*rate* % $\times$ *time*) = *prin.*

$\$406.25 \div (.06\frac{1}{2} \times 5) = \$1250.$

EXPLANATION. — Since \$.32½ is the interest of \$1 at 6½% for 5 yr., \$406.25 is the interest of as many dollars as the number of times \$.32½ is contained in \$406.25, which is 1250 times, or \$1250. Or,

Since the interest is the product of the three factors, principal, rate, and time, the quotient of the interest,

$406.25, divided by the product of the two factors, the rate, .06½, and the time, 5 years, must give $\$1250$, the principal required.

155. *Example* 2. — What sum of money will amount to $460.25 in 4 yr. 6 mo. at 7%?

SOLUTION.

$\$1 \times .07 \times 4\frac{1}{2} = \$.31\frac{1}{2}$.

$\$1 + \$.31\frac{1}{2} = \$1.31\frac{1}{2}$, *Amt. of* $\$1$.

$\$460.25 \div \$1.31\frac{1}{2} = \$350$, *Prin.*

Or, $Amt. \div (1 + \overline{rate\ \%} \times time) = prin.$

$\$460.25 \div (1 + .07 \times 4\frac{1}{2}) = \$350.$

EXPLANATION. — Since $1.31½ is the amount of $1 at 7% for 4½ years, $460.25 is the amount of as many dollars as the number of times $1.31½ is contained in $460.25, or $350. Or,

Since the interest of $1 is the product of $1, the rate, and the time, the amount of $1 is the sum of $1 plus the product of the rate % and the time; and $460.25 is the amount of as many dollars as the number of times 1 + rate % × time is contained in $460.25, or $\$350$, the principal required.

156. Rule to Find the Principal.

I. *Divide the interest by the interest of* $1 *at the given rate for the given time.* Or,

II. *Divide the amount by the amount of* $1 *at the given rate for the given time.*

157. FORMULAS. — 1. $Int. \div (rate\ \% \times time) = principal.$

2. $Amt. \div (1 + rate\ \% \times time) = principal.$

Note.— When the interest, the rate %, and the time are given to find the principal, the product of three factors and two of the factors are given to find the third factor.

Problems.

What principal will gain —

1. $33.75 in 2 yr. 3 mo. at 5%? $78.20 in 2 yr. 9 mo. at 6%?

2. $129.15 in 3 yr. 5 mo. at 7%? $223.38 in 4 yr. 6 mo. at 8%?

3. $181.50 in 4 yr. 7 mo. at 5½%? $310.62½ in 5 yr. 11 mo. at 6¾%? $743.68⅛ in 6 yr. 6 mo. at 7½%?

4. $658 in 4 yr. 6 mo. 25 da. at 8%? In 5 yr. 5 mo. 24 da.?

What principal will amount to —

5. $483 in 2 yr. 6 mo. at 6%? $617.50 in 3 yr. 9 mo. at 8%?

6. $605.80 in 4 yr. 3 mo. at 7%? $1092.91 in 5 yr. 2 mo. at 10%? 1435.26\frac{7}{8}$ in 7 yr. 6 mo. at 8$\frac{1}{2}$%?

7. $1220 in 3 mo. 10 da. at 6%? $1645 in 4 mo. 15 da. at 7$\frac{1}{2}$%?

8. $1330.79 in 1 yr. 72 da. at 6$\frac{1}{2}$%? In 3 yr. 2 mo. 12 da. at 8%?

9. $3352.25 in 4 yr. 5 mo. 25 da. at 5%? At 6%? At 7$\frac{1}{2}$%?

10. What sum of money, at 6%, will yield an annual income of $900? Of $1125? Of $1350?

11. If the semi-annual interest of a mortgage, at 7%, is $175, what is the amount of the mortgage? If $525?

12. A note bearing interest for 93 days, at 6%, amounted to $1462.32. What was the interest? For 4 mo. 4 da.?

13. What principal on interest at 7% from Dec. 20, 1900, to Nov. 12, 1901, would amount to $4467.05, reckoning by days?

14. On the 13th of Nov., 1901, I paid 4312\frac{2}{3}$, the amount due on a mortgage given Aug. 1, 1900. What was the face of the mortgage? What interest was due?

15. What sum must I invest in bonds bearing 7$\frac{3}{10}$% interest, to yield me $157.50 semi-annually?

CASE III.

To Find the Rate when the Principal, the Time, and the Interest are given.

1. At what rate will $60 in 5 yr. gain $18 interest?

158. Analysis. — Since the interest of $60 for 5 years at 1% is $3, $18 is the interest at as many per cent. as the number of times $3 are contained in $18, or 6%.

2. At what rate % will $40 gain $6 in 3 years?

3. $60 gain $9 in 3 yr.? $75 gain $20.25 in 4 yr. 6 mo.?

4. $80 amount to $104 in 5 yr.? $136 in 8 yr.? $144 in 10 yr.?

5. $100 amount to $121 in 3 yr.? To 152.31\frac{1}{4}$ in 7 yr. 9 mo.?

6. $150 double itself in 10 years? In 20 years? Any principal in 16$\frac{2}{3}$ years? In 8 yr. 6 mo.? In 12 yr. 9 mo.?

Written Exercises.

159. *Example.*—At what rate will \$625 gain \$100 in 2 yr. 8 mo.?

SOLUTION.

$\$625 \times .01 \times 2\frac{2}{3} = \$16.66\frac{2}{3}$.

$\$100 \div \$16.66\frac{2}{3} = 6$, *or* 6%.

Or, *Int.* $\div$ (*prin.* $\times$ *time*) = *rate* %.

$\$100 \div (\$625 \times 2\frac{2}{3}) = .06$, *or* 6%.

EXPLANATION.— Since \$16.66⅔ is the interest of \$625 at 1% in 2⅔ years, \$100 is the interest at as many per cent. as the number of times \$16.66⅔ are contained in \$100, which is 6 times. Hence, the rate is 6%. Or,

Since the interest is the product of the three factors, principal, rate, and time, the rate must be the quotient of the interest, \$100, divided by the product of the two factors, principal, \$625, multiplied by the time, 2⅔ yr., which is *.06*, or *6%*, the rate required.

160. Rule to Find the Rate of Interest.

Divide the interest by the interest of the given principal for the given time at 1%.

161. FORMULA. — *Interest* $\div$ (*principal* $\times$ *time*) = *rate* %.

When the interest and the principal and the time are given to find the rate per cent., the product of three factors and two of the factors are given to find the third factor.

Problems.

At what rate per cent. will —

1. \$80 gain \$3.66⅔ in 5 mo. 15 da.? \$22.40 in 4 yr.?
2. \$497 gain \$149.10 in 3 yr. 9 mo.? \$13.80⅘ in 6 mo. 20 da.?
3. \$1382.40 gain \$246.24 in 2 yr. 4 mo. 15 da.? Gain \$41.04?
4. \$2253.75 gain \$653.58¾ in 3 yr. 7 mo. 15 da.? Gain \$326.79⅜?
5. \$1300.50 amount to \$1590.94½ in 4 yr. 5 mo. 18 da.?
6. \$3300 amount to \$4004 in 3 yr. 6 mo. 20 da.? To \$3945.33⅓? To \$3515.11⅓ in 7 yr. 1 mo. 10 da.?
7. \$225 double itself in 4 yr.? 8 yr.? 10⅔ yr.? 12½ yr.? 20 yr.?
8. What rate of interest does a man receive if he invests \$7500, and obtains from it an annual income of \$562.50?
9. A note of \$3456, dated Feb. 5, 1899. was paid Aug. 20, 1901, when the amount due was \$4114.80. What was the rate %?

CASE IV.

To Find the Time when the Principal, the Rate, and the Interest are given.

1. In what time will \$40 gain \$5 at 5%?

162. Analysis. — Since the interest of \$40 at 5% for 1 yr. is \$2, \$5 is the interest for as many years as the number of times \$2 are contained in \$5, or $2\frac{1}{2}$ yr., or 2 yr. 6 mo. Hence, etc.

In what time will —

2. \$60 gain \$12 at 5%? Gain \$18 at 6%? \$42 at 7%?
3. \$80 gain \$18 at $4\frac{1}{2}$%? Gain \$20 at $6\frac{1}{2}$%? \$36 at $7\frac{1}{2}$%?
4. \$90 amount to \$117 at 6%? To \$135 at 7%? \$138 at 8%?
5. \$100 amount to \$144 at 8%? To \$154 at 8%? \$173 at $7\frac{3}{10}$%?
6. \$150 double itself at 4%? At 5%? Any principal at 6%? At 10%?

Written Exercises.

163. Example. — In what time will \$600 gain \$86 at 6%?

SOLUTION.

\$600 × .06 = \$36 Int. for 1 yr.
\$86 ÷ \$36 = $2\frac{7}{18}$.
$2\frac{7}{18}$ yr. = 2 yr. 4 mo. 20 da.
Int. ÷ prin. × rate % = time.
\$86 ÷ \$600 × .06 = $2\frac{7}{18}$
$2\frac{7}{18}$ yr. = 2 yr. 4 mo. 20 da.

EXPLANATION. — Since \$36 is the interest of \$600 at 6% for 1 yr., \$86 is the interest for as many years as the number of times \$36 are contained in \$86, which is $2\frac{7}{18}$ times, or 2 yr. 4 mo. 20 da. Or,

Since the interest is the product of the three factors, principal, rate, and time, the time must be the quotient of the interest, \$86, divided by the product of the two factors, the principal, \$600, multiplied by the rate %, .06, which is $2\frac{7}{18}$ times, or *2 yr. 4 mo. 20 da.*, the time required.

164. Rule to Find the Time.

Divide the interest by the interest of the given principal at the given rate for 1 *year.*

165. FORMULA. — *Interest ÷ (principal × rate %) = time.*

When the interest and the principal and the rate are given to find the time, the product of three factors and two of the factors are given to find the third factor.

Problems.

In what time will —

1. \$125 gain \$25 at 5%? \$33.75 at 6%?
2. \$350.50 gain \$78.512 at 7%? \$143.70½ at 10%?
3. \$530.60 gain \$145.91½ at 7½%? \$249.382 at 8%?
4. \$6700 gain \$1648.53½ at 7$\frac{3}{10}$%? \$3266.25 at 10%?
5. \$1200 amount to \$1404 at 8%? To \$1470 at 12½%?
6. \$7250 amount to \$8555 at 1% a month? At ½% a mo.?
7. Any principal double itself at 4%? 5%? 10%? 12%?
8. How long must a note of \$750 run to give an amount of \$879.37½ at 6%? To give \$923.25 at 7%?
9. At ½% a month, how long would it take \$2500 to yield a banker \$225 interest? To yield \$337.50?
10. At 6%, in what time will the interest of \$250 equal one-half the principal? Twice the principal?

Section V.

DISCOUNT.

166. Discount is a percentage deducted from the price of goods, or from the face of a bill or an account.

167. Discount is of two kinds: *Commercial Discount* and *True Discount.*

CASE I.

Commercial Discount.

168. Commercial Discount is a certain percentage deducted from the price of goods.

Time is *not* an element in the operations of Commercial Discount.

169. The ***Net Price*** of goods is the selling price less the discount.

The ***Net Proceeds*** of an obligation is its face less the discount.

170. The operations in Commercial Discount are the same as those in Percentage in which the base and the rate are given.

171. The corresponding terms of *Commercial Discount* and *Percentage* are —

1. The ***Selling Price*** or the ***Face of an Obligation*** is the *Base.*

2. The ***Rate %*** of deduction is the *Rate.*

3. The ***Commercial Discount*** is the *Percentage.*

4. The ***Net Price,*** or ***Net Proceeds,*** is the *Difference.*

172. FORMULAS.—1. *Selling price* × *rate %* = *com. discount.*
2. *Selling price* — *com. discount* = *net proceeds.*

Problems.

1. If goods marked $350 are sold at $2\frac{1}{2}\%$ off, what is the commercial discount? What is the net price?

2. What is the net value of a bill amounting to $2250, at 5% discount? What is the commercial discount?

3. Find the net proceeds of a bill amounting to $4500, at 5% discount, and 5% additional for cash.

4. What is the commercial discount of a bill of goods invoiced at $425.50, sold on 30 days' time, at 5% off for cash?

5. Find the net cash price of flour invoiced at $7.50 per barrel, on 30 days' time, or 5% off for cash.

6. What is the net cash value of goods amounting as per bill to $1375, less discount off 15%, and 5% off for cash?

CASE II.

True Discount.

173. The ***Present Worth*** of a debt payable at a future date without interest, is such a sum as, being placed at interest, will amount to the given debt when it becomes due.

Thus, ***$100*** is the *present worth* of $106 due in 1 year, at 6%, since $100, at interest for 1 year, at 6%, will amount to ***$106.***

174. The ***True Discount*** is the difference between the face of the debt and the present worth.

Thus, \$6 is the true discount on \$106 due in 1 year, at 6%.

Time is *always* an element in the operations of True Discount.

Note. — The *true discount* is the *interest* on the *present worth* for the time between the date of the debt and the date of its maturity.

175. The operations in True Discount are the same as those in Interest in which the amount and the time and the rate are given to find the principal.

176. The corresponding terms of *Discount* and *Interest* are—

1. The ***Face*** of the debt is the *Amount.*
2. The ***Present Worth,*** or ***Proceeds,*** is the *Principal.*
3. The ***True Discount*** is the *Interest* on the present worth.

Written Exercises.

177. *Example.* — Find the present worth and the true discount of \$362.25, payable in 6 mo., at 7%.

SOLUTION.

\$1 × .07 × $\frac{1}{2}$ = \$.03$\frac{1}{2}$ *int. of* \$1.

\$1 + \$.03$\frac{1}{2}$ = \$1.03$\frac{1}{2}$ *amt. of* \$1.

\$362.25 ÷ \$1.03$\frac{1}{2}$ = \$350 *pres. worth.*

Or, *Debt* ÷ (1 + $\overline{\text{rate} \times \text{time}}$) = *pres. worth.*

\$362.25 ÷ (1 + .07 × $\frac{1}{2}$) = \$350.

\$362.25 − \$350 = \$12.25 *discount.*

EXPLANATION. — Since \$1.03½ is the amount of \$1 at 7% for 6 months, \$362.25 is the amount of as many dollars as the number of times \$1.03½ is contained in \$362.25, or \$350.

Or,

Since the amount of \$1 is the sum of \$1 plus the product of the rate and the time (619), \$362.25 is the amount of as many dollars as the number of times \$1.03½ is contained in \$362.25, or \$350, the present worth required.

178. Rule to Find the Present Worth and the True Discount.

I. *Divide the debt by the amount of* \$1 *at the given rate for the given time. The quotient will be the present worth.*

II. *Subtract the present worth from the given debt. The remainder will be the true discount.*

179. FORMULAS. — 1. *Face of debt* ÷ (1 + $\overline{\textit{rate \% × time}}$) = *pres. worth.*

2. *Face of debt — pres. worth = true discount.*

Problems.

What is the present worth, and what is the true discount of —

1. $153 due in 4 mo., at 6%? $178.50 due in 3 mo., at 8%?

2. $248 due in 5 mo., at 8%? $310.50 due in 6 mo., at 7%?

3. $469.50 due in 8 mo., at 6½%? $57.15 due in 7 mo., at 10%?

4. $1224.80 to be paid in 3 mo. 3 da., at 8%? At 1% a mo.?

5. $1590.40 payable 1 yr. 3 mo. 20 da. hence, at 8%? At 10%?

6. Of a note given Dec. 10, 1900, payable Sept. 16, 1901, amount $2210.10, at 8%?

7. What sum of ready money is equivalent to $2486.25, due 1 yr. 6 mo. hence, when money is worth 7% a year?

8. What discount should be allowed me, if I pay a debt of $3660, 2 yr. 6 mo. before it is due, money being worth 8%?

9. A merchant offered to sell goods for $4250 cash, or for $4415.25 on 3 mo. Which was the better offer, money at 6%?

10. What is the difference between the interest and the discount of $2652 for 1 yr. 6 mo., at 7%? For 3 yr. 25 da.?

11. A builder bought some lumber for $450, giving his note for 9 mo. In 3 mo. 20 da. he paid the note, the holder allowing discount at 6%. How much did he pay?

12. A merchant bought goods for $4500 cash, and sold them for $4944, one-half cash, and the balance in 9 mo. How much ready money did he gain, discount at 6%?

13. Which would be the better, to buy flour at $7.80 a barrel on 6 mo. credit, or at $7.65 on 3 mo. time, money worth 8%?

14. What is the present worth of $5000, one-fourth due in 9 mo., one-third in 1 yr., and the balance in 1 yr. 6 mo., money at 6%?

15. I held two notes, one for $741.28, due Oct. 15, 1900, and the other for $669.68, due Jan. 1, 1901. What was the cash value of both notes, Aug. 10, 1900, at 8%?

Section VI.

BANKING.

180. A ***Note,*** or ***Promissory Note,*** is a written promise to pay a sum of money at a given time, for value received.

Note. — The person who signs the note is called the *Maker*. The person to whom or to whose order a note is made payable is the *Payee*. The owner of the note is the *Holder*. The person who writes his name on the back of the note as security for its payment is the *Indorser*.

Form of a Promissory Note.

$\$275\frac{50}{100}$. PHILADELPHIA, *Nov. 1, 1901.*

Three months after date I promise to pay to James Graham, or order, two hundred seventy-five $\frac{50}{100}$ dollars, for value received.

Chas. J. Raymond.

Form of a Note Payable at a Bank.

$\$750\frac{25}{100}$. PHILADELPHIA, *Nov. 1, 1901.*

Thirty days after date I promise to pay to the order of James Graham, at the First National Bank, seven hundred fifty $\frac{25}{100}$ dollars, without defalcation, for value received. *Chas. J. Raymond.*

181. A ***Negotiable Note*** is a promissory note made so that it may be sold or transferred.

182. ***Days of Grace*** are three days allowed in some of the States for the payment of a note after the specified time has expired.

To secure uniformity, interest for the days of grace has been calculated in all the problems under Banking. In States in which days of grace are not allowed the three days' interest should be omitted.

183. The ***Maturity*** of a note is the day on which it is legally due; that is, the last day of grace.

184. The ***Face*** of a ***Note*** is the sum for which the note is drawn.

185. A ***Bank*** is a corporation authorized by law to receive and loan money, or to issue bills to circulate as money.

186. ***Bank Discount*** is interest charged in advance by a bank for loaning money on a note.

187. The ***Proceeds*** of a note is the face or amount of the note less the bank discount.

188. The ***Term of Discount*** is the time from the date of discount to the date of maturity.

Note 1. — When a bank discounts a note, it retains the discount, or interest, keeps the note, and pays the proceeds to the *Indorser*. On the maturity of the note, the *Maker* pays both the proceeds and the interest, or the face, and receives back his note.

2. If the maker of a note fails to pay it at maturity, a written notice called a *Protest* is made out, and sent to the indorser by an officer called a *Notary Public*. In order to hold the indorser for the payment of the note, the protest should be served on the last day of grace.

189. The principles in Bank Discount are the same as those in Simple Interest, in which the principal, the rate, and the time are given.

190. The corresponding terms of Bank Discount and Interest are —

1. The ***Face of the Note*** is the *Principal.*
2. The ***Term of Discount*** is the *Time,* including days of grace.
3. The ***Bank Discount*** is the *Interest.*

The ***Bank Proceeds*** is the *Principal less the Interest.*

CASE I.

To Find the Bank Discount and the Bank Proceeds.

Written Exercises.

191. *Example.* — Find the bank discount and the proceeds of a note for $600 for 3 mo., dated Sept. 15, and discounted Oct. 16, at 7%.

SOLUTION.

$\frac{1}{6}$ *of* .063 = .010$\frac{1}{2}$.

$600 × .010$\frac{1}{2}$ = $6.30 *Dis. at* 6%.

$6.30 ÷ 6 = 1.05 " 1%.

$7.35 *Bank dis.* Or,

Face × *rate* × $\overline{\textit{time} + 3\ \textit{da.}}$ = *bank dis.*

$600 × .07 × $\frac{63}{360}$ = $7.35.

Face — bank discount = proceeds.

$600 − $7.35 = $592.65.

EXPLANATION.—3 mo. 3 da. from Sept. 15 is Dec. 18, the date of maturity. The *term of discount*, from Oct. 16 to Dec. 18, is 63 days.

The interest of $600 for 63 days at 6% is .010½ of $600, or $6.30; and at 7% is ⅙ more, or $7.35, the bank discount. Or,

The product of the face $600, by .07, the rate %, and the time, $\frac{63}{360}$, is $7.35, the bank discount required; and $600 − $7.35 is $592.65, the proceeds required.

192. Rules to Find the Bank Discount and the Proceeds.

I. *Find the interest on the face of the note at the given rate for three days more than the given time. The result will be the bank discount required.*

II. *Subtract the bank discount from the face of the note. The remainder will be the proceeds.*

193. FORMULAS.—1. *Face* × *rate* % × $\overline{\textit{time} + 3\ \textit{da.}}$ = *bank discount.*

2. *Face — bank discount = bank proceeds.*

Note 1. — If the note bears interest, find the discount on the *amount* of the note at maturity; from which subtract the discount for the proceeds.

2. If no mention of interest is made in a note, it draws interest only from the day on which it becomes due.

Problems.

Find the bank discount and the proceeds of—

1. $600 for 60 da., at 6%; $875 for 3 mo., at 7%.

2. $1200 for 4 mo., at 8%; $1250 for 3 mo. 12 da., at 7½%.

3. $1375 for 90 da., at 8½%; $1400 for 4 mo. 8 da., at 9%.

4. $1437.50 for 4 mo. 22 da., at 8½%; $1625 for 6 mo., at 10%.

5. What is the difference between the true discount and the bank discount of $1325, due in 1 yr. 6 mo., at 8%?

6. Find the proceeds of a note for $450, dated May 15, payable in 90 days, discounted 15 days after date, at 6%.

7. A note of $1000, dated Aug. 15, 1901, and payable in 6 mo., was discounted Nov. 27, 1901, at 10%. Find the bank discount and the proceeds.

8. Find the bank discount and the proceeds of a note for $1250.50, dated Nov. 20, 1901, payable in 9 mo., and discounted 30 days after date, at 1% a month.

Find the *date of maturity*, the *term of discount*, the *bank discount*, and the *proceeds* of the following: —

9. NEW YORK, *Aug.* 27, 1901.

637\frac{50}{100}$. Two months after date, I promise to pay to N. B. Boyd, or order, six hundred thirty-seven and $\frac{50}{100}$ dollars, for value received. EBEN H. WHITE.

Discounted Sept. 10, at 7%.

10. CINCINNATI, *Sept.* 15, 1901.

$1500. Sixty days after date, we promise to pay to the order of J. H. Butler & Co., fifteen hundred dollars, without defalcation. Value received.

Discounted Oct. 16. WILSON, HINKLE & Co.

11. NEW YORK, *Oct.* 1, 1901.

745\frac{25}{100}$. Ninety days after date, I promise to pay to the order of Eldredge & Bro., seven hundred forty-five and $\frac{25}{100}$ dollars, without defalcation. Value received.

Discounted at JOHN E. BYRON.

First National Bank, Oct. 15, at 6%.

12. PHILADELPHIA, *Nov.* 15, 1901.

1200\frac{50}{100}$. Four months after date, we jointly and severally promise to pay to the order of J. L. Claghorn, at the Commercial Bank, twelve hundred and $\frac{50}{100}$ dollars, without defalcation. Value received. JAMES L. HORN,
WILLIAM BROWN.

CASE II.

To Find the Face of a Note.

Written Exercises.

194. *Example.* — For what sum must a note be drawn at 4 mo., at 8%, that the proceeds may be $875.40?

SOLUTION.

Bank disc. of $1 for 123 da. $= .027\frac{1}{3}$.

Proceeds of $1 $= \$1 - \$.027\frac{1}{3} = \$.972\frac{2}{3}$.

$\$875.40 \div \$.972\frac{2}{3} = \$900$, *Face of note.* Or,

Proceeds $\div [1 - (rate\ \% \times \overline{time + 3\ da.})] = face.$

$\$875.40 \div (1 - .08 \times \frac{123}{360}) = \$900.$

EXPLANATION.— Since the bank discount of $1 for 123 da. is $.027\frac{1}{3}$, the proceeds of $1 is $1 —$.027\frac{1}{3}$, or $.972\frac{2}{3}$; and $875.40 are the proceeds of as many dollars as the number of times $.972\frac{2}{3}$ are contained in $875.40, or $900, the face required.

195. Rule to Find the Face of a Note.

Divide the proceeds by the proceeds of $1 *at the given rate for the given time plus three days.*

196. FORMULA. — *Proceeds* $\div$ [1 — (*rate* % $\times$ $\overline{time + 3\ da.}$)] = *face.*

Note. — If the bank discount only is given, divide the bank discount by the discount of $1, at the given rate, for the given time plus three days.

Problems.

Required the face of a note that the proceeds may be —

1. $397.80 for 30 da., at 6%. $886 for 2 mo. 17 da., at 7%.
2. $1175.20 for 90 da., at 8%. $1000 for 3 mo. 27 da., at $7\frac{1}{2}$%.
3. $1169.25 for 4 mo., at $7\frac{1}{2}$%. $1351.35 for 4 mo. 12 da., at 10%.
4. $1449.36 for 2 mo., at 5%. $2225.50 for 5 mo. 27 da., at 10%.
5. $2500 for 6 mo., at 6%. $7250.25 for 7 mo. 22 da., at 8%.
6. For what sum must a note dated Dec. 1, 1901, be drawn, at 90 da., to realize $3250, at 6%? At 7%? 8%?
7. Find the face of a 4 mo. note, which, discounted at 2% a month, gives $750 proceeds. $875 proceeds. $1000.

8. For what sum must a merchant give his note at 90 da., at 8%, to enable him to obtain from bank enough to pay for $87\frac{2}{3}$ yd. of cloth, at $\$1\frac{3}{4}$ per yard?

9. What must be the face of a note dated Nov. 10, 1901, payable in 4 mo., which, when discounted at 1% a month, will yield a manufacturer $1500 with which to pay his men?

10. A merchant bought a bill of goods, amounting to $3750, on 60 days' time, or 5% off for cash. If he borrowed the money from bank at 6%, how much did he gain or lose?

Section VII.

COMPOUND INTEREST.

197. ***Compound Interest*** is interest on both principal and interest combined at regular intervals.

Interest is generally compounded or made part of the principal annually, semi-annually, or quarterly.

Written Exercises.

198. *Example.* — What is the compound interest on $750 for 2 yr. 4 mo., at 6%?

SOLUTION.

$\$750$, *Prin. for 1st year.*
1.06
$\$795$, *Amt. for 1st year.*
1.06
$\$842.70$, *Amt. for 2d year.*
.02
$\$16.854$, *Int. for 4 mo.*
842.70
$\$859.554$, *Amt. for 2 yr. 4 mo.*
750
$\$109.554$, *Comp. int.*

EXPLANATION. — Since at 6% the amount for 1 yr. is 1.06 of the principal, the amount of $750 at the end of the first year is 1.06 times $750, or $795.

Since the amount is 1.06 of the principal, the amount of $795 at the end of the second year is 1.06 times $795, or $842.70.

Since the rate is 6% a year, the rate for 4 mo., or $\frac{1}{3}$ of a year, is $\frac{1}{3}$ of .06, or .02. And the interest of $842.70 for 4 mo. is .02 of $842.70, or $16.854, which, added to $842.70, gives $859.554, the amount for 2 yr. 4 mo.

Hence, $859.554 less the principal, $750 gives $\$109.554$, the compound interest required.

199. Rule to Find Compound Interest.

I. *Find the amount of the principal at the given rate for the first period of time at the end of which interest is due.*

II. *Make the amount for the first period of time the principal for the second period, and find the amount as before; and so continue till the end of the given time.*

III. *Subtract the given principal from the last amount, and the remainder will be the compound interest required.*

Note 1. — If the interest is compounded semi-annually, the rate % is one-half of the yearly rate; if quarterly, one-fourth of the yearly rate, etc.

2. If the time contains months and days, find the amount to the end of the last period of time; then find the simple interest of that amount for the remaining time, and add it to the last amount for the total amount.

Problems.

Find the amount and the compound interest of —

1. $500 for 2 yr., at 5%; for 3 yr., at 6%.

2. $750 for 2 yr., at 7%; for 2 yr., at 8%; 3 yr., at 9%.

3. $800 for $2\frac{1}{2}$ yr., at 6%; for 3 yr. 6 mo., at 7%, and at 8%.

4. $1000 for $3\frac{1}{4}$ yr., at 7%; for 3 yr. 9 mo., at 8%, and at 9%.

5. Find the amount of $325.50 for 2 yr. 6 mo., at 6%, interest compounding semi-annually.

6. Find the amount of a note of $400 due in 2 yr. 8 mo., bearing semi-annual compound interest, at 6%.

7. Find the difference between the simple and the compound interest of $1250 for 4 years, at 8%.

8. What sum will pay May 1, 1902, $2000 borrowed Nov. 1, 1900, at 6% compound interest?

9. Find the compound interest on a mortgage for $2500 from July 1, 1900, to Nov. 1, 1902, at 6%, payable semi-annually.

10. To how much more would $1250 amount at compound interest than at simple interest, in 4 yr. 6 mo., at 6%?

11. If I deposit $750 in a savings-bank, and let it compound at 5%, how much will be due to me at the end of 6 years?

Section VIII.

PARTIAL PAYMENTS.

200. *Partial Payments* are payments in part of a note or other obligation bearing interest.

201. *Indorsements* of partial payments are acknowledgments of the time and the amount of each payment written on the back of the note or obligation.

The Supreme Court of the United States and most of the States adopt a rule for partial payments based upon the following—

202. Principles.

I. *Payments must be applied to the discharge of interest due; and the balance, if any, to the discharge of the principal.*

II. *Interest must accrue only on unpaid principal.*

III. *Interest must not draw interest.*

Written Exercises.

$1000. PHILADELPHIA, *Sept.* 1, 1899,

Six months after date, I promise to pay Eldredge & Bro., or order, one thousand dollars, with interest. Value received.

EDWIN T. MURRAY.

Indorsed as follows: March 1, 1900, $200; June 1, 1900, $10; Sept. 16, 1900, $300. What balance was due Jan. 1, 1901?

FIRST SOLUTION.

Face of note, or principal,		*$1000.00*
Interest to March 1, 1900, 6 mo.,		*30.00*
Amount due at first payment,		*$1030.00*
First payment, March 1, 1900,		*200.00*
Second principal,		*$830.00*
Int. on $830 to June 1, 1900, 3 mo.,	*$12.45*	
Int. on $830 to Sept. 16, 1900, 3 mo. 15 da.,	*14.53*	*26.98*
		$856.98

Second payment, June 1, 1900, ...	*$10.00*	
Third payment, Sept. 16, 1900, ...	*300.00*	*$310.00*
Third principal, ...		*$546.98*
Interest to Jan. 1, 1901. 3 mo. 15 da., ...		*9.57*
Amount due Jan. 1, 1901, ...		*$556.55*

EXPLANATION. — The interest on the face of the note to March 1, the date of the first payment, is $30, which, added to the face, gives $1030, the amount due.

Since the first payment, $200, is greater than the interest due, it discharges the interest and a part of the principal, leaving a balance of $830 due.

The interest on $830, the second principal, to June 1, the date of the second payment, is $12.45; but since the second payment, $10, is less than the interest due, it cannot be deducted from the total amount due without leaving a portion of unpaid interest as part of the new principal, which would be compounding interest, and, therefore, illegal.

The interest, therefore, continues on $830 to Sept. 16, the date of the third payment, giving $14.53, and making the total amount due $856.98.

Since the sum of the two payments, $310, is greater than the interest due, it discharges the interest and a part of the principal, leaving $546.98.

The interest on $546.98, the third principal, to Jan. 1, the date of settlement, is $9.57, which, added to the principal, gives *$556.55*, the total amount due Jan. 1, 1901.

SECOND SOLUTION.

1899, 9 mo. 1 da. (date)	*Time.*	*Prin.*	*Int.*	*Amt. due*	*Payt.*	*Bal. due.*
1900, 3 " 1 " (1st p't.)	*6 mo.*	*$1000*	*$30.00*	*$1030*	*$200*	*$830.00*
1900, 6 " 1 " (2d ")	*3 "*	*$830*	*12.45*		*10*	
1901, 9 " 16 " (3d ")	*3 " 15 d.*	*$830*	*14.53*		*300*	
			$26.98	*$856.98*	*$310*	*$546.98*
1901, 1 " 1 " (settl'nt)	*3 " 15 "*	*$546.98*	*$9.57*	*$556.55*		*$556.55*

EXPLANATION. — Similar to that given to first solution.

Note.—The advantages of the arrangement in the second solution are—

1. It shows at a glance the date of each payment, and the time for which interest is to be computed.

2. It shows each principal upon which interest is to be computed, the interest due, and the total amount due.

3. It shows whether or not the payment exceeds the interest; and if

any payment is less than the interest due, it shows when the sum of the payments exceeds the total interest due.

4. It shows the balance due after each payment, and the balance due at the time of settlement.

203. United States Rule for Partial Payments.

I. *Find the amount of the given principal to the date of the first payment.*

II. *If this payment equals or exceeds the interest then due, subtract it from the amount, and regard the remainder as a new principal.*

If the payment is less than the interest due, find the amount of the same principal to the date when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from the total amount due.

III. *Proceed in the same manner with each of the remaining payments. The last result will be the amount due at the date of settlement.*

Problems.

1. On June 15, 1900, the face of a mortgage on a house in Philadelphia was $2500. On Jan. 1, 1901, $250 were paid, and July 1, 1901, $100. What amount was due on Nov. 11, 1901?

2. I bought a house for $3000, paying down $1000, six months later $1000, and a year later $100. How much was then due?

3. On a note of $500, at 7%, there were paid $100 annually for 4 years. What balance was due 4 yr. 6 mo. from the date of the note?

4. Date of note May 10, 1898, face $1200, rate of interest 8%; payments Oct. 5, 1898, $50; March 15, 1899, $100; Sept. 25, 1899, $75; July 1, 1900, $200. What remained due Jan. 11, 1901?

5. What balance was due Jan. 17, 1902, on the following note?—

$800. BOSTON, *Oct.* 1, 1900,

On demand, for value received, I promise to pay Louis Edwards, or order, eight hundred dollars, with interest, at 7%.

CHARLES WILLIAMS.

Indorsements: December 1, 1900, received fifty dollars: June 16, 1901, received twenty-five dollars; October 2, 1901, received one hundred fifty $\frac{75}{100}$ dollars.

6. Find the balance due Feb. 3, 1902, on this note: —

$1800. BOSTON, *Feb.* 14, 1900.

Nine months after date, we promise to pay to Edgar Watson, one thousand eight hundred dollars, with interest. Value received.

WILSON, THOMAS & Co.

Indorsements: Received Aug. 1, 1900, $50; received May 11, 1901, $400; received Dec. 16, 1901, $20.

Business men often settle notes and interest accounts running a year or less by the following —

204. Merchants' Rule for Partial Payments.

I. *Find the amount of the note or debt from its date to the date of settlement.*

II. *Find the amount of each payment from its date to the date of settlement.*

III. *Subtract the sum of the amounts of the several payments from the amount of the note or debt. The difference will be the balance required at the date of settlement.*

Problems.

7. Find the balance due at the end of a year, on a note for $750, given January 1, 1901, on which were paid April 10, $150; June 15, $20; Sept. 20, $75.

8. On a debt of $1500 due May 1, 1900, payments were made as follows: May 31, $400; July 15, $50; Sept. 1, $75; Dec. 5, $250. What was due March 10, 1901, interest at 6%?

Review Problems.

1. What sum must be invested at 7% to yield enough to pay $250 annual ground-rent for a house?

2. A merchant bought goods for \$6000 on 4 mo. credit, and was offered a discount of 5% for cash. What was the difference between the prices, money at 7%?

3. Find the difference between the true discount and the bank discount of \$1200, due in 6 mo., at 7%. The difference between the simple interest and the bank discount.

4. In what time will the interest of \$750, at 8%, equal the principal? Twice the principal? One-half the principal?

5. I am offered goods for \$2075 cash, or \$2116.50 on 3 mo. Which is the better offer, and how much?

6. For what sum must a merchant draw his note, at 60 days, to pay for $125\frac{1}{2}$ yd. of cloth at $\$2.37\frac{1}{2}$ a yd., money being worth 7%?

7. If I invest \$12500 in a business that pays $\$156\frac{1}{4}$ a month, what annual rate of interest do I receive?

8. An invoice of goods at retail prices amounts to \$950. If the commercial discount is 25% off, and $2\frac{1}{2}$% for cash, find the net proceeds of the sale.

9. A note of \$1000, dated Aug. 15, 1901, and payable in 6 mo., was discounted Nov. 27, 1901, at 7%. Find the proceeds of the note at bank.

10. I paid $\$1392.93\frac{3}{4}$, Jan. 1, 1902, the amount due on a note given June 25, 1900. Find the face of the note.

11. I sold a house for \$4500, and received \$2000 cash, and the balance in a note payable in 9 mo., at 7% interest, which I had discounted at 8%. What was the ready money selling price?

12. I bought a bill amounting to \$2500, on 60 da., or 5% off for cash. How much would I have gained or lost by borrowing the money at bank on a 60-day note to cash the bill?

13. A person bought a farm of 175 A. 40 sq. rd., at \$37.50 per A. He paid one-half of the purchase-money in cash, and the balance in 9 mo. Find the ready money cost of the farm, money at 8%.

14. What is the cash value of a legacy of \$5000, inherited July 4, to be paid on the 1st of January following?

15. Find the difference between the simple and the compound interest of $7500, for 3 yr., at 6%.

16. June 15, 1901, a broker offered me $300 for a note of $350, dated Oct. 8, 1900, payable March 21, 1902, bearing 6% interest. If money was worth 10%, did he offer me the full value of the note?

17. On a note of $400, at 7%, there were paid $100 annually for 3 yr. How much remained due 3 yr. 4 mo. from the date of the note?

18. I bought a house for $7500, and paid $3000 cash on delivery, $2000 in 9 mo., and the remainder in 1 year 4 mo. What was the whole amount paid, interest 6%?

19. Sold goods for $1500, to be paid Jan. 1, 1900. Received June 10, $250; Sept. 13, $151; Oct. 3, $420. What was due Dec. 31, 1900, at 6%, the goods having been bought Jan. 1, 1899?

20.

CINCINNATI, *Jan.* 1, 1900.

$1250. On demand, I promise to pay John Sherman, or order, twelve hundred fifty dollars, with interest, for value received.

HENRY WILSON.

Indorsements: Received Sept. 15, 1900, $300; Jan. 21, 1901, $12; Oct. 16, 1901, $20; Nov. 12, 1901, $12. What amount was due Jan. 2, 1902?

Review Questions.

1. Define ***Interest.*** What is the principal? The amount? Simple interest? The rate of interest? The legal rate of interest? What is usury? What is the legal rate on debts due the U. S.? The rate in Canada? In England? In France?

2. What ***elements*** are considered in Interest? What new element is introduced in the application of percentage in Interest? Name the corresponding terms of Interest and Percentage. Of what factors is interest the product? Repeat the general rule to find interest. What is the formula? How is interest found by aliquot parts? Explain the principle of the six per cent. method, and repeat the rule. Repeat the rule, and give the formula to find the principal. To find the rate. To find the time.

3. Define ***Discount.*** Name two kinds of discount. What is Commercial Discount? What is meant by net price? By net proceeds?

Name the corresponding terms of Commercial Discount and Percentage. Give the formulas to find the commercial discount, and the net price.

4. Define Present Worth. What is ***True Discount?*** To what does true discount correspond? To what do operations in True Discount correspond? Name the corresponding terms of True Discount and Interest. Repeat the rules, and give the formulas to find the true discount and the present worth.

5. What is a ***Promissory Note?*** Name the different parties concerned in a promissory note. Give the wording of a note. What is meant by a negotiable note? By days of grace? The maturity of a note? The face of a note? The term of discount? Define ***Bank Discount.*** What is meant by the proceeds? Explain the process of borrowing money on a note from a bank. What course is taken if a note is not paid at maturity? What principles are involved in Bank Discount? Name the corresponding terms of Bank Discount and Interest. Repeat the rules, and give the formulas to find the bank discount of a note. The proceeds. The face.

6. Define ***Compound Interest.*** Explain the process of finding compound interest. What are ***Partial Payments?*** Indorsements? Upon what principles is the U. S. rule for partial payments based? Explain the rule. Give the merchants' rule for partial payments on notes and accounts running a year or less.

Section IX.

INSURANCE.

205. Insurance is indemnity secured against loss or damage.

By the operation of insurance, one party, for a specified sum, guarantees to make good any loss or damage that may be sustained by another within a given time.

206. Insurance is of two general kinds: *Property Insurance* and *Life Insurance.*

CASE I.

Property Insurance.

207. Property Insurance includes *Fire Insurance,* or indemnity for loss by fire; *Marine Insurance,* for loss by sea; and *Stock Insurance,* for the loss of horses, cattle, etc.

208. The ***Policy*** is the contract or agreement between the insurer and the insured.

209. The ***Valuation*** is the amount for which property is insured.

210. The ***Premium*** is the sum paid for the insurance.

In property insurance, the premium is computed at some rate per cent. on the valuation as the base.

Note 1. — The business of insuring is generally conducted by Insurance Companies which are either *Stock Companies* or *Mutual Companies.*

A ***Stock Insurance Company*** is one in which the capital to secure the payment of losses is paid in by individuals called stockholders, who only share the profits and the losses.

A ***Mutual Insurance Company*** is one in which every person insured is a member, sharing in the profits and the losses.

2. The valuation of property insured is generally made less than the real value, to prevent the owner from being tempted to destroy the property for the purpose of obtaining the amount insured.

211. The corresponding terms of Insurance and Percentage are —

1. The ***Valuation,*** or ***Amount Insured,*** is the *Base.*
2. The ***Rate*** of ***Premium*** is the *Rate.*
3. The ***Premium*** is the *Percentage.*

1. How much must be paid for insuring some furniture for \$400, at $1\frac{1}{2}\%$ premium?

212. ***Analysis.*** — Since the premium is $1\frac{1}{2}\%$, or $\frac{3}{200}$ of the valuation, the premium on \$400 is $\frac{3}{200}$ of \$400, or \$6. Hence, etc.

2. What must be paid for insuring a case of merchandise worth \$600, at $1\frac{1}{2}\%$? At $1\frac{2}{3}\%$? At $2\frac{1}{2}\%$?

3. What premium must be paid on \$400, at $\frac{1}{4}\%$? At $\frac{1}{8}\%$?

4. If \$6 are paid for insuring \$300, what is the rate %?

213. ***Analysis.*** — Since on \$300 the premium is \$6, the rate % of premium is $\frac{6}{300}$, or $\frac{1}{50}$ of 100%, which is 2%. Hence, etc.

5. At what rate will the premium on \$400 amount to \$5?

6. What rate % on \$500 insured gives a premium of \$7?

7. At 2%, what sum can be insured for $8 premium?

214. ***Analysis.*** — Since the rate of premium is 2%, $\frac{2}{100}$, or $\frac{1}{50}$ of the sum insured, equals the premium, which is $8. If $8 is $\frac{1}{50}$ of the sum insured, $\frac{50}{50}$ of the sum insured is 50 times $8, or $400. Hence, etc.

8. What sum can be insured for $12 premium, at $1\frac{1}{2}$%?

9. At $\frac{3}{4}$%, what amount of insurance can be obtained for $6?

10. What amount of insurance can be obtained on a boat-load of flour for $50, at $\frac{3}{4}$%? For $60, at $1\frac{1}{2}$%?

11. If it costs $75 to insure a store for one-half its value, at 2%, how much is the store worth?

Written Exercises.

215. *Example.* — What will it cost to insure a store for $2500, at $2\frac{1}{2}$%?

SOLUTION.

Sum insured × *rate %* = *premium.*

$\$2500 \times .02\frac{1}{2} = \$62.50.$

EXPLANATION. — Since the premium is $2\frac{1}{2}$% of the sum insured, it is $.02\frac{1}{2}$ of $2500, or the product of $2500 × $.02\frac{1}{2}$, which is $\$62.50$.

216. FORMULA. — *Sum insured* × *rate %* = *premium.*

Problems.

Find the premium for insuring —

1. A house for $3500, at 2%; for $3750, at $\frac{3}{4}$%.

2. A cargo of dry-goods for $8000, at $\frac{5}{8}$%; for $12500, at $\frac{7}{8}$%.

3. A drove of horses for $7500, at $\frac{1}{4}$%; for $9000, at $\frac{3}{8}$%.

4. A factory worth $10000 insured for $\frac{2}{3}$ of its value, at $1\frac{1}{2}$%.

5. What must a merchant pay for $7500 insurance on his store at 2%, and $10250 on his goods at $\frac{1}{2}$%?

6. A factory worth $12000 is insured for $\frac{3}{4}$ of its value at $2\frac{1}{2}$%, and the cost of the survey and policy is $1.50. What is the entire cost of the insurance?

7. At $\frac{1}{5}$ of 1% a month, what will be the cost of insuring goods worth $3800, which remain in store 3 months?

217. If an insurance of $2800 costs $42, what is the rate?

SOLUTION.

Premium ÷ sum insured = rate %.

$42 ÷ $2800 = .015, or 1½ %.

EXPLANATION. — Since the premium is the product of the sum insured and the rate %, the rate % must be the quotient of the premium, $42, divided by the sum insured, $2800, which is .01½, or *1½ %*, the rate required.

218. FORMULA. — *Premium ÷ sum insured = rate %.*

Problems.

Find the rate of insurance —

8. If an insurance of $1800 costs $36; if $24; if $15.

9. If $100 are paid for an insurance of $12500 on goods.

10. If it costs $237.50 to insure a planing-mill for $4750.

11. If the premium for insuring ⅔ of a house valued at $7800 is $39. If the premium for a perpetual insurance is $87.75.

219. What is the sum insured which cost $24, at 1½ %?

SOLUTION.

Prem. ÷ rate % = sum insured.

$24 ÷ .015 = $1600.

EXPLANATION. — Since the premium is the product of the sum insured and the rate %, the sum insured must be the quotient of the premium, $24, divided by the rate %, .015, or *$1600.*

220. FORMULA. — *Premium ÷ rate % = sum insured.*

Problems.

Find the sum insured —

12. For $9, at 1⅕ %. For $10, at 1¼ %. For $21, at 1¾ %.

13. For $21, at ⅞ %. For $18, at ½ %. For $24, at ¼ %. $32.50, at ⅛ %. For $62.87½, at ¾ %.

14. If the annual premium for insuring a house at ⅝ % is $20, what is the amount covered by the policy? If the premium is $32?

15. A grain dealer paid $35.62½ at ⅜ % for insuring a cargo of grain from Buffalo to Chicago. What was the amount insured?

16. A grocer paid $41.25 for having his store insured for ¾ of its value, at 55 cents per $100. What was the value of the store?

CASE II.

Life Insurance.

221. Life Insurance is indemnity received for loss of life.

By the operation of life insurance, a company agrees to pay a certain sum to those interested in the life of the insured, if it occurs within a specified time.

Note. — Life Insurance policies are of different kinds, the premium varying with the age of the insured.

A ***Life Policy*** is one in which the insurance continues during the whole life of the person insured.

An ***Endowment Policy*** is one in which the insurance is payable to the person insured at the end of a specified time, or to his heirs if his death occurs before that time.

222. In Life Insurance the premium is computed at a certain rate or sum per \$100 or \$1000 insured.

Written Exercises.

223. *Example.* — What is the annual premium for a policy of \$5000 on the life of a person 30 years old, at \$23.40 per \$1000?

SOLUTION.

\$5000 ÷ \$1000 = 5, No. of thousands.
\$23.40 × 5 = \$117, Amt. of premium.

EXPLANATION.—Since the annual premium on \$1000 is \$23.40, the premium on \$5000 is 5 times \$23.40, which are *\$117.*

Problems.

Find the annual premium for a policy of —

1. \$2000 at \$25.98 per \$1000, age 35 yr.; \$3250 at \$26.28.

2. \$4000 at \$30.90 per \$1000, age 40; \$4500 at \$31.20.

3. A man 30 years old had his life insured for \$5000, at \$12.28 per \$1000, semi-annually. What annual premium did he pay?

4. A man 35 years old took out a policy of life insurance for \$3000, the annual premium being \$26.38 per \$1000. If he died at the age of 50, what did his heirs receive above the amount paid in?

5. If a man aged 25 insures his life for \$7500 on the endowment plan, and pays an annual premium of \$64.37½ per \$1000, for ten years, what amount above the premium paid in will he receive at the age of 40?

Section X.

STOCKS AND INVESTMENTS.

224. A ***Corporation*** is a company of persons authorized by law to transact business as an individual.

The law forming and regulating a corporation is called a *Charter*.

225. The ***Capital Stock*** of a corporation is the property or funds invested in the business of the company.

226. A ***Share*** is one of the equal parts into which the stock of a corporation is divided.

227. A ***Certificate of Stock*** is the paper or document issued by a corporation, stating the number and the value of shares owned by the holder of the certificate.

228. The ***Par Value*** of stock is the original value, or value mentioned in the certificate.

The par value of shares of stock is usually $100.

229. The ***Market Value*** of stock is the sum for which it can be sold.

Stock is *at par* when it sells for its original or face value, or 100%; it is *above par*, or *at a premium*, when it sells for more than its face value, or is above 100%; it is *below par*, or *at a discount*, when it sells for less than its face value, or is below 100%.

230. *Premium* and *discount* are usually computed at a certain rate % on the par value of the stock as the base.

231. ***Stock Quotations*** are published statements of the market value of stock.

Thus, if stock is at par, it is quoted at *100;* if at 5% above par, it is quoted at *105;* and if 5% below par, it is quoted at *95*.

232. An ***Installment*** is a portion of the capital stock of a corporation paid by the stockholders at a particular or set time.

233. A ***Dividend*** is a sum paid to stockholders from the profits of the business.

234. An ***Assessment*** is a sum required of stockholders to meet losses or to make up deficiencies.

235. Dividends and assessments are usually computed at a certain rate % on the par value as the base.

236. The ***Gross Earnings*** of a company is the entire amount of money received by the company.

237. The ***Net Earnings*** of a company is the amount of money left after deducting all expenses, losses, interest, etc.

238. A ***Bond*** is a written obligation securing the payment of a given sum of money at or before a specified time.

Bonds are issued by Government or by corporations, and bear interest payable at specified dates.

239. ***Treasury Notes*** are notes issued by the Government, payable on demand *without interest*, or at a specified time *with interest*.

A ***Coupon*** is a certificate of interest attached to a bond, to be cut off and presented for payment when the interest is due.

240. ***United States Government Securities*** are the several kinds of bonds issued by the United States Government for money which it has borrowed or for obligations which it has assumed. The bonds now in existence are those bearing interest respectively at two per cent., four per cent. and five per cent.

241. The principal United States securities are the following:

Consols of 1930, authorized March 14, 1900; rate of interest, 2 per cent.; issued in 1900; due after April, 1930.

Loan of 1908–1918, authorized June 13, 1898; rate of interest, 3 per cent.; issued in 1898; due after August 1, 1908.

Funded loan of 1907, authorized July 14, 1870, and January 20, 1871; rate of interest, 4 per cent.; issued 1877–1879; due after July 1, 1907

Refunding certificates, authorized February 26, 1879; rate of interest, 4 per cent.; issued in 1879.

Loan of 1925, authorized January 14, 1875; rate of interest, 4 per cent.; issued in 1895–1896; due after February 1, 1925.

Loan of 1904, authorized January 14, 1875; rate of interest, 5 per cent.; issued in 1894–1895; due after February 1, 1908.

The 5 per cent. bonds were issued at such a premium that the interest received on them by the purchasers amounts to only about 3 per cent.

The bonds issued by States, counties, cities, and corporations are usually

named according to their rate of interest; as, Pennsylvania 6's are bonds issued by the State of Pennsylvania, bearing 6% interest.

242. ***Stocks*** is a general term applied to corporation stock, and to various Government securities, State bonds, etc.

243. A ***Stock-broker*** is a person who deals in stocks, bonds, gold, etc.

244. ***Brokerage*** is a percentage allowed for buying and selling stocks, etc.

Brokerage is usually computed at a certain rate % on the par value of stock.

Note. — The rate of brokerage in large cities is generally $\frac{1}{8}$% for U. S. bonds; $\frac{1}{4}$% for other bonds; $\frac{1}{4}$% for corporation shares of $50, or more; $\frac{1}{8}$% for shares of $5 to $50, and $6\frac{1}{4}$ cents for shares of $5 or less.

245. In all ordinary transactions, the corresponding terms of Stock and Percentage are —

1. The ***Par Value*** is the *Base.*
2. The ***Rate of Premium*** or ***Discount*** is the *Rate.*
3. The ***Premium*** or ***Discount*** is the *Percentage.*
4. The ***Market Value*** is the *Amount,* or the *Difference.*

Note. — In finding the cost of stocks, bonds, etc., the rate % of brokerage may be added to the rate of premium or discount, as both are calculated upon the same sum, — the par value.

Mental Exercises.

1. What is the market value of 10 shares of bank stock, quoted at 90? What is the cost, brokerage being $\frac{1}{4}$%?

246. ***Analysis.*** — Since at 90% the market value of one share is $\frac{90}{100}$ of $100, or $90, the market value of 10 shares is 10 times $90, or $900; and since at $\frac{1}{4}$% the brokerage is $\frac{1}{400}$ of $900, or $2.25, the entire cost is the sum of $900 and $2.25, or $902.25. Hence, etc.

Find the market value and the cost of —

2. 20 shares of R. R. stock at par, brokerage $\frac{1}{8}$%, at 104; at 95.
3. $200 bonds at 110, brakerage $\frac{1}{8}$%, at 112; at 115.
4. How much currency can be bought for $500 in gold, when gold is at a premium of 9%? Of 10%? Of $12\frac{1}{2}$%?

5. If 40 shares of stock cost $3810, including brokerage, at what rate does it sell?

247. ***Analysis.*** — Since the par value is 40 times $100, or $4000, at ¼% the brokerage is $\frac{1}{40}$ of $4000, or $10. $3810 less $10 is $3800, the market value of 40 shares; and $\frac{1}{40}$ of $3800 is $95, which is .95, or 95% of the par value. Hence, etc.

Find the market value, rate %, and premium or discount of—

6. 8 shares of stock which cost $754, ¼% brokerage.

7. 7 shares of stock for which I pay $780, brokerage ¼%.

8. Of bonds which sell for $1155, brokerage ⅛%.

9. If a broker receives $875 to invest in canal stock at 12¼% discount, how many shares can he buy, brokerage ¼%?

248. ***Analysis.***—Since the stock sells at 12¼% discount, each share with brokerage will cost $100 less $12¼, or $87½; and for $875 he can buy as many shares as the number of times $87½ are contained in $875, or 10 shares. Hence, etc.

How many shares can be bought—

10. For $750 in mining stock at 87½, brokerage ¼%?

11. For $1145 at $114⅜, brokerage ⅛%?

12. When gold is at a premium of 14%, how much can be bought for $458? For $687? $912?

Written Exercises.

249. *Example.* — How much will 75 shares of railroad stock cost at 4¼% premium, brokerage ¼%?

SOLUTION.

(104¼% + ¼%) of $100 = $104.50, cost 1 share.

Cost of 1 share × no. shares = entire cost.

$104.50 × 75 = $7837.50.

EXPLANATION.— Since the stock sells at 4¼% premium, each share with the brokerage costs 104½% of $100, or $104.50; and the cost of 75 shares is the product of $104.50 × 75, or *$7837.50.*

250. FORMULAS. — 1. *Par value × rate % = premium or discount.*

2. *Par value ± premium or discount = market value.*

3. *(Market val. of 1 share + brok.) × no. shares = entire cost.*

Problems.

Find the premium or discount, the market value, and the entire cost of —

1. 25 shares canal stock at $2\frac{1}{4}\%$ premium; at $3\frac{3}{4}\%$ discount.

2. 75 shares R. R. stock, at $103\frac{1}{2}$, brokerage $\frac{1}{4}\%$.

3. \$500 in bonds, quoted at 112, brokerage $\frac{1}{4}\%$; at $111\frac{1}{2}$.

4. Three \$1000 bonds at $123\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

5. A broker bought 70 shares of insurance stock at $2\frac{1}{2}\%$ discount, and afterwards sold them at $101\frac{3}{8}\%$. Find the gain.

6. If a manufacturing company calls for an installment of $12\frac{1}{2}\%$, what must a stockholder pay who owns 25 shares?

7. A railroad declared a dividend of $4\frac{1}{2}\%$. How much did a stockholder owning 37 shares receive?

8. When gold is quoted at $116\frac{5}{8}$, what is the currency value of \$3750 in gold? What is the amount of premium? Of brokerage, at $\frac{1}{8}\%$?

251. \$625 premium are paid for 50 shares of telegraph stock. What is the rate per cent.?

SOLUTION.

\$100 × 50 = \$5000, par value.
Premium ÷ par value = rate %.
\$625 ÷ \$5000 = .12½, or 12½ %.

\$5000, which is .12½, or *12½ %*.

EXPLANATION. — Since the premium is the product of the par value and the rate, the rate % must be the quotient of the premium, \$625, divided by the par value of the shares,

252. FORMULA. — *Premium ÷ par value = rate %.*

Find the rate per cent. of premium, discount, dividend, or installment —

9. If \$450 premium are paid for 100 shares R. R. stock.

10. If 50 shares stock sell at \$550 discount. If at \$62.50 premium; at \$125 discount.

11. If 80 shares bank stock sell for \$600 above par, brokerage $\frac{1}{4}\%$. If for \$200 below par; for \$220 above par.

12. If three \$1000 bonds sell for \$3555, brokerage $\frac{1}{8}\%$.

13. The net earnings of a company with a capital of $125000 are $8500. If it reserves $3500 as surplus for expenses, what rate of dividend can it declare? What is the dividend on 20 shares?

253. When railroad stock is quoted at $103\frac{3}{4}$, how many shares can be bought for $3120, brokerage $\frac{1}{4}\%$?

SOLUTION.

($103\frac{3}{4}\% + \frac{1}{4}\%$) of $100 = $104, cost of 1 share.
$3120 ÷ $104 = 30, no. of shares.

EXPLANATION. — Since the stock sells at $103\frac{3}{4}$, each share with the brokerage will cost 104% of $100, or $104; and for $3120 there can be bought as many shares as the number of times $104 are contained in $3120, or *30 shares.*

254. FORMULAS. — 1. *Market value ÷ $\overline{1 \pm rate}$ = par value.*

2. *Sum invested ÷ cost of* 1 *share = no. of shares.*

3. *Gain or loss ÷ gain or loss on* 1 *share = no. of shares.*

Find the par value, and the number of shares —

14. Of bonds bought for $2925, at $97\frac{1}{2}$, brokerage $\frac{1}{2}\%$.

15. Of stock sold at $6150, at $102\frac{1}{2}$, brokerage $\frac{1}{2}\%$.

16. Of stock bought at 18% discount, and sold at $16\frac{1}{4}\%$ discount at a gain of $140, brokerage each way $\frac{1}{4}\%$.

17. How many $100 bonds, at $17\frac{3}{4}\%$ premium, can be bought for $9430, brokerage $\frac{1}{8}\%$? For $11787.50?

18. A broker bought 84 shares of mining stock at $3\frac{1}{2}\%$ discount, and sold it at $5\frac{1}{8}\%$ discount. What did he lose?

19. A stock jobber bought 120 shares of stock at $82\frac{1}{2}$. He sold 50 shares at $77\frac{3}{4}$, and the balance at $91\frac{1}{2}$. What did he gain or lose?

255. How much must be invested in a 6% stock, at 95, to yield an annual income of $1200?

SOLUTION.

$100 × .06 = $6, income of 1 share.
$1200 ÷ $6 = 200, no. of shares.
$95 × 200 = $19000, investment.

EXPLANATION. — Since the interest on 1 share of 6% stock is $6, to yield $1200 interest requires as many shares as the number of times $6 are contained in $1200, or 200 shares; and since the sum invested in each share is $95, the sum invested in 200 shares is 200 times $95, or *$19000.*

256. FORMULAS.—1. *Income ÷ income of* 1 *share = no. of shares.*

2. *Price of* 1 *share × no. of shares = sum invested.*

What sum must be invested in —

20. 6% stock at 85, to yield an annual income of $900?

21. U. S. bonds at 110⅜, brokerage ⅛%, to give $1000?

22. Stock at 90, brokerage ¼%, to give $350? $840?

23. U. S. bonds are quoted at 117⅝, brokerage ⅛%. What must be invested to give a yearly income of $900? If at 118¼?

24. What sum must I invest in railroad stock, at 106¼, which pays 4% semi-annual dividends, to realize a yearly income of $800?

257. What per cent. of income will 8% bonds pay when bought at $120?

SOLUTION.

$\$100 \times .08 = \$8$, *int. on 1 share, at par.*

$\$8 \div \$120 = .06\frac{2}{3}$, *or* $6\frac{2}{3}\%$.

EXPLANATION. — The interest on one share at par as principal, at 8%, is $8.

Since 1 share at 120 yields $8 interest, the rate % is the quotient of the interest, $8, divided by the principal, $120, which is .06⅔, or $6\frac{2}{3}\%$, the rate of income required.

258. FORMULA. — *Income of* 1 *share ÷ cost per share = rate % of income.*

Find the rate % of income realized from —

25. 6% stock at 95. At 102½. At 97½. At 105. At 110.

26. Railroad stock paying 10 % annual dividend bought at 120.

27. Stock bought at 114⅛, brokerage ⅛%. At 116⅞.

28. Bank stock at 89¾, brokerage ¼, annual dividend 8%.

29. If I invest in five $100 bonds, at 97¾, what interest do I receive? What rate % on my investment, brokerage ¼%? If I buy at 98½, brokerage ¼%?

30. What per cent. of income will be realized on an investment in stock paying 10% dividends, if bought at a premium of 12⅜, brokerage ⅛%?

259. At what price must 6% stock be bought to yield an income of 7½% on the investment?

SOLUTION.

$100 × .06 = $6, int. on 1 share.
$6 ÷ .075 = $80, price per share.
$80 per share = .80, or 80%.

EXPLANATION.—Since the interest on 1 share at par, at 6%, is $6, and the rate of income to be realized is 7½%, the price or principal must be the quotient of the interest, $6, divided by the rate %, .075, or $80, the price per share, which is .80, or *80%* of par.

260. FORMULA. — *Income from* 1 *share* ÷ *rate % of income = cost of stock.*

At what price must —

31. 6% stock be bought to yield an income of 8%? 9%?

32. 8% stock be bought to yield 6% interest? To yield 7%?

33. Bonds to yield 5% in gold? 6% in currency, gold being at 115? To yield 6½% in currency, gold at 112½?

34. What premium must be paid on stock paying 10% dividend, to realize 7½% on the investment?

35. What rate of interest does an investment in 4½% bonds at 90% pay? In 5% bonds, at 118, gold being at 15% above par?

36. If I invest $10000 in bonds at 115, when gold is worth 112½, what rate % do I receive on my investment?

37. Which is the better investment and how much%, 4% bonds at 89, or 5% bonds at 119, brokerage ⅛%?

Section XI.

EXCHANGE.

261. A ***Draft*** is a written order directing one person to pay a certain sum of money to a third person at a specified time.

The person who makes the order or draft is called the *Drawer*. The person who is ordered to pay the money is the *Drawee*. The person to whom, or to whose order, the sum is payable, is the *Payee*.

262. A ***Sight Draft,*** or ***Bill,*** is a draft which requires payment to be made when it is presented.

263. A ***Time Draft*** is a draft which requires payment to be made at a time named in it.

264. The ***Acceptance*** of a draft is the agreement by the drawee to pay the draft at its maturity.

Note 1.— The drawee *accepts* a draft by writing the word "Accepted" across the face of it, with the date, and his name. He then becomes responsible for its payment.

2. Three days of grace are generally allowed on time drafts, as on promissory notes, but usually not on sight drafts.

265. ***Exchange*** is a method of making payments or of remitting value from one place to another by means of drafts or bills instead of money.

Form of a Draft.

$525 $\frac{50}{100}$. PHILADELPHIA, *Dec. 1, 1901.*

Ten days after date, pay to the order of Wilson & Co., five hundred twenty-five $\frac{50}{100}$ *dollars, value received, and charge to the account of*

To Browning Bros.,
Trenton, N. J.

William Tempest.

The words "*At sight,*" instead of "--- *days after sight,*" or "--- *days after date,*" are used in a sight draft. When the time is *after sight,* it means after *acceptance.*

266. The ***Rate*** or ***Course of Exchange*** is the current or market rate on the face of a draft or bill.

Exchange is *at par* when a draft sells for its face; at a *premium* when it sells for more than its face; and at a *discount* when it sells for less than its face.

267. When *time* is *not an element,* the terms of Exchange correspond to those of Percentage, as follows: —

1. The ***Face of the Draft*** is the *Base.*
2. The ***Rate of Exchange*** is the *Rate.*
3. The ***Premium*** or the ***Discount*** is the *Percentage.*
4. The ***Cost of the Draft*** is the *Amount* or the *Difference.*

268. When *time* is *an element*, the terms and the principles correspond to those of Percentage and Bank Discount combined.

Written Exercises.

269. *Example* 1. — Find the cost of a sight draft on New York, for $1250, at $1\frac{1}{4}\%$ premium.

SOLUTION.

$\$1 + \$.01\frac{1}{4} = \$1.01\frac{1}{4}$, *cost of* $\$1$.

Cost of $\$1 \times$ *face* = *cost of draft.*

$\$1.01\frac{1}{4} \times \$1250 = \$1265.62\frac{1}{2}$.

EXPLANATION.—Since at $1\frac{1}{4}\%$ premium, $1 of exchange costs $1.01¼, the cost of a draft of $1250 is 1250 times $1.01¼, or $\$1265.62\frac{1}{2}$.

270. *Example* 2.—What is the cost of a draft on Boston for $1500, payable in 60 days, exchange at a premium of $1\frac{1}{2}\%$?

SOLUTION.

Bank discount of $\$1$ *for 63 days* = $\$.0105$.

$\$1 - \$.0105 = \$.9895$, *proceeds of* $\$1$.

$\$.9895 + .015 = \$1.0045$, *cost of* $\$1$.

Cost of $\$1 \times$ *face* = *cost of draft.*

$\$1.0045 \times \$1500 = \$1506.75$.

EXPLANATION.—Since the bank discount of $1 for 63 days, at 6%, is .0105, the bank proceeds is $.9895; and at $1\frac{1}{2}\%$, or .015 premium, the cost of $1 of exchange is $1.0045, and the cost of $1500 is 1500 times $1.0045, or $\$1506.75$, the value of the draft required.

271. FORMULA. — *Cost of* $1 $\times$ *face* = *cost of draft.*

Problems.

Find the cost of —

1. A sight draft on St. Louis for $1200, at 2% premium.

2. A sight draft on Louisville for $2130, at $2\frac{1}{2}\%$ discount.

3. A sight draft on Pittsburg for $2375, at $1\frac{3}{4}\%$ premium.

4. How much must be paid in Philadelphia for a draft of $1275.50 on Richmond, exchange being at $1\frac{1}{2}\%$ discount?

5. A broker sold some uncurrent money at $2\frac{1}{2}\%$ discount. How much did he receive for it? How much did he lose?

6. What is the cost of a draft on Cincinnati, at 30 days, for $3000, interest 6%, and premium 1%? At $1\frac{1}{2}\%$?

7. Find the cost of a draft on New Orleans, at 90 days, for $7275, interest 5%, and discount $1\frac{1}{2}\%$. $2\frac{1}{2}\%$ discount.

8. What must be paid in New York for a draft on Boston for $3500 at 90 days, the course of exchange being $100\frac{3}{4}\%$?

272. What is the face of a sight draft which cost $492.50, exchange $1\frac{1}{2}\%$ discount?

SOLUTION.

$1 − $.01½ = $.985, cost of $1.
Cost of draft ÷ cost of $1 = face.
$492.50 ÷ $.985 = $500.

EXPLANATION. — Since at $1\frac{1}{2}\%$ discount $1 of face costs $.985, to cost $492.50 requires as many dollars as the number of times $.985 is contained in $492.50, or *$500*, the face required.

273. Find the face of a draft on Washington at 90 days, purchased for $2500, interest 6%, premium $1\frac{1}{2}\%$.

SOLUTION.

Bank dis. of $1 for 93 da. = $.0155.
$1 − $.0155 = $.9845, proceeds of $1.
$.9845 + .015 = $.9995, cost of $1.
Cost of draft ÷ cost of $1 = face.
$2500 ÷ $.9995 = $2501.25.

EXPLANATION. — Since the bank discount of $1 for 93 days is $.0155, the bank proceeds is $.9845; and at $1\frac{1}{2}\%$, or $.015 premium, the cost of $1 face exchange is $.9995; and to cost $2500 requires as many dollars of face as the number of times $.9995 are contained in $2500, or *$2501.25*, the face required.

274. FORMULA. — *Cost of draft ÷ cost of* $1 = *face.*

Problems.

Find the face of —

9. A sight draft bought for $627, at $1\frac{3}{8}\%$ premium.

10. A sight draft which cost $636.25, discount $1\frac{3}{8}\%$.

11. A sight draft drawn on Mobile for $903.24, exchange $96\frac{1}{2}$.

12. How large a sight draft can be bought for $1256.25, exchange at $\frac{1}{2}\%$ premium? At $\frac{1}{2}\%$ discount?

13. Find the face of a draft on Baltimore at par, 30 da., which was sold for $502.25, interest 6%, premium 1%.

14. How large a draft, payable 60 da. after sight, can be bought for $1597.60, interest 8%, exchange $101\frac{1}{4}$?

15. What is the face of a draft, exchange at $101\frac{1}{4}$, which will discharge a note of $3000, for 1 yr., interest at 7%?

Section XII.

TAXES.

275. A ***Tax*** is a sum of money imposed on the person or the property of an individual to meet public expenses.

CASE I.

General Taxes.

276. A ***Poll Tax*** is a tax imposed on every male citizen not exempt by law.

277. A ***Property Tax*** is a tax imposed on property according to its estimated or assessed value.

278. Property is of two kinds: *Real Property* and *Personal Property.*

279. ***Real Property,*** or ***Real Estate,*** is *fixed* property; such as *houses* and *lands.*

280. ***Personal Property*** is *movable* property; such as *money, stocks, furniture, cattle, merchandise,* etc.

281. An ***Assessor*** is an officer appointed to estimate or appraise the taxable value of property, to make tax lists, and to apportion taxes among the tax payers.

282. The ***Tax Rate*** is the rate per cent. on the valuation of the property on which a certain amount of taxes is to be raised.

The tax rate is usually fixed at so many mills on $1, or so many dollars on $100.

283. ***General Taxes*** are taxes imposed for State, county, city, town, or district purposes.

284. General taxes are computed upon the assessed value of property as the base.

285. The corresponding terms of Taxes and Percentage are—

1. The ***Valuation*** of the property is the *Base.*
2. The ***Tax Rate*** or ***Tax on $1*** is the *Rate.*
3. The ***Sum to be raised*** is the *Percentage.*
4. The ***Sum Collected*** less the commission is the *Difference.*

Problems.

286. What amount of tax is raised by a district having property assessed at $69500, the rate being 3 mills on the dollar?

SOLUTION.
Valuation × *tax rate* = *tax.*
$\$69500 \times .003 = \$208.50$.

EXPLANATION. — Since the rate is 3 mills on the dollar, it is .003 of $1; and the tax on $69500 is the product of $69500 × .003, or $\$208.50$.

287. FORMULA. — *Valuation* × *tax rate* = *tax.*

1. How much tax must a man pay whose property is assessed at $6250, if he pays $1\frac{3}{4}\%$ city tax, and $\frac{1}{2}\%$ State tax?

2. One year a man whose property was assessed at $3275, paid $2\frac{1}{2}\%$ school tax, road tax 1%, county tax $1\frac{1}{4}\%$, and $1 poll tax. What was the amount of his taxes?

288. What tax rate will raise $8500 for school-houses in a county having property assessed at $3400000?

SOLUTION.
Tax ÷ *valuation* = *tax rate.*
$\$8500 \div \$3400000 = .002\frac{1}{2}$, *or* $.2\frac{1}{2}\%$, *or* $2\frac{1}{2}$ *mills on the dollar.*

EXPLANATION.—Since the amount of tax raised is the product of the valuation multiplied by the rate, the tax rate is the quotient of the tax, $8500, divided by the valuation, $3400000, which is $.002\frac{1}{2}$, or $2\frac{1}{2}$ *mills* on the dollar.

289. FORMULA. — *Tax* ÷ *valuation* = *tax rate.*

3. What rate of school tax will give $13.80 on property assessed at $6000? $7.43\frac{3}{4}$ on property assessed at $4250?

4. A tax of $1562.50 for building a bridge was levied upon a county having an assessed valuation of $1250000. What is the tax on a farm assessed at $3275?

290. If the rate is $\$.001\frac{1}{2}$ on a dollar, and the tax is $\$10.87\frac{1}{2}$, what is the valuation?

SOLUTION.
Tax ÷ *tax rate* = *valuation.*
$\$10.87\frac{1}{2} \div \$.0015 = \$7250$.

EXPLANATION. — Since the amount of tax is the product of the valuation multiplied by the tax rate, the valuation is the quotient of the tax, $10.875, divided by the rate, $.0015, or $\$7250$, the valuation required.

291. FORMULA. — *Tax ÷ tax rate = valuation.*

5. What is the valuation of property which pays a tax of $73.50, at the rate of $1\frac{3}{8}$ mills on a dollar?

6. If a tax of $292.20 is levied on property assessed at $24350, what is the valuation of property that pays tax of $55.50?

292. What sum must be assessed to raise $3600 after deducting the cost of collection, at 4%?

SOLUTION.

Sum raised ÷ (1 — rate %) = sum assessed.

$3600 ÷ (1 — .04 = .96) = $3750.

EXPLANATION.—Since at 4% for collection the sum raised is but .96 of the sum assessed, to raise $3600 requires as many dollars to be assessed as the number of times .96 is contained in 3600, or *$3750*

7. What sum must be assessed in a district to build a bridge at a cost of $1413.75, and to pay 2% for collection?

8. A school-house costing $4784.85 was built by a tax upon the property of a town. The rate of taxation was $2\frac{1}{4}$ mills on a dollar, and the cost of collection $2\frac{1}{2}$%. Find the valuation.

293. Rule for General Taxes.

I. *Subtract the amount of poll tax, if any, from the whole amount to be raised, and the remainder will be the property tax.*

II. *Divide the property tax by the valuation of taxable property, and the quotient will be the rate of taxation.*

III. *Multiply the valuation of each man's property by the rate, and to the product add his poll tax, if any. The sum will be the whole amount of his tax required.*

Problems.

9. A tax of $11440.50 was levied on a certain township. The valuation of taxable property was $875240, and the number of polls at $1 each was 500. What was the rate of property tax, and what was A's tax, his property being valued at $6252, and who pays for 4 polls?

10. A tax of $24500 is to be imposed on a certain town. The

valuation of taxable property is $2000000, and there are 400 polls, each assessed $1.25. What will be the tax on $1 of valuation? What is B's tax, whose valuation is $7500, and who pays for 1 poll?

CASE II.

National Taxes.

294. ***Duties,*** or ***Customs,*** are taxes levied on imported goods and on the tonnage of vessels.

295. A ***Custom House*** is a Government building or office at which duties are collected, vessels entered, etc.

296. An ***Invoice*** is a written statement of a ship's cargo, with the original cost of the articles, the charges, etc.

297. Before calculating duties, certain allowances, or deductions, are made, called *Tare*, *Leakage*, *Breakage*, etc.

Tare is an allowance for the weight of the box, bag, or cask containing the goods.

Leakage is an allowance for the loss of liquor imported in casks.

Breakage is an allowance for the loss of liquor imported in bottles.

298. ***Gross Weight*** or ***Value*** is the weight or value of goods before any allowances or deductions are made.

299. ***Net Weight*** or ***Value*** is the weight or value of goods after all allowances or deductions are made.

300. ***Duties*** are of two kinds: *Specific Duties* and *Ad Valorem Duties.*

301. A ***Specific Duty*** is a tax on the net number or quantity of goods imported without regard to the value.

302. An ***Ad Valorem Duty*** is a tax of a certain rate per cent. of the net cost of goods in the country from which they are imported.

A list of the rates of duties fixed by the Government on all kinds of imported goods is called a *Tariff.*

303. ***National Taxes*** are taxes imposed to meet the expenses of the National Government.

304. The corresponding terms of Ad Valorem Duties and Percentage are —

1. The ***Net Value*** is the *Base.*

2. The ***Rate*** % ad valorem is the *Rate.*

3. The ***Duty*** is the *Percentage.*

Problems.

305. *Example.* — What is the duty, at 5 cents a pound, on 100 bags of coffee, each 75 lb. gross weight, tare 5%?

SOLUTION.

75 lb. × *100* = *7500 lb. gross.*
7500 lb. × *.05* = *375 lb. tare.*
7500 lb. — *375 lb.* = *7125 lb. net.*
$.05 × *7125* = *$356.25, duty.*

EXPLANATION. — From the gross weight subtract the tare for the net weight upon which duty is levied. The duty on 1 lb., $.05, multiplied by the number of pounds taxed, 7125, gives *$356.25*, the duty required.

306. FORMULA. — *Rate* × *net quantity* = *specific duty.*

1. Find the duty, at 4½ cents a pound, on 50 casks of raisins, each 120 lb. gross weight, tare 1%.

2. What is the duty on 25 hhd. of molasses at 9 cents per gallon, an allowance of 5% leakage being made?

307. A merchant imported a lot of silk invoiced at $17500. What was the duty at 50% ad valorem?

SOLUTION.

$17500 × *.50* = *$8750.*

EXPLANATION. — Since the duty was 50% of the invoice price of the silk, the product of the invoice price, $17500, multiplied by the rate %, .50, gives *$8750*, the amount of the duty required.

308. FORMULA. — *Net inv. price* × *rate %* = *ad val. duty.*

3. A dealer imported 25 clocks invoiced at $22.50, and 50 Geneva watches at $75. What was the duty, the rate on the clocks being 20%, and on the watches 30%?

4. What is the ad valorem duty on 125 chests of tea, each 70 lb., invoiced at $.50 a pound, the tare 7 lb. per chest, and the duty 40% ad valorem?

5. Find the duties on 25 cases of tobacco invoiced 60 lb. each, and 10000 Havana cigars weighing 150 lb., invoiced at $50 per M, the duty on tobacco being $.35 per lb., and on cigars $3 per lb. specific, and 50% ad valorem.

6. A merchant imported from England 24 sacks of wool weighing 2500 lb., invoiced at 1 s. 3 d. per lb. What duty did he pay in the United States money, the rate being 10 cents per pound and 11% ad valorem, 3% tare allowed?

Section XIII.

AVERAGE OF PAYMENTS.

1. The interest of $10 for 1 year equals the interest of $1 for how many years at the same rate?

2. The interest of $20 for 2 years equals the interest of $10 for how many years at the same rate %?

309. Analysis. — At the same rate % for 2 years, the interest of $20, which is twice $10, equals the interest of $10 for twice the time, or twice 2 years, which are 4 years. Hence, etc.

3. The interest of $30 for 3 years equals the interest of $1 for how many years? Of $10? Of $15? Of $25?

4. The interest of $1 for 10 years equals the interest of $10 for how many years, at the same rate?

5. The interest of $10 for 4 years equals the interest of $20 for how many years? Equals the interest of $8?

6. If I borrow $25 for 6 months, for how many months should I lend $50 to balance the interest?

310. Analysis. — Since the interest of $25 for 6 months equals the interest of $1 for 25 times 6 months, or 150 months, the interest of $50 equals the interest of $1 for $\frac{1}{50}$ of 150 months, or 3 months. Hence, etc.

7. If A has the use of $100 of B's money for 8 months, for how long a time should B have the use of $50 of A's money to balance the favor?

Definitions.

311. Average of Payments is the process of finding the average time for the payment of several sums of money due at different times, without loss to either debtor or creditor.

312. The ***Average*** or ***Equated Time*** is the date at which the several sums due may be paid together without loss to either party.

313. The ***Term of Credit*** is the time which must elapse before a debt becomes due.

314. The ***Average Term of Credit*** is the time which must elapse before several sums of money due at different times may all be paid together without loss to debtor or creditor.

315. Principle.

The rate per cent. being the same, the less the principal the longer the time; and the greater the principal the shorter the time, to give the same interest.

CASE I.

When the Terms of Credit begin at the Same Time.

316. *Example.* — I owe \$200 cash, \$300 in 4 mo., and \$400 in 6 mo. In what time would one payment discharge all?

SOLUTION.

\$200 cash.
300 for 4 mo. = *\$1 for 1200 mo.*
400 for 6 mo. = *\$1 for 2400 mo.*
\$900 for ? mo. = *\$1 for 3600 mo.*
3600 mo. ÷ *900* = *4 mo.*

EXPLANATION. — Since \$200 are paid in cash, it has no term of credit.

The credit of \$300 for 4 mo. is the same as the credit of \$1 for 300 times 4 mo., or 1200 mo.

The credit of \$400 for 6 mo. is the same as the credit of \$1 for 400 times 6 mo., or 2400 mo.

Hence, the credit on the whole debt, \$900, is the same as the credit of \$1 for 3600 mo.; and the credit of \$1 for 3600 mo. is the same as the credit of \$900 for $\frac{1}{900}$ of 3600 mo., which is *4 mo.*

317. Rule to Find the Average Term of Credit.

Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient will be the average term of credit required.

Note. — If the average or equated *date* of payment is required, add the average term of credit to the date at which the several terms begin.

Problems.

1. Find the average term of credit of $400 due in 1 mo., $600 due in 3 mo., and $800 due in 4 mo.

2. What is the average or equated term of credit of $600 due in 2 mo., $900 due in 4 mo., and $1000 due in 6 mo.?

3. A merchant owes $600 due in 3 mo., and $800 due in 10 mo. In what time would one payment discharge both debts?

4. A grocer owes $1000, of which $\frac{1}{2}$ is payable in 2 mo., $\frac{1}{4}$ in 4 mo., and the remainder in 6 mo. What is the average term of credit? Find the equated date of payment.

5. A merchant bought goods, July 1, 1901, as follows: $500 in 2 mo., $600 in 3 mo., and $800 in 9 mo. Find the average term of credit and the equated time of payment.

6. Find the average term of credit and the equated time of payment, from October 15, of $250 due in 30 da., $400 due in 60 da., and $500 due in 90 da.

7. If I owe $800 due in 6 mo., and pay cash $200, how long may I keep the balance as an equivalent?

8. On a debt of $2000, due in 9 mo. from January 1, there were paid April 1, $400, June 1, $500, and August 1, $600. When was the balance due?

CASE II.

When the Terms of Credit begin at Different Dates.

318. *Example.* — Find the average or equated time for the payment of the following bills: Sept. 15, 1901, $250 on 4 mo.; Oct. 10, $500 on 2 mo.; and Nov. 20, $600 on 3 mo.

SOLUTION.

Oct. 10 + 2 mo. = Dec. 10; $500 cash.
Sept. 15 + 4 mo. = Jan. 15; 250 for 36 da. = $1 for 9000 da.
Nov. 20 + 3 mo. = Feb. 20; 600 for 72 da. = $1 for 43200 da.
$1350 for ? da. = $1 for 52200 da.
52200 da. ÷ 1350 = $38\frac{2}{3}$ da.; Dec. 10 + 39 da. = Jan. 18, 1902.

EXPLANATION. — By adding the term of credit to the date of each, the several bills are found to fall due Jan. 15, Dec. 10, and Feb. 20, respectively.

Since the first payment falls due Dec. 10, it is taken as the standard date from which to reckon the average term of credit.

$500 due Dec. 10 has no credit; $250 from Dec. 10 to Jan. 15 has a credit of 36 da.; and $600 to Feb. 20 has a credit of 72 da. Hence the average term of credit is 39 da. nearly; and 39 da. after Dec. 10 is *Jan.* 18, 1902, the average time required.

319. Rule to Find the Average Time when the Credits begin at Different Dates.

I. *Find the date at which each debt becomes due.*

II. *Take the earliest date at which any of the debts becomes due as a standard date, and from it find the term of credit of each debt.*

III. *Multiply each debt by its term of credit from the standard date, and divide the sum of the products by the sum of the debts for the average term of credit.*

IV. *Add the average term of credit to the standard date for the average time of payment required.*

Problems.

1. Find the average term for the payment of the following bills: June 10, $325 on 3 mo.; Feb. 20, 2 mo.; April 15, 4 mo.

2. I bought goods as follows: Oct. 5, 1901, on 4 mo., $450; Nov. 8, on 2 mo., $600; Dec. 16, on 3 mo., $700; and Jan. 5, 1902, on 3 mo., $800. Find the equated time for one payment.

3. Find the equated time for the payment of three notes, one $750, dated Oct. 25, due in 90 da.; one for $600, Nov. 10, due in 30 da.; and one for $500, Dec. 20, due in 60 da.

4. A merchant has the following charges against a customer: Sept. 8, 1901, $875 on 90 da.; Oct. 6, $750 on 30 da.; Nov. 10, $1000 on 60 da.; and Dec. 1, $500 on 4 mo. Find the equated time of maturity.

5. A farmer bought fertilizers on a credit of 90 days: Aug. 8, a bill of $400, and Sept. 17, a bill of $300. Find the average date of purchase, and the average time of payment.

Review Problems.

1. What % is made by buying stocks at 10% below par, and selling them at 10% above par?

2. If I invest $4500 in bonds at 12½% below par, what % upon my investment will the interest be?

3. I bought a house, and insured it for $4500, paying $80.25 premium, and cost of policy $1.50. Find the rate.

4. How much must be paid for a draft of $1250 payable 10 days after sight, exchange at ½% discount, interest 6%?

5. A's property is assessed at $5750, and B's at $7500. If A's tax is $86.25, how much is B's?

6. I owe $3000 payable in 6 mo. If I pay down one-third of the amount, how long may I retain the balance?

7. How many shares of bank stock at 2¼% discount can a broker buy for $4611.50, less his brokerage of ¼%?

8. What sum may be invested in 4½% bonds at 95, brokerage ¼%, to secure an annual income of $1000?

9. A house which had been insured for $3500 for 10 years at ¾% a year was destroyed by fire. How much did the money received from the company exceed the amount of premium?

10. What is the cost in U. S. money for a bill on London of £2000, exchange at $4.87½ per £, and gold 112½?

11. On June 1, 1901, I bought a bill of goods as follows: $500, cash; Aug 1, $600; Oct. 1, $800; Dec. 1, $100. What was the average date of purchase?

12. A district school-house cost $3000, and the property of the district was assessed at $60000. What was the rate, and what was A's tax, his property being valued at $3500?

13. A broker exchanged 75 shares of railroad stock at 90 for insurance stock at 112½. How many shares did he receive?

14. I bought some Government securities, par value $2000, at 105⅛, and sold them at 109⅞, brokerage each way ⅛%. What did I gain?

15. A flour dealer had 400 bbl. flour insured for 75% of their cost at $2\frac{3}{4}$%, paying \$68.75 premium. At what price per bbl. must he sell to clear 20%?

16. Find the face of a bill on London which cost \$2500, exchange being \$4.88½ to the £.

17. Find the ad valorem duty, at 40%, on 75 chests of tea, each containing 70 lb., invoiced at 60 cents per lb., tare being 6 lb. to the chest.

18. I bought on a credit of 90 days, Dec. 6, a bill of \$200, and January 15, \$200. Required the average date of purchase and the average time of credit.

19. A broker bought 50 shares of stock at 95, and sold them at par, after having received a 6% dividend. What did he gain?

20. The capital of a mining company is \$225000. The gross receipts are \$50450, and the expenses are \$32950. What rate of dividend can it declare after reserving a surplus of \$6250?

21. At \$105.50 for \$1000, what amount of premium must a man aged 35 years pay to receive an endowment policy for \$5000, payable to himself in 10 years, or his heirs if he die sooner?

22. The duty on 1200 yd. of silk at 25% ad valorem was \$625. What was the invoice price per yard? For how much must the importer sell the silk to clear 20%?

Review Questions.

1. Define ***Insurance.*** Explain the operation of insurance. What does property insurance include? What is the policy? The valuation? The premium? Upon what is premium computed? Name the corresponding terms of Insurance and Percentage. Give the formulas to find the premium, the rate, and the valuation.

Explain the operation of life insurance. Explain the difference between life and endowment policies. Upon what is the premium in life insurance computed?

2. Define ***Corporation.*** What is meant by capital stock? A share? A certificate? The par value of stock? The market value? When is stock at par? At a premium? At a discount? Upon what are premium and discount computed? What are stock quotations? What is an installment? An assessment? A dividend? Upon what are assessments and dividends computed? What are gross earnings? Net earnings?

What is a ***Bond?*** By whom are bonds issued? What are treasury notes? Government securities? Name and explain the principal U. S. securities. How are bonds issued by states, counties, etc., named? What are stocks? Who is a stock-broker? What is brokerage? Name the corresponding terms of Stocks and Percentage.

Give the formula to find the premium or discount. Assessment or dividend. To find the rate % of premium or discount. To find the par value or number of shares. To find the sum to be invested to yield a certain sum. To find the rate % of income from an investment. To find the cost of stock to yield a certain rate %.

3. Define ***Draft.*** Name the parties interested in a draft. What is the difference between a sight and a time draft? What is meant by acceptance? Define ***Exchange.*** Give the form of a draft. What is the rate or course of exchange? Name the corresponding terms of Exchange and Percentage. Give the formulas to find the cost of a draft. The face.

4. What is a Tax? A poll tax? A property tax? What is real estate? Personal property? An assessor? The tax rate? How is the tax rate fixed? What are general taxes? Upon what are taxes computed? Name the corresponding terms of Taxes and Percentage. Give the formula to find the tax, the rate, and the valuation. The rule for general taxes.

What are ***Duties,*** or ***Customs?*** What is a custom-house? A port of entry? An invoice? What allowances are made? Explain each. What is gross weight? Net weight? Explain the difference between a specific and an ad valorem duty. What are national taxes? Name the corresponding terms of ad valorem duties and percentage. Explain how to find specific duties. Give the formula to find ad valorem duties.

5. Define ***Average of Payments.*** What is the average or equated time? The term of credit? The average term of credit? State the principle. Give the rule to find the average term of credit, when the terms begin at the same time, and at different dates.

CHAPTER VI.

RATIO AND PROPORTION.

Section I.

RATIO.

1. William has 15 dollars, and Charles has 5 dollars. William has how many times as much money as Charles?

2. A 20-dollar note is worth how many times as much as a 10-dollar note? 20 is how many times 10?

3. The value of a 10-dollar note is what part of the value of a 20-dollar note? 10 is what part of 20?

4. How does a chain 30 feet long compare in length with a chain 6 feet long? What is the relation of 30 to 6?

5. What is the relative value of 8 yards compared with 24 yards? What is the relation of 8 to 24? 8 is what part of 24?

6. What is the relation of 8 to 40? 25 to 10? \$9 to \$45? 50 feet to 20 ft.? $12\frac{1}{2}$ to 75? $12\frac{1}{2}$ to $6\frac{1}{4}$? 8 to 4? $\frac{1}{10}$ to 20?

Definitions.

320. ***Ratio*** is the relative value of one number compared with another number of the same kind.

321. Ratio is expressed by the quotient of the first number divided by the second number.

Thus, the ratio of *12 to 4* is $12 \div 4 = 3$; of 5 to 15 is $5 \div 15 = \frac{5}{15}$, or $\frac{1}{3}$.

322. The ***Terms of a Ratio*** are the two numbers whose values are compared.

323. The ***Antecedent*** is the first term, or the *dividend.*

324. The ***Consequent*** is the second term, or the *divisor.*

The two terms of a ratio considered together are called a *Couplet.*

325. The ***Sign of Ratio*** is the colon ***(:)*** placed between the two numbers. It is the sign of division ($\div$) with the horizontal line omitted.

Thus, $8 : 4$ denotes the ratio of 8 to 4, or $8 \div 4$, or $\frac{8}{4}$, or 2; and $4 : 8$ denotes the ratio of 4 to 8, or $4 \div 8$, or $\frac{4}{8}$, or $\frac{1}{2}$.

326. Ratios are of two kinds: *Simple Ratios* and *Compound Ratios.*

327. A ***Simple Ratio*** is the ratio of one number to another number of the same kind.

Thus, \$9 : \$5 is a *simple ratio;* so, also, is 8 ft. : 3 ft.; $\frac{1}{2} : 4$.

328. A ***Compound Ratio*** is the ratio of the products of the corresponding terms of two or more simple ratios.

Thus, $\left.\begin{matrix} 8:4 \\ 9:3 \end{matrix}\right\}$, or $(8:4) \times (9:3)$, or $8 \times 9 : 4 \times 3$ is a *compound ratio.*

329. The ***Value of a Ratio*** is the quotient of the antecedent divided by the consequent. It is always an abstract number.

Thus, in the ratio of *15 ft. : 5 ft.*, the terms of the ratio are 15 ft. and 5 feet, and the quotient of 15 feet ÷ 5 ft. is *3*, the value of the ratio.

330. Principles of Ratio.

1. *The terms of a ratio must be similar numbers.*

2. *The value of the ratio of two numbers is the quotient of the antecedent divided by the consequent.*

331. Since the antecedent is a dividend, and the consequent is a divisor, any change in the antecedent or the consequent will affect the value of the ratio according to the general principles of division and the general principles of fractions. Hence, also,

332. General Principles of Ratio.

1. A ratio is multiplied by { *Multiplying the antecedent.* Or, *Dividing the consequent.*
2. A ratio is divided by { *Dividing the antecedent.* Or, *Multiplying the consequent.*
3. A ratio is not changed by { *Multiplying or dividing both antecedent and consequent by the same number.*

Exercises.

Analyze the ratio 21 : 7.

333. *Model.* — The ratio *21 : 7* is read *21 to 7;* the *antecedent*, or dividend, is *21;* the *consequent*, or divisor, is *7.*

Since it is a ratio between two numbers, it is a simple ratio; and since the quotient of the antecedent by the consequent is 3, the value of the ratio is 3.

Analyze the following ratios: —

1. *4 : 2; 3 : 6.*
2. *6 : 3; 4 : 8.*
3. *5 : 10; 12 : 6.*
4. *16 : 4; 5 : 20.*
5. *$10 : $30; 40½ : 8.*
6. *12½ sq. ft. : 5 sq. ft.*
7. *10½ gal. : 33⅓ gal.*
8. *16.5 yd. : 5.5 yd.*
9. { *3 : 6* ; { *½ : 4* / *7 : 2* ; { *8 : ¼*
10. { *2½ ft. : 12½ ft.* / *11⅓ yd. : 3⅓ yd.*

Written Exercises.

334. *Example* 1. — The antecedent is $16\frac{1}{2}$ feet, and the ratio 3. Find the consequent.

SOLUTION.

Antecedent ÷ *ratio* = *consequent.*

$16\frac{1}{2}$ *ft.* ÷ 3 = $5\frac{1}{2}$ *ft.*

EXPLANATION. — Since the antecedent is the dividend, and the consequent is the divisor, the quotient of the antecedent, $16\frac{1}{2}$ ft., divided by the ratio, 3, gives $5\frac{1}{2}$ *ft.*, the consequent required.

335. *Example* 2. — Simplify the compound ratio $\begin{cases} 3 : 8 \\ 3\frac{1}{3} : 4\frac{1}{2} \end{cases}$

SOLUTION.

$$\left.\begin{matrix} 3 : 8 \\ 3\frac{1}{3} : 4\frac{1}{2} \end{matrix}\right\} = (3 \times 3\tfrac{1}{3}) : (8 \times 4\tfrac{1}{2}) = \tfrac{10}{36} = \tfrac{5}{18}.$$

EXPLANATION. — Since a compound ratio is the ratio of the product of the corresponding terms of two or more simple ratios, find the product of the two antecedents, which is *10;* and the product of the consequents, which is *36.* Hence, the value of the ratio is the quotient of 10 ÷ 36, or $\frac{10}{36}$, or $\frac{5}{18}$.

336. FORMULAS. — 1. *Antecedent* ÷ *consequent* = *ratio.*
2. *Antecedent* ÷ *ratio* = *consequent.*
3. *Consequent* × *ratio* = *antecedent.*

Problems.

Find the value of the ratio —

1. 16 : 8; $2\frac{1}{2}$: 10.
2. 5 : 25; 9 : $2\frac{1}{4}$.
3. 2 : 15; $2\frac{2}{3}$: $7\frac{1}{2}$.
4. $5\frac{1}{2}$ yd. : 5 rods.
5. 3 gal. 2 qt. : 15 gal.
6. 40.5 bu. : 8 bu. 2 pk.
7. $\begin{cases} 3 : 4 \\ 7 : 8; \\ 9 : 2 \end{cases}$ $\begin{cases} 2\frac{1}{2} : 4\frac{1}{4} \\ 17 : 9 \\ 2.5 : 7.5 \end{cases}$

8. 10 T. : 7 T. 15 cwt.; 30 gal. 2 qt. 1 pt. to 46 gal. 1 qt. $1\frac{1}{2}$ pt.

9. 40 mi. 4 rd. 3 yd. to 30 mi. 3 rd. $2\frac{1}{4}$ yd.; 50.5 A. to 32 sq. rd.

10. The consequent is \$6.50, and the antecedent is \$50. What is the ratio? If the antecedent is $32\frac{1}{2}$ cents?

11. The consequent is 7.5 bu., and the ratio 7. Find the antecedent. If the ratio is .5?

12. The ratio is $\frac{5}{7}$, and the antecedent is $\frac{2}{3}$ of $7\frac{1}{2}$. What is the consequent? If the ratio is $\frac{3}{5}$ of $.62\frac{1}{2}$?

13. Find the ratio of $\begin{cases} 7\frac{1}{2} \text{ A.} : 4\frac{1}{2} \text{ sq. rd.} \\ 3\frac{1}{2} \text{ sq. rd.} : 8 \text{ A.} \end{cases}$; of $\dfrac{8\frac{1}{3}}{\frac{2}{5}}$ to $\dfrac{\frac{3}{4} \text{ of } \frac{6}{5}}{\frac{2}{3}}$.

Section II.

PROPORTION.

1. Name two numbers that have a ratio equal to that of 2 : 6; of 10 to 5; of $\frac{1}{2}$ to 4; of 6 to $1\frac{1}{2}$.

2. What number divided by 10 gives the same quotient as $15 \div 5$? As $21 \div 7$? As $6 \div 18$?

3. What number has the same ratio to 10 that 15 has to 5? That 6 has to 18? 24 to 8? $4\frac{1}{2}$ to $1\frac{1}{2}$?

4. To what number has 8 the same ratio that 5 has to 20? That 12 has to 48? That $2\frac{1}{2}$ has to 10?

5. The ratio of 27 to 9 is the same as the ratio of 21 to what number? The same as 18 to what?

6. Express the ratio of any two numbers equal to the ratio of 5 to 10. Equal to the ratio of 16 to 4. Of 4 to 20.

7. 21 is to 7 as 15 is to what number? 24 is to 8 as what number is to 3? 25 : 5 = 15 : what?

8. If two oranges cost 10 cents, what will 8 oranges cost at the same rate? 2 oranges : 8 oranges as 10 cents : how many cents?

Definitions.

337. A ***Proportion*** is an equality of ratios.

Thus, *6 : 2 = 9 : 3* is a *proportion* in which the ratio of 6 to 2 equals the ratio of 9 to 3.

338. The ***Sign of Proportion*** is the sign of equality (=), or the double colon (::), written between the two equal ratios.

Thus, *6 : 2 :: 9 : 3* denotes that the ratio of *6 to 2 equals* the ratio of *9 to 3*, or that 6 is to 2 as 9 is to 3.

339. Since proportion is an equality of two or more ratios, and each ratio consists of two terms, every proportion must have at least *four terms.*

Each ratio in a proportion is called a *Couplet*, and each term a *Proportional.*

340. The ***Antecedents*** of a proportion are the first term and the third term, or the antecedents of the two ratios.

341. The ***Consequents*** of a proportion are the second term and the fourth term, or the consequents of the two ratios.

342. The ***Extremes*** of a proportion are the first term and the fourth.

343. The ***Means*** of a proportion are the second term and the third.

Thus, the proportion 12 : 6 :: 8 : 4 consists of *four terms* or proportionals, *12*, *6*, *8*, and *4*; the ratio *12 : 8* is the *first couplet*, and *8 : 4* is the *second couplet*; *12* and *8* are the *antecedents*, and *6* and *4* are the *consequents*; *12* and *4* are the *extremes*, and *6* and *8* are the *means*.

344. Since every ratio may be expressed in the form of a common fraction, the proportion 12 : 6 :: 8 : 4 may be expressed $\frac{12}{6} = \frac{8}{4}$. Changing these fractions to similar fractions, $\frac{12 \times 4}{6 \times 4} = \frac{8 \times 6}{4 \times 6}$, or $\frac{12 \times 4}{24} = \frac{8 \times 6}{24}$; and

Since the resulting fractions have a common denominator, and are equal to each other, their numerators must be equal, and $12 \times 4 = 8 \times 6$; but 12 and 4 are the extremes, and 8 and 6 are the means. Hence,

345. Principles of Proportion.

I. *The product of the extremes equals the product of the means.*

II. *The product of the extremes divided by either mean gives the other mean.*

III. *The product of the means divided by either extreme gives the other extreme.*

Exercises.

Analyze the proportion 15 : 5 :: 24 : 8.

346. *Model.* — The proportion 15 : 5 :: 24 : 8 is read 15 is to 5 as 24 is to 8, or as 15 is to 5 so is 24 to 8.

It is composed of two ratios: the first ratio is 15 : 5, and the second ratio is 24 : 8; the antecedents are 15 and 24, and the consequents are 5 and 8; the four terms or proportionals are 15, 5, 24, and 8; the extremes are 15 and 8, and the means are 5 and 24.

Since the value of the first ratio is $\frac{15}{5}$, or 3, and the value of the second ratio is $\frac{24}{8}$, or 3, the ratios are equal to each other; and since the product

of the extremes is 15×8, or 120, and the product of the means is 5×24, or 120, the product of the extremes equals the product of the means.

Analyze the following proportions: —

1. $4 : 2 :: 6 : 3$.
2. $6 : 2 :: 9 : 3$.
3. $5 : 10 :: 6 : 12$.
4. $15 : 3 :: 20 : 4$.
5. $\$50 : \$10 :: \$60 : \$12$.
6. 5 ft. $: 2\frac{1}{2}$ ft. $:: 7$ yd. $: 3\frac{1}{2}$ yd.
7. 5.5 ft. $: 22$ ft. $:: 7.5$ mi. $: 30$ mi.
8. $7\frac{1}{3}$ T. $: 29\frac{1}{3}$ T. $:: 8\frac{1}{8}$ bu. $: 32\frac{1}{2}$ bu.

Written Exercises.

347. *Example.* — Find the fourth term in the proportion $8 : 32 :: 10 : —$.

SOLUTION. $(32 \times 10) \div 8 = 40$, *and* $8 : 32 :: 10 : 40$.

EXPLANATION. — Since the product of the extremes equals the product of the means, and the product of the means is 320, the fourth term must be the quotient of $(30 \times 12) \div 8$, which is 40; for, the product of two factors divided by one of the factors gives the other factor.

348. FORMULAS FOR PROPORTION.

1. *Product of means ÷ given extreme = required extreme.*
2. *Product of extremes ÷ given mean = required mean.*

Problems.

Find the omitted or missing term in the following proportions: —

1. $75 : 25 :: 9 : ?$.
2. $18 : 54 :: ? : 72$.
3. $? : 63 :: 32 : 96$.
4. $70 : ? :: 80 : 16$.
5. $2.5 : 22.5 :: 7\frac{1}{2} : ?$.
6. $? : \frac{3}{4} :: \$5 : \$10$.
7. $2\frac{1}{2}$ yd. $: 6\frac{1}{4}$ yd. $:: \$8 : \$?$.
8. $?$ ft. $: 16.5$ ft. $:: .5$ T. $: 1.5$ T.
9. $\frac{7}{8}$ bu. $: ? :: \$5 : \$3.75$.
10. $3\frac{1}{3}$ lb. $: 5$ lb. $:: \$? : \$2\frac{1}{2}$.
11. 7.5 T. $: 2.50$ T. $:: 7\frac{1}{2} : ?$.
12. $9.37\frac{1}{2}$ A. $: 625$ A. $:: 3\frac{1}{2} : ?$.

13. The extremes are $37\frac{1}{2}$ and $11\frac{1}{3}$, and one of the means is $4\frac{1}{4}$. What is the other mean?

14. If the means are .625 and 12.5, and one of the extremes is 1.875, what is the other extreme?

15. If the first three terms of a proportion are $17\frac{1}{2}$, $2\frac{1}{2}$, and 875, what is the fourth term?

CASE I.

Simple Proportion.

349. ***Simple Proportion*** is the equality of two simple ratios.

Thus, *8 yd. : 12 yd. :: $10 : $15* is a *simple proportion.*

350. Simple Proportion is used chiefly to solve problems in which three terms are given to find the fourth term which shall be proportional to the other three.

Of the three given numbers, two must always be similar numbers to form *one of the ratios*, and the third number must be of the same kind as the required term to form *the other ratio.*

351. A ***Statement*** is the arrangement of the terms in the form of a proportion.

Written Exercises.

352. *Example.* — If 10 tons of coal cost $65, what is the cost of 8 tons at the same rate?

SOLUTION.

10 T. : 8 T. :: $65 : —.

$$\frac{\$65 \times 8}{10} = \$52.$$

EXPLANATION.—At the same rate, the ratio of $65, the *cost* of 10 T., to the *required cost* of 8 T., must be the same as the ratio of 10 T. to 8 T.

Since 8 T. are less than 10 T., the cost of 8 T. must be less than the cost of 10 T.; therefore, since the fourth term is less than the third, the second term, or consequent of the first ratio, must be less than the first term. Hence, the first ratio is 10 T. to 8 T., and the second is $65 : $—, which in the form of a proportion, is 10 T. : 8 T. :: $65 : $—.

Since the product of the extremes is equal to the product of the means, $65 × 8, or $520, is also the product of the extremes; and $520, the product of two factors, ÷ 10, one of the factors, gives *$52*, the other factor, or the fourth term required.

353. Rule for Simple Proportion.

I. *Take for the third term that number which is of the same kind as the required or fourth term.*

II. *Write the remaining two numbers for the first couplet, placing the greater number for the second term when the fourth term is to*

be greater than the third term; and the less number for the second term when the fourth term is to be less than the third term.

III. *Multiply the third term by the second, and divide the product by the first term. The quotient will be the fourth term required.*

Note. — If the first and the second term are of different denominations, they must be changed to the same denomination.

354. From the preceding, it appears that there are three steps in the solution of problems by Simple Proportion: —

1. *To find the two ratios.* | 2. *To make the statement.*

3. *To find the missing term.*

Problems.

1. What will 75 A. of land cost, if 45 A. cost $3937.50?

2. How many days will 15 men require to do the work that 12 men can do in $5\frac{1}{2}$ days?

3. If $\frac{1}{3}$ bu. of wheat costs $\$\frac{3}{8}$, what is the price of $\frac{1}{4}$ bu.?

4. If I pay $350 rent for 7 months' rent of my house, what amount must I pay in 2 years 9 months?

5. If $10\frac{1}{2}$ bushels of oats feed 8 horses $3\frac{1}{2}$ days, how long will $57\frac{3}{4}$ bu. last them? How long will 76 bu. $\frac{1}{2}$ pk. last?

6. If $8\frac{8}{9}$ cd. of wood cost $\$44\frac{4}{9}$, find the cost of $284\frac{4}{9}$ cu. ft.

7. If 12 lb. 3 oz. of gold are worth $8268.75, what is the value of 30 oz.? What is the value of 1 lb. $\frac{1}{2}$ oz.?

8. If a property worth $4875 is taxed $\$53.62\frac{1}{2}$, what should be the tax on a property worth $26025?

9. When 5 bu. 1 pk. of wheat cost $\$9.84\frac{3}{8}$, how many bushels can be bought for $\$107.81\frac{1}{4}$?

10. If .75 cd. of wood cost $\$5.62\frac{1}{2}$, what will $2\frac{3}{8}$ cd. cost?

11. How much is $\frac{3}{4}$ of $\frac{7}{8}$ of a vessel worth, when $\frac{5}{8}$ of it is worth $2016? Find the value of $2\frac{1}{3}$ times $\frac{6}{7}$ of it.

12. If a pole 30 ft. 6 in. high casts a shadow $15\frac{1}{2}$ ft. long, how high a tree will cast a shadow 18.75 ft. long, at the same time?

13. At the rate of $\$104\frac{1}{4}$ for $16\frac{2}{3}$ tons of coal, what must be paid for $\frac{3}{4}$ of $\frac{8}{9}$ of a ton? For $\frac{4}{5}$ of $.2\frac{1}{2}$ of a ton?

14. If 5 yd. of cloth $2\frac{1}{4}$ yd. wide will make a cloak, how much cloth that is only $\frac{3}{4}$ of a yard wide would be needed to line it?

15. If $\frac{5}{7}$ of a piece of work can be performed in 14 days, how much of it can be done in $17\frac{1}{2}$ days? In $8\frac{1}{8}$ days?

16. A tradesman failed, having property worth \$1750, while his debts amounted to \$5212. What can he pay on a dollar?

17. A piece of iron beaten into a plate $\frac{3}{8}$ of an inch thick covers 16 sq. ft. What surface would it cover if it were but $\frac{2}{9}$ of an inch thick? If but .18$\frac{3}{4}$ in. thick?

18. If a pound English money is worth \$4.866$\frac{1}{2}$, what is the value of £25 10 s. 2 d. at the same rate?

19. If a ship has provision to last 36 men 60 days, how long would the provisions last if 8 men more were taken on board?

20. What amount of interest is due semi-annually on a \$1000 bond paying \$.07$\frac{3}{10}$ on \$1?

21. If a business yields \$1500 net profits in 1 yr. 4 mo., in what time will it yield \$1750, at the same rate?

22. By selling carpets for \$4000, a dealer clears \$666$\frac{2}{3}$. What amount must he sell to clear \$1200?

23. How many yards of carpeting $\frac{3}{4}$ of a yard wide must be bought to cover a floor 15 ft. long 12 ft. wide?

24. A grocer has a false gallon which measures only 3 qt. $1\frac{1}{2}$ pt. What is the real value of the molasses sold from it for \$.64?

25. If a weight of 15$\frac{3}{4}$ ounces be used as a pound, what will be the actual weight of sugar paid for as being 53$\frac{1}{3}$ pounds? What is overcharged in the quantity sold for \$112.

26. When money is worth 6 per cent. per annum, how much cheaper is it to borrow \$3500, and buy a house, than to pay \$20$\frac{1}{4}$ a month rent for it?

27. If the interest on an investment of \$2500 is $6\frac{1}{2}\%$, what investment will yield the same interest at $7\frac{3}{10}\%$?

28. If a man's annual income from bonds is \$1500 in currency when gold is 112$\frac{1}{2}$, how much is it when gold is 118$\frac{3}{4}$?

29. The length of a croquet-ground is 40 ft., and its width is to its length as 3 : 4. How wide is it?

30. Divide 60 into two parts whose ratio shall be equal to the ratio 4 to 6.

SOLUTION.

$4 + 6 = 10$.

$10 : 4 :: 40 : —$, *smaller part.*

$\frac{60 \times 4}{10} = 24$.

$10 : 6 :: 60 : —$, *larger part.*

$\frac{60 \times 6}{10} = 36$.

EXPLANATION. — Now 4 + 6, or 10, is the sum of the terms of the ratio.

Since 60 bears the same ratio to the smaller of the parts into which it is divided that 10 does to 4, the first proportion is 10 : 4 :: 60 to the smaller part, which is 24. Also,

Since 60 bears the same ratio to the larger of the parts into which it is divided, etc.

31. Divide 90 into two parts whose ratio shall be as 9 : 6.

32. Divide 4122 into two parts which shall be as 19 to 17.

33. Divide 9875 into two parts proportional to 27 and 18.

34. Divide $280 into three parts proportional to 3, 5, and 6.

35. A and B start from places 75 miles apart, and travel towards each other, A going 7 miles an hour, and B 8 miles. How far will each go before meeting?

36. An estate of $7800 was divided between two children in proportion to their ages, which were 15 years and 11 years respectively. What did each receive?

CASE II.

Compound Proportion.

355. Compound Proportion is an equality of two compound ratios, or of a compound and a simple ratio.

Thus, $\left\{\begin{matrix} 4:6 \\ 7:8 \end{matrix}\right\} :: \left\{\begin{matrix} 7:10 \\ 5:6 \end{matrix}\right\}$, or $\left\{\begin{matrix} 8:9 \\ 3:6 \end{matrix}\right\} :: 20 : 45$ is a *compound proportion.*

356. Compound Proportion is used chiefly to solve problems in which two or more terms in pairs and one single term are given to find a term which shall be proportional to the other terms.

357. The principles of Compound Proportion are the same as those which apply to Simple Proportion; also, the order of the terms of each ratio, and the order of the ratios, are the same as in Simple Proportion.

Written Exercises.

358. *Example.* — If 18 men earn \$270 in 12 days, how much will 10 men earn in 30 days?

SOLUTION.

18 men : 10 men :: \$270 : \$ —.
12 days : 30 days.

$$\frac{\$270 \times 10 \times 30}{18 \times 12} = \$375.$$

EXPLANATION. — At the same rate, the ratio of \$270 to the required number of dollars must depend on two conditions: 1st. The number of men employed, and 2d, the number of days they work.

Since the required term is \$—, write \$270 as the first term of the second ratio, or the third term of the proportion.

Consider first the number of men employed. Since 10 men are less than 18 men, 10 men will earn less than 18 men; therefore, etc.

Consider next the number of days they work. Since 30 days are more than 12 days, more will be earned in 30 days than in 12 days; therefore, etc.

Since the product of the extremes equals the product of the means, \$270 × 10 × 30 is also the product of the three extremes; that is, the product of three factors and two of them are given to find the third; and the product of the means, \$270 × 10 × 30, divided by the product of the given extremes, 18 × 12, gives *\$375*, the required extreme or fourth term.

359. Rule for Compound Proportion.

I. *Write for the third term that number which is of the same kind as the required or fourth term.*

II. *Form each pair of the other numbers as a couplet of the compound ratio, in the same manner as in simple proportion, as if the fourth term depended on each couplet alone.*

III. *Multiply the third term and the second terms together, and divide their product by the product of the first terms. The quotient will be the required fourth term.*

Problems.

1. If 4 men can do 24 rods of ditching in 12 days, how many rods can be done by 16 men in 24 days?

2. If 7 horses eat 42 bu. of oats in 16 days, how many bushels will 15 horses eat in 12 days?

3. How much will 8 boys earn in $11\frac{1}{4}$ days, if 5 boys earn $\$7\frac{1}{5}$ in one day?

4. If 8 yd. of muslin $1\frac{1}{4}$ yd. wide cost \$1.25, how many yards $1\frac{1}{8}$ yd. wide can be bought for $\$1.40\frac{5}{8}$?

5. What sum of money will give \$15 interest in 9 mo., if \$800 yield \$75 in 15 mo.?

6. What is the cost of $24\frac{1}{2}$ yards of cloth $1\frac{1}{2}$ yd. wide, if $3\frac{1}{2}$ yd. $1\frac{1}{5}$ yd. wide cost $\$5.62\frac{1}{2}$?

7. If it costs $\$7\frac{1}{2}$ to haul 10 tons $2\frac{1}{2}$ miles, what will it cost to carry 200 tons $\frac{1}{2}$ of a mile?

8. How much tea would a family of 6 persons use in $\frac{1}{4}$ of a year, if 8 persons use $1\frac{7}{8}$ lb. per month?

9. What sum of money on interest for 7 yr. 6 mo., at 6%, will yield the same income as \$3000 for 3 yr. 9 mo., at $7\frac{3}{10}\%$?

10. If a building lot $6\frac{1}{2}$ yd. by $1\frac{4}{5}$ rd. sells for \$900, what is the selling price of a lot 20 ft. by $41\frac{1}{4}$ yd.?

11. If 6 men can remove 288 cu. ft. of earth in 4 da. of $9\frac{1}{2}$ hr. each, how much can 9 men remove in $4\frac{1}{2}$ da. of 10 hr. each?

12. If a bin $6\frac{1}{3}$ ft. long, $2\frac{4}{5}$ ft. wide, and 2 ft. deep contains 30 bu. of corn, how wide must another bin $12\frac{5}{6}$ ft. long and $3\frac{2}{3}$ ft. deep be to contain 150 bu.?

13. If it costs \$125 to build a wall 40 ft. long, $6\frac{1}{2}$ ft. high, 1 ft. 6 in. thick, how much will it cost to build a wall 60 ft. long, $8\frac{5}{8}$ ft. high, 2 ft. 3 in. thick?

14. How much will it cost to carpet a room 17 ft. 11 in. by 16 ft., at $\$1.87\frac{1}{2}$ a yard, if 39 yd. of the same kind of carpet are needed to cover a room 16 ft. 3 in. by 13 ft.?

15. If the expenses of 9 horses for oats be \$45 for $4\frac{1}{2}$ mo., when the price of oats is $\$.36\frac{2}{3}$ per bu., how many horses will, at the same rate, consume the value of \$216 in 6 mo., when the price of oats is \$.33?

16. A builder undertook to finish some work in 16 days with 24 men. At the end of 12 days he found but $\frac{3}{5}$ of it completed. How many more men were needed to finish it in the time set?

17. What sum will keep 9 horses and 20 sheep for 9 days, if 3 horses and 31 sheep can be kept 10 days for \$18, supposing that 3 horses eat as much as 50 sheep?

18. A contractor engaged to remove 800 cu. yd. of earth in 6 days with 18 carts. If at the end of 4 days he had removed 450 cu. yd., how many carts were required to remove the balance within the given time?

19. If 15 men in 20 hours dig 40 cu. yd. of a cellar, how many yards will 40 men, stronger in the ratio of 5 to 4, dig in 45 hours, if the ground is harder in the ratio of 5 to 4?

20. If 12 men can build a wall 40 ft. long, 6 ft. high, 1 ft. 6 in. thick in 16 da., how many men are needed to build a wall 400 ft. long, 8 ft. high, 2 ft. 3 in. thick, in 80 da.?

21. If a barrel of flour lasts a family of 6 persons for 20 days, how many barrels will last 3 times as many during a leap year?

22. If a man makes 6 steps, each 2 ft. 6 in., while a boy makes 8 steps, each 1 ft. 9 in., how far will the boy walk while the man is walking $2\frac{3}{4}$ miles?

23. If 25 men can do a piece of work in 12 days, how many men would be required to perform another piece of work 3 times as hard in $\frac{3}{4}$ of the time?

24. How many men will be required to dig a cellar 40.25 ft. long, 28.5 ft. wide, 8.25 ft. deep in 10 da. of 9.8 hr. each, if 8 men can dig a cellar 27 ft. long, 14.25 ft. wide, 5.5 ft. deep in 7.5 da. of 8.4 hr. each?

25. If it requires 15 yd. of silk $\frac{7}{8}$ yd. wide to line a cloak made of 12.25 yd. of cloth $1\frac{3}{8}$ yd. wide, how many yards of silk $\frac{5}{8}$ yd. wide will be required to line a cloak made of $8.33\frac{1}{3}$ yd. of cloth $1\frac{3}{4}$ yd. wide?

Section III.

PARTNERSHIP.

1. If Charles gets 2 apples out of 5 apples, what part of 5 apples does he get? If he gets 3 out of 5?

2. If 10 cents are divided so that John receives 4 cents, and James receives 6 cents, what part of 10 cents does each receive?

3. Two men bought a ton of coal for $7, the first paying $4, and the second $3. What part of the ton belongs to each?

4. Divide 20 cents between two boys so that one shall have 2 cents as often as the other shall have 3 cents. What part of the 20 cents will each have? How many cents will each receive?

5. Two men hired a pasture for $36, one putting in 3 cows for 4 weeks, and the other 2 cows for 3 weeks. How much of the rent should each pay?

6. If $40 be divided between two persons so that of every $8 divided one shall have $5 and the other $3, how many dollars will each of them receive?

Definitions.

360. A ***Partnership*** is an association of two or more persons for the transaction of business.

Partners are the persons associated in business. A partnership association is called a *Company*, *Firm*, or *House*.

361. ***Capital*** or ***Stock*** is money and other property invested in business.

362. The ***Assets*** or ***Resources*** of a company are the debts due to it, and all the property belonging to it.

363. The ***Liabilities*** of a company are the debts which it owes.

364. ***Net Capital,*** or ***Net Proceeds,*** is the assets less the liabilities.

365. ***Bankruptcy*** is a failure in business with inability to pay debts.

A person who fails in business and has not property enough to pay all his debts is called a *Bankrupt*, and is said to be *insolvent*.

366. Principles.

I. *The longer the time or the greater the capital, the greater each partner's share of the gains or losses.*

II. *The shorter the time or the less the capital, the less each partner's share of the gains or losses.*

CASE I.

To Apportion Gains and Losses according to Capital or Time.

Written Exercises.

367. *Example.* — A and B entered into partnership. A furnished $1500 capital, and B $1000. If they gained $750, what were the profits of each?

FIRST SOLUTION.

$1500, *A's capital.*	$\frac{1500}{2500} = \frac{3}{5}$, *A's share of the gain.*
1000, *B's capital.*	$\frac{1000}{2500} = \frac{2}{5}$, *B's share of the gain.*
$2500, *whole capital.*	

$\frac{3}{5}$ *of* $750 = $450, *A's gain;* $\frac{2}{5}$ *of* $750 = $300, *B's gain.*

SECOND SOLUTION.

$1500, *A's capital.*	$750 *gain* $= \frac{750}{2500} = \frac{3}{10} = 30\%$ *of capital.*
1000, *B's capital.*	$1500 $\times$ .30 = $450, *A's share of gain.*
$2500, *whole capital.*	$1000 $\times$ .30 = $300, *B's share of gain.*

EXPLANATIONS. — (*Solution by Fractions.*) Since the whole capital is $2500, A's capital is $\frac{1500}{2500}$, or $\frac{3}{5}$, of the whole capital, and he is entitled to $\frac{3}{5}$ of the gain; and since B's capital is $\frac{1000}{2500}$, or $\frac{2}{5}$, of the whole capital, he is entitled to $\frac{2}{5}$ of the gain.

Solution by Percentage. — Since the whole capital is $2500, and the gain is $750, the gain is $\frac{750}{2500}$, or $\frac{3}{10}$, or 30% of the capital, and each partner's share of the profits is 30% of the capital.

368. Rule to Apportion Gains and Losses according to Capital or Time.

I. *Add the partners' shares of the capital for the whole capital.*

II. *Find what fractional part of the whole capital each partner's capital is; then find that fractional part of the whole gain or loss for his share of the gain or loss.* Or,

Find what per cent. of the whole capital the gain or loss is; then find that per cent. of each partner's capital for his share of the gain or loss. Or,

State by proportion, as the whole capital is to each partner's share of the capital, so is the whole gain or loss to each partner's share of the gain or loss.

Problems.

1. A and B formed a partnership in business. A put in $8500, and B $6500, and their loss the first year was $2600. What was each partner's loss?

2. Three persons bought a house in company, the first furnishing $2500 of the purchase-money, the second $1800, and the third $1700. They sold the house for $7500. What did each gain?

3. The capital of two partners was as 3 to 4. Their profits were $6000, and their expenses $1800. What was each partner's share of the net profits?

4. A bankrupt whose liabilities are $15000 and whose assets are $6000, owes A $1625 and B $1075.50. What amount will each of these two creditors receive?

5. An estate valued at $8750 was divided between two heirs so that one received $\frac{1}{4}$ more than the other. What amount did each receive? If one received 20% less than the other?

6. Three stock-growers rent a farm of 175 A. 40 sq. rd. at $6\frac{1}{2}$ per acre. A put in 124 cattle, B put in 186, and C 279. What rent should each of them pay?

CASE II.

To Apportion Gains and Losses according to Capital and Time.

Written Exercises.

369. *Example.* — A went into business with a capital of $3000. Six months afterwards he admitted B with $8000. If the profits at the end of the year were $2400, what was the share of each?

SOLUTION.

A's $3000 for 12 mo. = $36000 for 1 mo.
B's $8000 for 6 mo. = $48000 for 1 mo.
A's and B's together = $84000 for 1 mo.
$\frac{36000}{84000}$, or $\frac{3}{7}$, *of* $2400 = $1028.57\frac{1}{7}$, *A's gain.*
$\frac{48000}{84000}$, or $\frac{4}{7}$, *of* $2400 = $1371.42\frac{6}{7}$, *B's gain.*

EXPLANATION.—Since each partner's capital, and the time during which it was invested, are both unequal, his share of the profits must depend upon two conditions: 1. The *amount* of his capital; and 2, the *time* it was used in business.

Since A had \$3000 invested for 12 mo., the use of his capital was equivalent to the use of 12 times \$3000, or \$36000, for 1 mo.

Since B had \$8000 invested for 6 mo., the use of his capital was equivalent to the use of 6 times \$8000, or \$48000, for 1 mo., and the use of A's capital and B's was equivalent to the use of \$84000 for 1 mo.

Since the whole capital for one month is \$84000, and A's capital for 1 mo. is \$36000, he is entitled to $\frac{3}{8}\frac{6}{4}\frac{0}{0}\frac{0}{0}$, or $\frac{3}{7}$, of the gain, \$2400, or \$1028.57$\frac{1}{7}$; and since B's capital is \$48000, he is entitled to $\frac{4}{8}\frac{8}{4}\frac{0}{0}\frac{0}{0}$, or $\frac{4}{7}$, of the gain, etc.

370. Rule to Apportion Gains and Losses according to Capital and Time.

I. *Multiply each partner's share of the capital by the time it is invested.*

II. *Find what fractional part each partner's capital is of the whole capital, and take such part of the whole gain or loss for his share of the gain or loss.*

Note. — Each partner's share of the gain or loss may also be found by percentage, or by proportion.

Problems.

1. Two men rented a farm of 25 A. 8 sq. ch. at \$6 an acre. The first put in 12 cattle for 10 weeks, and the second 18 cattle for 8 weeks. What rent should each pay?

2. Three persons began business, Jan. 1, 1901, with \$1000 furnished by A. April 1, B put in \$1200; July 1, C put in \$1500. The profits for the year were \$900. Apportion them.

3. July 1, 1901, John Williams commenced business with \$5000; Oct. 1, he took into partnership James Wilson with \$4000. At the end of the year they were in debt \$750. Apportion the assessment. What % did each lose?

4. Three grain dealers shipped from Chicago 9500 bu. of wheat, A furnishing 4000 bu., B 2500, and C the balance. In a storm 3000 bu. were damaged. At \$1.12$\frac{1}{2}$ per bushel, what was the loss to each? If the amount had been 1250$\frac{1}{8}$ bu.?

5. A and B formed a partnership for 2 years. A furnished \$3000 at first, and 8 mo. after withdrew \$500, which he returned 6 mo. later. B put in \$2500, and 4 mo. after \$1000 more; but 6 mo. later drew out \$1000. The whole gain was \$2500. Find the share of each.

Review Problems.

1. A has sugar worth $12\frac{1}{2}$ cents per lb., and B has tea worth $\$.87\frac{1}{2}$. If in an exchange A rates his sugar at $13\frac{1}{2}$ cents, at what should B put his tea?

2. If a difference of 15° in longitude makes a difference of 1 hour in time, what is the difference in time between Boston, in 71° 3′ 30″ W. long., and Washington, in 77° 0′ 15″ W.?

3. Two kinds of coffee are mixed in the ratio of 25 lb. of Java to 4 lb. of Mocha. How much of each is there in a mixture weighing 116 lb.? In a mixture weighing 137 lb. 10 oz.?

4. If 3 lb. of coffee are worth \$1.20, and 10 lb. of coffee are worth 4 lb. of tea, how much are 50 lb. of tea worth?

5. If a clock set at 12 o'clock on Saturday gains 5 min. by Tuesday noon, what will be the true time when it strikes 12 o'clock at noon on the following Saturday?

6. An apple-woman bought 100 apples at 2 for a cent, and 100 at 3 for a cent. If she sells them at 5 for 2 cents, how much will she gain or lose? What % will she gain or lose?

7. A, B, and C are partners, and B has invested $\frac{1}{4}$ as much capital as A, and C $\frac{2}{3}$ as much as B. If their profits are \$7200, what is each partner's share?

8. The capital of two partners is in the ratio of 3 to 4. If their profits are \$8000, and their expenses are \$1700, what is each partner's share of the net profits?

9. If 10 oxen or 14 cows eat $4\frac{5}{16}$ T. of hay in 58 da., in what time will 4 oxen and 6 cows eat the same quantity?

10. Smith & Co. went into bankruptcy owing \$50000, and having \$12500 assets. If the expenses of settlement were \$2500, how much did A receive on a claim of \$7500?

11. A mechanic agreed to work 100 days for \$2.50 a day, and was to forfeit 50 cents for every day he was idle. At the end of the time he received \$190. How many days did he work?

12. A store was insured in the Home Insurance Co., \$4000; in the Mutual, \$5000; and in the Independent, \$6000. It was damaged by fire to the amount of \$7250. What did each pay?

13. If a ditch $272\frac{8}{11}$ yd. long, $1\frac{1}{11}$ yd. wide, $\frac{6}{11}$ yd. deep be dug by 280 men in 5 da. of $9\frac{3}{5}$ hr. each, how many men will be required to dig a ditch $\frac{6}{11}$ mi. long, $\frac{8}{11}$ yd. wide, $1\frac{1}{11}$ ft. deep, in $20\frac{5}{6}$ da. of 8 hr. each?

14. The joint stock of a company was $7500. A put in $\frac{1}{2}$ for 8 mo., B $\frac{1}{5}$ for 5 mo., and C the remainder for 1 yr. At the end of the year the stock was doubled. What was each partner's share at the beginning of the second year?

Review Questions.

1. Define ***Ratio.*** How is ratio expressed? Name the terms of a ratio, and define each. What is a couplet? What is the sign of ratio? What is a simple ratio? A compound ratio? State the principles and the general principles of ratio. Give the formulas to find the ratio, the antecedent, and the consequent.

2. What is a ***Proportion?*** Give an example. What is the sign of proportion? How many terms are in a proportion? What is each ratio, and what is each term called? What are the antecedents of a proportion? The consequents? The means? The extremes? Give an example. State the principles of proportion. Give the formulas.

Define ***Simple Proportion.*** For what is it used? What is a statement? Repeat the rule for simple proportion. What steps are taken in the solution of problems by simple proportion. Define ***Compound Proportion.*** For what is it used? To what do the principles of Compound Proportion correspond? Repeat the rule.

3. Define ***Partnership.*** What is capital or stock? What are assets or resources? Liabilities? Net capital or net proceeds? What is ***Bankruptcy?*** State the principles of partnership. State how to apportion gains and losses according to capital or time, and according to both capital and time.

CHAPTER VII.

INVOLUTION AND EVOLUTION.

Section I.

INVOLUTION.

1. What is the product of 3 used twice as a factor?

2. What is the result of using 4 three times as a factor?

3. What is the result of using $\frac{2}{3}$ twice as a factor? Of using $\frac{3}{4}$ three times as a factor? $\frac{4}{5}$ three times?

4. Find the product of .5 used twice as a factor. Of .6 used twice. Of .4 used three times. .07 used twice.

5. Each side of a square table is 4 feet. How many square feet are in the table? If each side is 5 ft. 6 in.?

6. How many cubic feet are in a cubical block of stone which measures 3 feet on each edge? If each edge is 3 ft. 4 in.?

Definitions.

371. A ***Power*** is the product of any given number used two or more times as a factor.

Thus, ***25*** is a *power* of ***5***, since it is the product of 5×5, or of 5 used twice as a factor. 64 is a power of 4, since $4 \times 4 \times 4 = 64$.

372. A ***Root*** is the number used two or more times as a factor.

Thus, ***25*** is a *power* of ***5***, and ***5*** is the *root*.

373. *Powers are named* from the number of times the root or given number is used as a factor.

374. The ***First Power*** is the root or given number itself.

375. The ***Second Power,*** or ***Square,*** is the product of any given number used twice as a factor.

Thus, the *second power* of ***3*** is the product of 3×3, or ***9***.

376. The ***Third Power,*** or ***Cube,*** is the product of any given number used three times as a factor.

Thus, the *third power* of ***3*** is the product of $3 \times 3 \times 3$, or ***27***.

377. The *Fourth Power* is the product of four equal factors; the *Fifth Power*, of five equal factors; etc.

378. The ***Index,*** or ***Exponent,*** of a power is a small figure placed at the right and a little above the root, to show how many times it is used as a factor to produce the power.

Thus, in the expression 3^2, the index or exponent is ^{2}, which indicates that 3 is to be used twice as a factor, or that the second power of 3 is to be found; in 3^3, the index is ^{3}; etc.

379. A ***Perfect Power*** is a number that is the product of two or more equal factors.

Thus, *25* is a *perfect power*, since it is the product of 5×5.

380. ***Involution*** is the process of finding any power of a given number.

Note. — The process of finding the *square* or *second power* of a number is called *squaring* it, or raising it to the second power; the process of finding the *cube* is called *cubing* it, or raising it to the third power; etc.

Written Exercises.

381. *Example* 1. — Raise 25 to the third power.

SOLUTION.

25, *1st power.*
25
625, *2d power.*
25
15625, *3d power.*

EXPLANATION.—Since the third power of any number is the product of that number used three times as a factor, $25^3 = 25 \times 25 \times 25$, or *15625*, the power required.

382. *Example* 2. — What is the value of $(\frac{3}{4})^4$?

SOLUTION. $(\frac{3}{4})^4 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$.

EXPLANATION. — Since the fourth power of any number is the product of that number used four times as a factor, $(\frac{3}{4})^4 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$, or $\frac{81}{256}$, the power required.

383. Rule to Raise a Given Number to any Required Power.

Use the given number as many times as a factor as there are ones in the index of the required power. The last product will be the power required.

Find the required power of —

1. 15^2; 25^2; 35^2.
2. 70^3; $.07^3$; $.78^3$.
3. 24^4; 2.4^4; $.24^4$.
4. 102^3; 1.02^3; $.102^3$.
5. 200^4; $.002^4$; 2.02^4.
6. 3.03^2; $.003^3$; 300^4.
7. $(\frac{2}{3})^3$; $(\frac{3}{4})^4$; $(\frac{4}{5})^5$; $(\frac{5}{6})^6$.
8. $(1\frac{1}{2})^6$; $(2\frac{1}{4})^5$; $(10\frac{1}{10})^4$.
9. $(.2\frac{1}{2})^3$; $(.05\frac{1}{5})^5$; $(8.00\frac{1}{4})^2$.
10. Find the result of 5^5 multiplied by $.01^5$; $20^4 \times (\frac{1}{10})^5$.
11. Find the quotient of 4^6 divided by $.005^3$. Of $(2\frac{1}{4})^3 \div .002^3$.
12. Multiply the quotient of $25^3 \div .005^2$ by $.25^4 \times 50^3 \div (\frac{1}{5})^6$.

384. To Find the Square of a Number composed of Tens and Ones.

385. Example. — Find the square of 25.

SOLUTION.

25	=	*20 + 5*	=	*25*
25	=	*20 + 5*	=	*25*
25 × 5	=	*(20 × 5) + 5²*	=	*125*
25 × 20	= *20²* +	*(20 × 5)*	=	*500*
25²	= *20²* + *2 times (20 × 5)* + *5²*		=	*625*

EXPLANATION. — Since 25 is 2 tens + 5 ones, or 20 + 5, the square of 25 is the square of $\overline{20 + 5}$.

The product of $\overline{20 + 5} \times 5$ is $(20 \times 5) + 5^2$; and the product of $\overline{20 + 5} \times 20$ is $20^2 + (20 \times 5)$. Hence, the sum of these two products is $2C^2$ + 2 times $\overline{(20 \times 5)} + 5^2$, which is the square of 20 + 5, or of 25.

Now $20^2 = 400$; 2 times $\overline{20 \times 5} = 200$; and $5^2 = 25$; or 400 + 200 + 25 = 625.

386. Principles.

The square of any number composed of tens and ones is the square of the tens, plus twice the product of the tens by the ones, plus the square of the ones.

Note 1. — Since every number can be regarded as being composed of tens and ones, the method of finding the square of a number by separating it into tens and ones can be applied to every number.

2. By writing *t* for tens, and *o* for ones, and omitting the figure denoting the number of tens and ones, every square may be expressed by the

387. GENERAL FORMULA.— $(t + o)^2 = t^2 + 2\,(t \times o) + o^2$.

Problems.

388. Example. — Find the square 125.

SOLUTION.

$(125)^2 = (120 + 5)^2$.	
120^2	$= 14400$
$2 \times 120 \times 5$	$= 1200$
5^2	$= 25$
125^2	$= 15625$

EXPLANATION.— Since every number is composed of a number of tens and ones, 125 may be expressed as 120 + 5.

Now t^2, or 120^2, equals 14400; $2 \times (t \times o)$, or 2 times 120×5, = 1200; and o^2, or 5^2, = 25; and $t^2 + \overline{2 \times (t \times o)} \times o^2 = 14400 + 1200 + 25 = 15625$, the square required.

Find by separating each into tens and ones, the square of—

1. 22; 24; 26.	3. 42; 45; 48.	5. 69; 78; 77.	7. 123; 134.
2. 33; 35; 37.	4. 52; 54; 66.	6. 84; 85; 96.	8. 234; 245.

389. *To Find the Cube of a Number composed of Tens and Ones.*

390. *Example.* — Find the cube of 25.

SOLUTION.

$25^3 = 20^3 + \overline{2 \times (20 \times 5)} + 5^2$

$25 = 20 + 5$

$25^2 \times 5 = (20^2 \times 5) + \overline{2 \times (20 \times 5^2)} + 5^3$

$25^2 \times 20 = 20^3 + \overline{2 \times (20^2 \times 5)} + (20 \times 5^2)$

$25^3 = 20^3 + \overline{3 \times (20^2 \times 5)} + \overline{3 \times (20 \times 5^2)} + 5^3$

EXPLANATION. — Since 25 is 2 tens 5 ones, or 20 + 5, the cube of 25 must be the cube of (20 + 5).

Now the square of 20 + 5 is $20^2 + \overline{2 \times (20 \times 5)} + 5^2$, which multiplied by (20 + 5) must give the cube of 25, which is $20^3 + 3 \times \overline{(20^2 \times 5)} + 3 \times \overline{(20 \times 5^2)} + 5^3$. But $20^3 = 8000$; 3 times $(20^2 \times 5) = 6000$; 3 times $(20 \times 5^2) = 1500$; $5^3 = 125$. Hence, $(20 + 5)^3 =$ 8000 + 6000 + 1500 + 125, or *15625*.

391. Principle.

The cube of any number composed of tens and ones is equal to the cube of the tens, plus three times the product of the square of the tens by the ones, plus three times the product of the tens by the square of the ones, plus the cube of the ones.

Note 1. — Since every number can be regarded as being composed of tens and ones, the method of finding the cube of a number by separating it into tens and ones can be applied to every number.

2. By writing t for tens and o for ones, and omitting the figure denoting the number of tens and ones, every cube may be expressed by the

392. GENERAL FORMULA. — $(t + o)^3 = t^3 + 3(t^2 \times o) + 3(t \times o^2) + o^3$.

Problems.

393. *Example.* — Find the cube of 125.

SOLUTION.

$125 = 120 + 5.$

$120^3 = 1728000$

$120^2 \times 5 \times 3 = 72000$

$120 \times 5^2 \times 3 = 9000$

$5^3 = 125$

1809125

EXPLANATION. — Since every number is composed of a number of tens and ones, 125 may be expressed as 120 + 5.

Now t^3, or 120^3, = 1728000; $3(t^2 \times o)$, or 3 times $120^2 \times 5$, = 72000; $3(t \times o^2)$, or 3 times $120 \times 5^2 = 9000$; t^3, or $5^3 = 125$; and $t^3 + 3(t^2 \times o) + 3(t \times o^2) + o^3 =$ 1728000 + 72000 + 9000 + 125 = *1809125*, the cube required.

Find by separating into tens and ones, the cubes —

1. Of 12; 14.	3. Of 42; 46.	5. Of 61; 64.	7. Of 82; 86.
2. Of 23; 35.	4. Of 53; 57.	6. Of 72; 79.	8. Of 93; 99.

Section II.

EVOLUTION.

1. What is the product of 4×4? Of what two equal factors is 16 the product?

2. What is the second power of 5? 25 is the second power of what number or root?

3. What are the three equal factors of 27? 27 is the third power of what number or root?

4. What are the three equal factors of 8? Of 64? Of 125?

5. Of what root is 125 the third power or cube? 216? 512?

6. Which is one of the two equal factors of 64? One of the three equal factors of 64?

7. If a square table contains 25 sq. ft., how long is each side? If a cube contains 125 cu. ft., how long is each edge?

Definitions.

394. A ***Root*** is one of the equal factors of any given number.

Thus, *3* is the *root* of *9*, since it is one of the two equal factors of 9; 3 is also a root of 27 and of 81.

395. If a number or root is used *twice* as a factor to produce any given number, it is called the *Second*, or *Square Root;* if used *three* times, it is called the *Third*, or *Cube Root;* if *four* times, the *Fourth Root;* etc.

Thus, *3* is the *second*, or *square root*, of *9*, since it is used twice as a factor to produce 9; 4 is the third, or cube root, of 64, since it is used three times as a factor to produce 64; etc.

Note. — A number is the second power of its square root, the third power of its cube root, etc. Also, a number is the second or square root of its second power, the third or cube root of its third power, etc.

396. The ***Radical Sign*** is the character, $\surd$, placed before a number to show that some root of it is to be found.

397. The ***Index of the Root*** is a small figure placed within the radical sign to show what root is to be found.

If no index is written within the radical sign, the index ^{2} is understood.

Thus, $\sqrt[2]{25}$, or $\surd 25$, denotes that the *square root* of 25 is to be found; $\sqrt[3]{125}$ denotes the cube root of 125; etc.

CASE I

Square Root.

398. The smallest number that can be expressed by one figure is 1, and the greatest is 9; the smallest that can be expressed by two figures is 10, and the greatest 99; the smallest by three is 100, and the greatest 999.

399. The squares of the smallest and the greatest numbers that can be expressed by one, by two, and by three figures are —

$1^2 = 1.$	$10^2 = 100.$	$100^2 = 10000.$
$9^2 = 81.$	$99^2 = 9801.$	$999^2 = 998001.$

400. The square of a number expressed by one figure consists either of one or of two figures; of a number expressed by two figures, either of three or four figures; of a number expressed by three, either of five or six figures; etc. That is —

401. The square of a number contains twice as many figures as the number itself, or one figure less than twice as many. Hence,

If a number be separated into periods of two figures each, beginning with ones, there will be as many figures in its square root as there are periods in the given number.

Note. — If the number of figures in the given number is *odd*, there will be only one figure in the left-hand period.

402. The squares of the smallest and the greatest number of *ones*, of *tens*, and of *hundreds* are —

$1^2 = 1.$	$10^2 = 100.$	$100^2 = 10000.$
$9^2 = 81.$	$90^2 = 8100.$	$900^2 = 810000.$

403. The square of a number of *ones* is found either in the order of ones alone, or in the orders of tens and ones together; the square of *tens*, in the order of hundreds alone, or in the orders of thousands and hundreds together; the square of *hundreds*, in the order of ten-thousands alone, or in the orders of hundred-thousands and ten-thousands together; etc. That is —

404. If the square of any number is separated into periods of two figures each beginning at ones, the first period contains the square of the ones of that number; the second period contains the square of the tens; the third period, the square of the hundreds; etc. Therefore,

405. The left-hand period of the square contains the square of the left-hand figure of the square root; the first two left-hand periods contain the square of the first two left-hand figures of the square root; etc.

Definitions.

406. The ***Square Root*** is one of the two equal factors of any given number.

Thus, *5* is the *square root* of *25*, since it is one of the two equal factors of 25; 6 is the square root of 36; etc.

407. A ***Perfect Square*** is a number which is the product of two equal factors, and which has an exact square root.

Thus, *49* is a *perfect square*, since it is the product of *two equal factors*, *7* and *7*; and since it has an exact *square root*, *7*.

408. The ***Extraction of the Square Root*** is the process of finding one of the two equal factors of a given number.

409. Principles of Square Root.

I. *The square of any number contains twice as many figures as the number itself, or one less than twice as many.*

II. *The first two orders of the square of any number contain the square of the ones of that number; the next two higher orders contain the square of the tens; etc.*

III. *The square of any number composed of tens and ones is equal to the square of the tens of that number, plus twice the product of the tens by the ones, plus the square of the ones.*

Exercises.

Separate 1225 into periods, tell of how many orders its square root is composed, and name the left-hand figure of its square root.

410. Model. — Begin at ones' place and separate it into periods of two figures each by placing a dot over ones and over the second figure to the left, or hundreds, giving $1\dot{2}2\dot{5}$; the square root is composed of two figures, or of tens and ones; the left-hand period is 12; the greatest square in 12 is 9. Since the square root of 9 is 3, the left-hand or tens' figure of the square root is 3.

Separate each of the following numbers into periods, tell of how many figures the square root is composed, and name the left-hand figure of the square root: —

1. *81; 144; 169.*
2. *196; 225; 256.*
3. *289; 324; 361.*
4. *400; 484; 776.*
5. *1089; 1296; 1936.*
6. *2601; 4225; 5476.*
7. *6561; 9701; 10201.*
8. *23421; 46656; 58504.*
9. *77284; 97344; 964169.*

Written Exercises.

411. Example 1. — Find the square root of 4489.

SOLUTION.

$$\begin{array}{r} \dot{4}4\dot{8}9(67 \\ 6^2 = 36 \\ \hline 12 \quad)889 \\ 127 \times 7 = 889 \\ \hline \end{array}$$

EXPLANATION.— Since 4489 is composed of four figures, it may be separated into two periods, and its square root will be composed of two figures.

Since the square of tens can give no order lower than hundreds, the tens of the root must be found from 44 hundreds, the left-hand period. The greatest square of a number of tens in 44 hundreds is 36 hundreds, and the square root of 36 hundreds is 6 tens, which is the first or left-hand figure of the root. Subtracting 36 hundreds from 44 hundreds, there is a remainder of 8 hundreds, to which annex 89, the figures of the next period.

Since the *square of the tens* (36 hundreds) has been taken from 4486, the remainder, 889, must be composed of *twice* the product of the *tens* by the *ones*, plus the *square* of the *ones;* and since the product of tens by ones can contain no order lower than tens, twice the product of the tens by the ones must be contained in 88 tens.

If 88 tens be divided by twice the 6 tens of the root, or 12 tens, the quotient, 7, must be the ones' figure of the root. Now to 12 tens, the first divisor, add 7 ones, giving 127, which is twice the tens plus the ones; and 127 multiplied by 7, the ones' figure of the root, gives 7 times 7, or the square of the ones, and 7 times 12 tens or twice the product of the tens by

the ones. The product is 889, the same as the dividend; hence, *67* is the square root required.

412. *Example* 2. — Find the square root of 294849.

SOLUTION.

$$\begin{array}{l} 294849(543 \\ 25 \\ \hline 104)448 \\ 416 \\ \hline 1083)3249 \\ 3249 \\ \hline \end{array}$$

EXPLANATION.— Since 294849 is composed of six figures, etc.

Since the number is composed of more than two periods, the square root may be found by regarding but two periods at a time, and then finding each figure of the root by the process given in Example 1.

For brevity each divisor may be completed at once by placing at the right of the trial divisor each figure of the root as it is found.

413. Rule for Square Root.

I. *Begin at ones' place and separate the given number into periods of two figures each.*

II. *Find the greatest square in the left-hand period, and place its root for the first figure of the required root. Subtract the square of this figure from the left-hand period, and to the remainder annex the next period, for a dividend.*

III. *Double the root already found, regarded as tens, for a trial divisor; find how often it is contained in the dividend, and place the result in the root, and also at the right of the trial divisor.*

IV. *Multiply the complete divisor by the last figure of the root; subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

V. *Proceed in the same manner with all the periods to the last. The result will be the square root required.*

Note 1. — If the given number contains a decimal, begin at the ones' place and point off into periods of two figures each toward the *left* in the *integer*, and toward the *right* in the *decimal*.

2. If any dividend is less than the trial divisor, place a cipher in the root, and also in the divisor; annex the next period to the dividend for a new dividend, and proceed as before.

3. If there is a remainder after all the periods have been used, annex periods of decimal ciphers, and carry the root to as many decimal places as may be required; four places being generally enough.

4. The square root of a fraction is found by extracting the square root of the numerator and of the denominator separately. If the terms of the fractions are not perfect squares, change the fraction to a decimal.

A mixed fraction must first be changed to an improper fraction.

Problems.

Find the square root of—

1. 484; 576; 625.	5. 10201; 23409.	9. 190096; 201601.
2. 676; 841; 961.	6. 37249; 41616.	10. 381924; 419904.
3. 1089; 2601; 3025.	7. 59536; 60516.	11. 514089; 692224.
4. 4225; 5476; 6561.	8. 77284; 96721.	12. 881721; 996004.

13. 110.0401; 261.4689; 379.8601; 648.2116; 1041.9984.

14. 13.972644; 20.421361; 6009.3504; .88981489.

15. 123; 2345; 4050; 5607; 6007; 8900; 9000; 10002.

16. Extract the square of 5 to four decimal orders. Of .5.

17. Find the value of $\sqrt{21.16}$; of $\sqrt{.0076}$; and of $\sqrt{.000625}$.

18. The area of a square lot of ground is 46656 sq. ft. What is the length of its equal sides?

19. A field is 196 rd. long and 16 rd. wide. What is the length of the side of a square containing an equal area?

Extract the square root of—

20. $\frac{9}{16}$; $\frac{16}{36}$; $\frac{25}{49}$.	23. $43\frac{14}{15}$; $125\frac{76}{84}$.	26. $(12\frac{1}{2})^2$; $144\frac{9}{64}$.
21. $\frac{121}{144}$; $\frac{169}{225}$; $\frac{196}{324}$.	24. $72\frac{1}{4}$; $365\frac{61}{84}$.	27. $(1.2\frac{1}{2})^3$; $225\frac{16}{25}$.
22. $\frac{256}{441}$; $\frac{361}{676}$; $\frac{484}{729}$.	25. $7.7\frac{3}{4}$; $1.87\frac{4}{5}$.	28. $(2\frac{3}{8})^4$; $(6\frac{1}{2})^4$; $9\frac{1}{3}$.

Find the value of—

29. $\sqrt{.3025} + \sqrt{42.25}$; $\sqrt{61\frac{1}{3}\frac{3}{6}} \times \sqrt{.64^2}$.

30. $\sqrt{.5^3} \times \sqrt{.0025}$; $\sqrt{.26^2} + \sqrt{\frac{2}{6}\frac{2}{2}\frac{6}{5}}$; $\sqrt{8\frac{1}{2}} \div \sqrt{\frac{1}{2}^4}$.

31. $\sqrt{.05^4} \div \sqrt{.02\frac{1}{2}^2}$; $(81 \div \sqrt{.00\overline{25}})^2 \times .36_{\frac{4}{100}}$; $\sqrt{(4.00_{\frac{1}{16}})^4}$.

32. $(64_{\frac{1}{64}} - \sqrt{26\frac{1}{12}\frac{0}{1}\frac{3}{1}}) \times \frac{2}{5}$ of $\sqrt{.005\frac{5}{8}}$; $\sqrt{10 - \sqrt{(60_{\frac{1}{8}})}}$.

33. Extract the square root of 5; $5\frac{1}{2}$; $6\frac{1}{4}$; $2.0\frac{1}{2}$; $3.00\frac{1}{4}$; $(\frac{4}{9})^3$.

34. Find the value of $\sqrt{15}$; $\sqrt{.015}$; $\sqrt{.0015}$; $\sqrt{15.00015}$.

35. The area of a circular field is 10160.64 sq. yd. What is the side of a square field of equal area?

CASE II.

Cube Root.

414. The cubes of the smallest and the greatest numbers that can be expressed by one, by two, and by three figures are —

$1^3 = 1.$	$10^3 = 1000.$	$100^3 = 1000000.$
$3^3 = 27.$	$33^3 = 35937.$	$333^3 = 36926037.$
$9^3 = 729.$	$99^3 = 970299.$	$999^3 = 997002999.$

415. The cube of a number expressed by one figure consists of one, two, or three figures; of a number expressed by two figures consists of four, five, or six figures; of a number expressed by three figures, of seven, eight, or nine figures. That is,

416. The cube of a number contains three times as many figures as the number itself, or one or two figures less than three times as many. Hence,

417. If a number be separated into periods of three figures each, beginning with ones, there will be as many figures in its cube root as there are periods in the given number.

Note.—The left-hand period may consist of three figures, or two, or one.

418. The cubes of the smallest and the greatest number of ones, tens, and hundreds are —

$1^3 = 1.$	$10^3 = 1000.$	$100^3 = 1000000.$
$3^3 = 27.$	$30^3 = 27000.$	$300^3 = 27000000.$
$9^3 = 729.$	$90^3 = 729000.$	$900^3 = 729000000.$

419. The cube of a number of *ones* is found in the orders of ones, tens, and hundreds, or in the ones' period.

The cube of a number of *tens* is found in the order of thousands, ten-thousands, and hundred-thousands, or in the thousands' period; of a number of *hundreds*, in the millions' period; etc. That is,

420. If the cube of any number is separated into periods of three figures each, beginning at ones, the first period contains the cube of the ones of that number; the second period contains the cube of the tens; the third period, the cube of the hundreds; etc. Therefore,

421. The left-hand period of the cube contains the cube of the left-hand figure of the cube root; the first two left-hand periods contain the cube of the first two figures of the cube root; etc.

Definitions.

422. The ***Cube Root*** is one of three equal factors of a given number.

Thus, *6* is the *cube root* of *216*, since it is one of the three equal factors of 216; 7 is the cube root of 343, etc.

423. A ***Perfect Cube*** is a number which is the product of three equal factors, and which has an exact cube root.

Thus, *512* is a *perfect cube*, since it is the product of the *three equal factors*, *8*, *8*, and *8*; and since it has an *exact cube root*, *8*.

424. The ***Extraction of the Cube Root*** is the process of finding one of the three equal factors of a given number.

425. Principles of Cube Root.

I. *The cube of any number contains three times as many figures as the number itself, or one or two figures less than three times as many.*

II. *The first three orders of the cube of any number contain the cube of the ones of that number; the next three higher orders contain the cube of the tens; the next three orders, the cube of the hundreds; etc.*

III. *The cube of any number composed of tens and ones is equal to the cube of the tens, plus three times the square of the tens by the ones, plus three times the tens by the square of the ones, plus the cube of the ones.*

Exercises.

Separate 85184 into periods, tell of how many orders its cube root is composed, and name the left-hand figure of its cube root.

426. ***Model.*** — Begin at ones' place, and separate it into periods of three figures each by placing a dot over ones, and over the third figure to the right, or thousands, giving $85\dot{1}84\dot{.}$

The cube root is composed of two figures, or of tens and ones; the left-hand period is 85; the greatest cube in 85 is 64, and the cube root of 64 is 4; therefore, the left-hand or tens' figure of the cube root of 85184 is 4.

Separate each of the following numbers into periods, tell of how many figures the cube root is composed, and name the left-hand figure of the cube root: —

1. *84; 125; 729.*	4. *175616; 300763.*	7. *456533; 12812904.*
2. *1728; 12167.*	5. *474552; 1030301.*	8. *681472; 41063625.*
3. *39304; 91125.*	6. *704969; 1860867.*	9. *970299; 997002999.*

Written Exercises.

427. *Example* 1. — Find the cube root of 91125.

SOLUTION.

$$\begin{array}{rl|l} & \dot{9}1\dot{1}2\dot{5}(45 \\ 4^3 = & 64 & \\ \hline & 27125 & \\ 3 \times 40^2 = & 4800 & \\ 3 \times 40 \times 5 = & 600 & \\ 5^2 = & 25 & \\ \hline & 5425 & 27125 \\ \hline \end{array}$$

EXPLANATION. — Since 91125 is composed of five figures, it may be separated into two periods, and its cube root will be composed of two figures.

Since the cube of tens can give no lower order than thousands, the tens of the root must be found from 91 thousands, the left-hand period.

The greatest cube of a number of tens in 91 thousands is 64 thousands, and the cube root of 64 thousands is 4 tens, which is the first, or left-hand figure of the root. Subtracting 64 thousands from 91 thousands, there is a remainder of 27 thousands, to which annex 125, the figures of the next period.

Since the *cube of the tens* (64 thousands) has been taken from 91125, the remainder, 27125, must be composed of *3 times* the square of the *tens by the ones*, plus *3 times* the *tens by the square of the ones*, plus the *cube of the ones;* and since the product of the square of tens by ones can give no order lower than hundreds, three times the product of the square of the tens by the ones must be contained in 271 hundreds.

If 271 hundreds be divided by three times the square of the 4 tens of the root, or 4800, the quotient, 5, must be the ones' figure of the root. Now to 48 hundreds, the trial divisor, add three times 4 tens, multiplied by 5, and the square of 5, giving 5425, which is three times the square of the tens, plus three times the tens by the ones, plus the square of the ones; and 5425 multiplied by 5, the ones' figure of the root, gives five times 5^3, or the cube of the ones, plus three times the tens by the square of the ones, plus three times the square of the tens by the ones.

The product, 27125, is the same as the dividend. Hence, *45* is the cube root required.

428. *Example* 2. — Find the cube root of 12812904.

SOLUTION.

$12\dot{8}12\dot{9}0\dot{4}(234$

$2^3 = 8$

	4812
$3 \times 20^2 = 1200$	
$3 \times 20 \times 3 = 180$	
$3^2 = 9$	
1389	4167
	645904
$3 \times 230^2 = 158700$	
$3 \times 230 \times 4 = 2760$	
$4^2 = 16$	
161476	645904

EXPLANATION. — Since the given number, 12812904, is composed of eight figures, etc.

Since the number is composed of more than two periods, the cube root may be found by regarding but two periods at a time, and finding each figure of the cube root as in Example 1.

429. Rule for Cube Root.

I. *Begin at the ones' place, and separate the given number into periods of three figures each.*

II. *Find the greatest cube in the left-hand period, and place its root for the first figure of the required root. Subtract the cube of this figure from the left-hand period, and to the remainder annex the next period, for a dividend.*

III. *Multiply the square of the root already found, regarded as tens, by* 3; *find how often it is contained in the dividend, and place the result in the root.*

IV. *To the partial divisor add* 3 *times the product of the first figure, regarded as tens, by the second figure, and also the square of the second figure. The sum will be the complete divisor.*

V. *Multiply the complete divisor by the last figure of the root; subtract the product from the dividend, and to the remainder annex the next period, for a new dividend.*

VI. *Proceed in the same manner with all the periods to the last. The result will be the cube root required.*

Note 1. — If the given number contains a decimal, begin at the ones' place, and point off into periods of three figures each toward the *left* in the *integer*, and toward the *right* in the *decimal*.

2. If any dividend is less than the trial divisor, place a cipher in the root, two ciphers at the right of the trial divisor, annex the next period for a new dividend, and proceed as before.

3. If there is a remainder after all the periods have been used, annex periods of decimal ciphers, and carry the root to as many decimal places as may be required; three places being generally enough.

4. The cube root of a fraction may be found by extracting the cube root of the numerator and of the denominator separately. If the terms of the fraction are not perfect cubes, change the fraction to a decimal, and extract the cube root of the decimal.

Problems.

Extract the cube root of—

1. 216; 343; 512.	5. 68921; 74088.	9. 41063625.
2. 729; 1728; 2197.	6. 85184; 91125.	10. 130323843.
3. 15625; 35937.	7. 195112; 274625.	11. 997002999.
4. 42875; 54872.	8. 300763; 405224.	12. 4080659192.

13. 12.167; 39.304; 474552; 259.694072.

14. .015625; .000166375; .00042875; 12.345678.

15. 144.203; .0304; 40.056; 567.008; .0090.

16. Extract the cube root of 5 to three decimal orders; of .5.

17. .000729 is the cube of what number? .091125?

18. Find one of the three equal factors of .001728; .001030301.

19. Find the length of the edge of a cubical block which contains 42875 cu. in.; 41063.625 cu. in.

20. Find the length of the inner edge of a cubical bin that holds 100 bu. Of a cubical bin that holds as much as a bin 5 ft. 4 in. long, 4 ft. 6 in wide, and 2 ft. 8 in. deep.

Extract the cube root of—

21. $\frac{27}{64}$; $\frac{64}{125}$.	23. $42\frac{7}{8}$; $34\frac{21}{64}$.	25. $10\frac{5}{12}$ times $1\frac{25}{144}$.
22. $\frac{729}{2744}$; $\frac{1728}{10648}$.	24. $166\frac{101}{216}$; $10\frac{1}{2}$.	26. $219\frac{7}{10}$ times $5\frac{3}{25}$.

Find the value of—

27. $\sqrt[3]{8} + \sqrt[3]{.8}$; $\sqrt[3]{(682.8^3)}$; $\sqrt[3]{(\frac{2}{3})^6} \times (\frac{4}{9})^2$.

28. $\sqrt[3]{1.025 + 14.6}$; $\sqrt[3]{.125 \times .729}$; $\sqrt[3]{.78\frac{1}{8} \div 50000}$.

29. $\sqrt[3]{8^6} \div \sqrt{16^3}$; $(\frac{3}{4})^2 + \sqrt{\frac{1}{25}} + \sqrt[3]{\frac{2}{25}}$.

30. Extract the cube root of the following expressions to three decimal places: $\frac{\sqrt{.04} \times \sqrt[3]{.216}}{\sqrt[3]{.008} \div \sqrt{.2\frac{1}{2}}}$; $\frac{\sqrt{.09} + \sqrt{.25} - \sqrt{.04} \times \sqrt{.36}}{\sqrt{.09} + (\sqrt{.25} - \sqrt{.04}) \times \sqrt{.36}}$.

31. Find the cube root of 2; 5; 7; $\frac{2}{3}$; $.8\frac{1}{3}$; 2.06; $(3\frac{3}{8})^2$.

32. What is the value of $\sqrt[3]{25}$? $\sqrt[3]{2.5}$? $\sqrt[3]{.25}$? $\sqrt[3]{.025}$? $\sqrt[3]{.0025}$? $(\sqrt[3]{15^6} \div \sqrt[3]{729}) - (1.44^{\frac{1}{3}} + \sqrt[3]{2\frac{1}{2}^2})$?

33. Compare the volumes of a 6-inch cube, one containing 6 cu. in., another containing 6 in.^{3}, and another $\sqrt[3]{6}$ cu. in.

34. Multiply $\sqrt[3]{(1\frac{1}{2})^6}$ by $\sqrt{(10\frac{1}{10})^4}$, and to the product add $\sqrt[3]{.5\frac{3}{25}} \times \sqrt{(.04)^3}$.

35. The volume of a cube is 91125 cu. ft. What is the length of the edge? Find the area of the entire surface.

36. What is the length of the inner edge of a cubical bin that holds 100 bu. of wheat? 100 bu. of corn in the ear?

CHAPTER VIII.

ALLIGATION.

1. If a pound of sugar worth 10 cents is mixed with a pound worth 8 cents, what is the entire value of the mixture? The average value?

2. If 2 pounds of tea at 75 cents are mixed with 3 pounds at 50 cents, how many pounds are in the mixture? What is the entire value? The average value?

3. If coffee worth 30 cents is mixed with coffee worth 45 cents, is the value of the mixture greater or less than the sum of the values of the kinds added?

4. What is the value of 5 gallons of wine at $2 a gallon? Of 6 gallons at $3 a gallon? What are both worth? If they be mixed, what will the mixture be worth?

5. If sugar at 10 cents be mixed with sugar at 15 cents, will the value of the mixture be more or less than 15 cents per pound? Than 10 cents per pound?

6. If a bushel of corn worth 60 cents a bushel is mixed with corn worth 80 cents, and the mixture is sold at 70 cents, how is the loss on the 80-cent corn balanced? The gain on the 60-cent corn?

Definitions.

430. Alligation treats of the mixing or compounding of two or more articles of different values or qualities.

Alligation is of two kinds; *Alligation Medial* and *Alligation Alternate.*

431. Alligation Medial is the process of finding the mean or average value or quality of several ingredients.

432. Alligation Alternate is the process of finding the proportional quantities to be used in any required mixture.

433. Principles.

I. *The value of a mixture equals the entire value of the ingredients at their respective values.*

II. *The average value of a mixture is greater than the lowest value and less than the highest value of any ingredient used in forming the mixture.*

III. *The gain or the loss on the value of any ingredient in a mixture must be balanced by the gain or the loss on the other ingredients.*

Section I.

ALLIGATION MEDIAL.

To Find the Mean Value when the Value and the Quantity of the Ingredients are given.

Written Exercise.

434. *Example.* — If a grocer mixes 3 lb. of sugar at 10 cents, 5 lb. at 8 cents, and 2 lb. at 15 cents, what is the average value per pound?

SOLUTION.

10 cents	×	*3* =	*30 cents.*
8 cents	×	*5* =	*40 cents.*
15 cents	×	*2* =	*30 cents.*
?	×	*10* =	*100 cents.*
100 cents	÷	*10* =	*10 cents.*

EXPLANATION. — Since 3 lb. of sugar at 10 cents are worth 30 cents, 5 lb. at 8 cents are worth 40 cents, and 2 lb. at 15 cents are worth 30 cents, and the mixture of 10 lb. is worth the sum of 30 cents and 40 cents and 30 cents, or 100 cents; hence, the value per pound of the mixture is $\frac{1}{10}$ of 100 cents, or 100 cents ÷ 10, which is *10 cents*, the average value required.

Note. — The processes in Alligation are best developed by the principles of analysis. No formal rules are, therefore, given for the solution of any of the problems.

Problems.

1. If 8 lb. of tea at 60 cents are mixed with 6 lb. at 70 cents, 2 lb. at $1.10, and 4 lb. at $1.20, what is a pound of the mixture worth?

2. If a goldsmith melts together 3 ounces of gold 22 carats fine, 15 ounces 20 carats fine, and 6 ounces 14 carats fine, how many carats fine is the mixture?

3. A drover bought 21 sheep at $5 a head, 24 at $4.75, and 25 at $7.12. What would be his average gain by selling them at $6.25 each?

4. If a grocer buys 30 gallons of molasses at 30 cents, 20 gallons at 25 cents, and mixes 4 gallons of water with them, how must he sell the mixture per gallon to gain 20%?

Section II.

ALLIGATION ALTERNATE.

To Find the Proportional Parts to be Used when the Mean Value of a Mixture and the Values of the Ingredients are given.

Written Exercise.

435. *Example.* — A dealer desires to mix flour worth $6, 7\frac{1}{2}$, and $10 a barrel, so as to sell the mixture at $9. What proportion of each kind must he use?

SOLUTION.

$\$9 - \$6 = \$3$, *gain.*
$\$9 - \$7\frac{1}{2} = \$1\frac{1}{2}$, "
$\$10 - \$9 = \$1$, *loss.*
L. c. m. of 3 and 1 = 3; of 1 and $1\frac{1}{2} = 3$.
1 bbl. at \$3 gain $= \$3 = 3$ *bbl. at \$1 loss.*
2 bbl. at $\$1\frac{1}{2}$, " $= \$3 = 3$ *bbl. at \$1,* "

EXPLANATION. — Since in every ingredient used whose value is less than the mean value there is a gain, and on every ingredient whose value is greater there is a loss, the entire gains in forming the mixture must balance the losses.

First, find the gain or the loss on a unit of each ingredient; then, compare the gain on one unit with the loss on the other, and find how many of each must be put in the mixture to balance the gain and the loss.

To give the least integral proportional parts, the sum required to balance the gains and the losses must be the least common multiple of the gain and the loss in the couplet of ingredients compared.

In the first couplet the gain is \$3 on a barrel and the loss is \$1; therefore it will take 1 bbl. at \$3 gain to balance 3 bbl. at \$1 loss; also, it will take 3 bbl. at \$1 loss to balance 2 bbl. at \$1½ gain. Hence, there must be taken *1 bbl.* at \$6, *2 bbl.* at \$7½, and *3 bbl.* + *3 bbl.*, or *6 bbl.* at \$10, the proportional quantities required.

Problems.

1. In what proportions must sugars worth 10 cents, 11 cents, and 14 cents be used to form a mixture worth 12 cents a pound?

2. What quantity of flour worth \$5½, \$6, and \$7¼, per barrel must be sold to realize an average price of \$6¼ per barrel?

3. A jeweler has gold 16 carats, 18, 22, and 24 fine. How much of each can he use to form gold 20 carats fine?

4. A wine merchant has wine worth \$3 and \$3.50 per gallon. What proportion of each and how much water can he mix with these two qualities to make a grade worth \$2.50 per gallon?

To Find the Proportional Parts when One of the Ingredients is Limited in Quantity.

436. *Example.*—A merchant bought 25 yards of dress goods at 24 cents a yard. How many yards at 16 cents, and at 30 cents, must he buy that the average cost may be 26 cents per yard?

SOLUTION.

26 cents — 16 cents = 10 cents, gain.
26 cents — 24 cents = 2 cents, " "
30 cents — 26 cents = 4 cents, loss.
L. c. m. of 10 and 4 = 20; of 2 and 4 = 4.
2 yd. at 10 cts. gain = 20 = 5 yd. at 4 cts. loss.
2 yd. " 2 cts. " = 4 = 1 yd. " 4 cts. "

EXPLANATION. — Since in every article or ingredient used, etc.

The proportional quantities are, therefore, 2 yd. at 16 cents, 2 yd. at 24 cents, and 6 yards at 30 cents.

But as the actual quantity of 24-cent goods is 25 yards, or $12\frac{1}{2}$ times the proportional part at that price, the quantity of each of the other proportional parts must be $12\frac{1}{2}$ times as great; giving *25 yards* at 16 cents, and *75 yards* at 30 cents, the quantities required.

5. A dairyman bought 10 cows at \$30 a head. How many must he buy at \$24, \$27, and \$36 a head, that all may average \$33 a head?

6. How many acres of land worth \$60 an acre must be added to a farm of 75 acres worth \$100 an acre that the average value may be \$70 an acre?

7. How many pounds of tea worth respectively \$.75, \$1, and \$1.50 per pound must be mixed with 16 pounds of tea worth \$1.25 that it may be sold at an average price of \$1.20?

8. If a druggist has 5 gallons of alcohol 90% strong, and some 85% strong, how much water and how much of each grade of alcohol can he use that the strength of the mixture may be 80%?

To Find the Proportional Parts when the Entire Mixture is Limited in Quantity.

437. *Example.* — How much sugar at 6 cents, 7 cents, 12 cents, and 13 cents must be used to make 105 pounds worth 10 cents a pound?

SOLUTION.

10 cents — 6 cents = 4 cents, gain.
10 " — 7 " = 3 " "
12 " — 10 " = 2 " loss.
13 " — 10 " = 3 " "
L. c. m. of 4 and 2 = 4; of 3 and 3 = 3.
1 lb. at 4 cts. gain = 4 cents = 2 lb. at 2 cts. loss.
1 lb. " 3 cts. " = 3 " = 1 lb. " 3 cts. "
2 + 3 = 5, sum.
105, entire quantity ÷ 5, sum of parts = 21.

EXPLANATION. — Since in every article or ingredient used, etc. (911).

The proportional quantities are, therefore, 1 lb. at 6 cts, 1 lb. at 7 cts., 2 lb. at 10 cts., and 1 lb. at 13 cts.

But as the entire quantity is 105 lb., while the

sum of the proportional parts is but 5, the quantity of each ingredient must be 21 times as great, giving *21 lb.* at 6 cents, *21 lb.* at 7 cents, *42 lb.* at 10 cents, and *21 lb.* at 13 cents, the quantities required.

9. How much water can be mixed with wine worth \$.90 a quart to make 100 quarts worth \$.60 a quart.

10. A grocer bought some butter at 28 cents, some at 30 cents, and some at 35 cents, — in all 50 pounds for \$16.50. How many pounds of each kind were there?

11. If a jeweler melt together gold 14, 18, and 24 carats fine to make 12 ounces 22 carats fine, how many ounces of each kind must he take?

12. A manufacturer paid \$396 to 33 mill-hands in two weeks. How many hands were there if to the men he paid \$10 a week, to the boys \$2, and to the girls \$1?

CHAPTER IX.

PROGRESSIONS.

Section I.

ARITHMETICAL PROGRESSION.

438. An ***Arithmetical Progression*** is a succession of numbers which increase or decrease by a common difference.

439. The ***Common Difference*** is the difference between any two consecutive terms of an arithmetical progression.

440. The ***Terms*** of an arithmetical progression are the numbers used to form the progression.

The first term and the last are called the *Extremes.*

The terms between the first and the last are called *Means.*

Thus, the progression 2, 5, 8, 11, 14, has six *terms;* the *extremes* are *2* and *14*, and the *means* are *5*, *8*, and *11*.

441. An Increasing Arithmetical Progression

is one in which the terms regularly increase from the left to the right.

442. ***A Decreasing Arithmetical Progression*** is one in which the terms regularly decrease from the left to the right.

Thus, *2, 5, 8, 11, 14* is an *increasing arithmetical progression* in which the *common difference* is *3*. And

9, 7, 5, 3, 1 is a *decreasing arithmetical progression* in which the *common difference* is *2*.

443. In Arithmetical Progression, five quantities are considered:

1. The first term (a).
2. The last term (l).
3. The common difference (d).
4. The number of terms (n).
5. The sum of all the terms (s).

If any three of the quantities of a progression are given, the other two may be found.

444. If the first term of an increasing arithmetical progression is 3, and the common difference is 5, the progression is formed as follows:

	1st Term.	2d Term.	3d Term.	4th Term.	5th Term.
(1)	*3*	*8*	*13*	*18*	*23, etc.*
(2)	*3*	$3+(5)$	$3+(5+5)$	$3+(5+5+5)$	$3+(5+5+5+5)$

And $3+8+13+18+23=65$, *sum of the series.*

445. The sum of an arithmetical progression may be found by adding the terms of the progression and the terms of the given progression reversed, as follows:

(1) *3 8 13 18 23*, an arithmetical progression.
(2) *23 18 13 8 3*, same progression reversed.

(3) $26+26+26+26+26$ = twice the sum of the terms.

Now in (3), which equals twice the sum of all the terms, each term equals the sum of the first term and the last.

446. Principles of Arithmetical Progression.

I. *The sum of the additions or subtractions equals the common difference taken as many times as the number of terms less* 1.

II. *Any term equals the given extreme increased or diminished by the sum of the additions or subtractions.*

III. *The common difference equals the quotient of the sum of the additions or subtractions divided by the number of terms less* 1.

IV. *The number of terms equals* 1 *more than the quotient of the sum of the additions or subtractions divided by the number of additions or subtractions.*

V. *The sum of all the terms equals the product of the average value of the terms, or half the sum of the extremes, multiplied by the number of terms.*

Written Exercises.

447. *Example* 1. — The first term of a decreasing arithmetical progression is 9, the common difference is $\frac{1}{2}$, and the number of terms is 10. What is the last term?

SOLUTION.

$10 - 1 = 9$, *no. of times com. diff. is subtracted.*

$\frac{1}{2} \times 9 = 4\frac{1}{2}$, *sum of subtractions from 1st term.*

$9 - 4\frac{1}{2} = 4\frac{1}{2}$, *last term.*

Or, $10 - 1 = 9$, *number of terms less 1.*

$9 - (\frac{1}{2} \times 9) = 9 - 4\frac{1}{2} = 4\frac{1}{2}$, *last term.*

EXPLANATION. — Since the first term is 9, and the common difference is $\frac{1}{2}$, the second term is $9 - \frac{1}{2}$, the third is $9 - (\frac{1}{2} \times 2)$, and so on; or the first term diminished by the sum of the subtractions, or the common difference taken as many times as the number of terms less 1, gives $4\frac{1}{2}$, the first term required.

448. *Example* 2. — The first term is $2\frac{1}{2}$, the last term is 35, and the number of terms is 14. What is the common difference?

SOLUTION.

$35 - 2\frac{1}{2} = 32\frac{1}{2}$, *sum of additions to 1st term.*

$14 - 1 = 13$, *number " " "*

$32\frac{1}{2} \div 13 = 2\frac{1}{2}$, *common difference.*

Or, $14 - 1 = 13$, *number of terms less 1.*

$(35 - 2\frac{1}{2}) \div 13 = 2\frac{1}{2}$, *common difference.*

EXPLANATION. — Since the last term diminished by the first terms equals the sum of the additions to the first term, and the number of additions equals the number of terms less 1, $32\frac{1}{2}$ is the product of the common difference by the number of terms less 1. Hence, $32\frac{1}{2} \div 13 = 2\frac{1}{2}$, the common difference required.

449. *Example* 3. — The first term is $31\frac{1}{4}$, the last term is $12\frac{1}{2}$, and the common difference is $1\frac{1}{4}$. Find the number of terms.

SOLUTION.

$31\frac{1}{4} - 12\frac{1}{2} = 18\frac{3}{4}$, *sum of subtractions from 1st.*

$18\frac{3}{4} \div 1\frac{1}{4} = 15$, *number " " "*

$15 + 1 = 16$, *number of terms.*

Or, $(31\frac{1}{4} - 12\frac{1}{2}) \div 1\frac{1}{4} = 15.$

$15 + 1 = 16$, *number of terms.*

EXPLANATION. — Since the first term diminished by the last term equals the sum of the subtractions from the first, and the number subtracted, or the common difference, is $1\frac{1}{4}$, the number of subtractions is the quotient of the sum of the subtractions divided by the common difference. Hence, $18\frac{3}{4} \div 1\frac{1}{4} = 15$, the number of subtractions, and $15 + 1 = 16$, the number of terms required.

450. *Example* 4. — If the first term is 8.5, the common difference 1.5, and the number of terms is 5, what is the sum of all the terms?

SOLUTION.

$5 - 1 = 4$, *number of terms less 1.*

$8.5 - (1.5 \times 4) = 2.5$, *last term.*

$8.5 + 2.5 = 11$, *two times average value.*

$11 \div 2 = 5.5$, *average value of terms.*

$5.5 \times 5 = 27.5$, *sum of series.*

Or, $\overline{(8.5 + 2.5) \times 5} \div 2 = 27.5$, *sum.*

EXPLANATION. — Since the first term is 8.5, etc. (925), the last term is 2.5.

Since the sum of the extremes is two times the average value of the terms, $11 \div 2$, or 5.5, is the average value of the terms. Hence, $5.5 \times 5 = 27.5$, the sum of the terms required.

451. From the preceding principles and explanations are deduced the following rules or formulas for the solution of the most important classes of problems in Arithmetical Progression:

1. $a + (n - 1)\, d = l.$
   $a - (n - 1)\, d = l.$
2. $l - (n - 1)\, d = a.$
   $l + (n - 1)\, d = a.$
3. $(l - a) \div (n - 1) = d.$
   $(a - l) \div (n - 1) = d.$
4. $\overline{(l - a) \div d} + 1 = n.$
   $\overline{(a - l) \div d} + 1 = n.$
5. $\overline{(a + l) \div 2} \times n = s.$

Note.— For each formula above given, the pupil should be required to substitute a rule for the solution of each problem to which the formula may be applicable.

Problems.

Find the required extreme

1. If the first term is 2, the common difference 3, and the number of terms 6. If the common difference is $1\frac{1}{2}$.

2. If the last term is 27, the common difference 3, and the number of terms 7. If the last term is $31\frac{1}{4}$.

3. If the first term is $3\frac{1}{2}$, the common difference $\frac{1}{4}$, and the number of terms 7. If the common difference is $-\frac{1}{4}$.

4. If the last term is $42\frac{1}{2}$, the common difference $-2\frac{1}{2}$, and the number of terms 10. If the first term is $42\frac{1}{2}$.

Find the common difference

5. If the first term is $2\frac{1}{2}$, the last term $17\frac{1}{2}$, and the number of terms 11. If the last term is $\frac{1}{5}$.

6. If the extremes are 56 and $6\frac{1}{2}$ and the number of terms 100 in a decreasing progression. If 50 terms.

7. If the first term is 37.5 and the 26th term is 6.25. If the 51st term is 25. If the 81st is 2.5. If the 112th is .5.

Find the number of terms

8. If the first term is 5, the common difference $2\frac{1}{4}$, and the last term $29\frac{3}{4}$. If the first term is $63\frac{1}{2}$.

9. If the last term is 1.5, the common difference 2.5, and the first term 39. If the last term is 76.5.

10. If the extremes are $\frac{1}{2}$ and $50\frac{1}{2}$, and the common difference is $\frac{1}{4}$. If the extremes are $16\frac{2}{3}$ and $\frac{2}{3}$. If 93.75 and 31.25.

Find the sum of the series

11. If the first term is $1\frac{1}{4}$, the last term $18\frac{3}{4}$, and the number of terms 15. If the common difference is $\frac{7}{8}$.

12. If the extremes are $\frac{1}{2}$ and $50\frac{1}{2}$, and the number of terms 50. If the extremes are 1.5 and $76\frac{1}{2}$.

13. If the extremes are $2\frac{1}{4}$ and 9, and the common difference $\frac{3}{4}$. If the common difference is $\frac{3}{8}$.

14. Find the 16th term of 1, $1\frac{1}{4}$, $1\frac{1}{2}$, etc. The 20th term of $5\frac{3}{4}$, $5\frac{1}{2}$, $5\frac{1}{4}$, etc. Find the sum of each.

15. What is the 25th odd number? The sum of the first 25 even numbers? Of the first 50 odd numbers?

16. A man walked 7 miles one day, 10 miles the next, 13 the next, and so on for 8 days. How far did he go?

17. If at the beginning of the year I have \$300, and at the end have \$740, what is my monthly salary?

18. The amount of \$100 at 5% interest is \$105 for the first year, \$110 for the second, etc. Find the amount for the 10th year. How much interest accrues in 21 years?

19. The amount of \$500 for 25 years is \$1375. What is the annual interest? The rate %?

20. Insert 12 arithmetical means between 20 and 59. Insert 13 between $4\frac{1}{2}$ and $32\frac{1}{2}$. Insert 18 between 6.25 and 125.

Section II.

GEOMETRICAL PROGRESSION.

452. A ***Geometrical Progression*** is a succession of numbers which increase or decrease by a common multiplier.

The *common multiplier* is usually called the *Rate.*

453. The ***Terms*** of a progression are the numbers used to form the progression.

The first term and the last are called the *Extremes.*

The terms between the first and the last are called the *Means.*

454. An Increasing Geometrical Progression is one in which the rate is greater than 1.

Thus, 1, 3, 9, 27, 81, is an *increasing geometrical progression*, in which the *rate* is *3*.

455. A Decreasing Geometrical Progression is one in which the rate is less than 1.

Thus, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, is a *decreasing geometrical progression*, in which the *rate* is $\frac{1}{2}$.

456. An *Infinite Decreasing Geometrical Progression* is one in which the rate is less than 1, and the number of terms unlimited.

Thus, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, and so on is an *infinite decreasing progression.*

457. In Geometrical Progression, five quantities are considered:

1. The first term (a).
2. The last term (l).
3. The rate (r).
4. The number of terms (n).
5. The sum of all the terms (s).

If any three of the quantities of a progression are given, the other two may be found.

458. If the first term of an increasing geometrical progression is 3, and the rate is 2, the progression is formed as follows:

	1st Term.	2d Term.	3d Term.	4th Term.	5th Term.
(1)	3	6	12	24	48, etc.
(2)	3	3×2	$3\times(2\times2)$	$3\times(2\times2\times2)$	$3\times(2\times2\times2\times2)$
(3)	3	3×2	3×2^2	3×2^3	3×2^4

And $3+6+12+24+48=93$, *sum of the series.*

459. The sum of a geometrical progression may be found by subtracting the terms of the progression from the terms of the same progression multiplied by the rate, as follows:

$$\left.\begin{array}{ll}(1) & \overline{2\times3}+\overline{6\times3}+\overline{18\times3}+\overline{54\times3}+\overline{162\times3}\\ (2) & 6+18+54+162+\overline{162\times3}\end{array}\right\} = \text{sum multiplied by the rate.}$$

Subtracting the terms of the given progression from the product of these terms by the rate, the results are:

$$\begin{array}{rl} s\times3= & 6+18+54+162+\overline{162\times3}\\ s= & 2+6+18+54+162\\ \hline 2s= & -2+\overline{162\times3}, \text{ or } \overline{162\times3}-2; \text{ that is,}\end{array}$$

The rate times sum less 1 time sum equals the product of the last term multiplied by the rate less the first term.

460. Principles of Geometrical Progression.

I. *The product of the multiplications or divisions equals the product of the rate used as a factor as many times as the number of terms less* 1.

II. *Any term equals the given extreme multiplied or divided by the product of the multiplications or divisions.*

III. *The rate equals such root of the product of the multiplications or divisions as is indicated by the number of terms less* 1.

IV. *The number of terms is* 1 *greater than the quotient of the multiplications or divisions divided by the rate.*

V. *The sum of all the terms equals the quotient of the rate times the greater extreme diminished by the less, divided by* 1 *less than the rate.*

Written Exercises.

461. *Example* 1.—The last term is $\frac{1}{2}$, the rate is 3, and the number of terms is 5. What is the first term?

SOLUTION.

$5 - 1 = 4$, *number of times rate is used as a divisor.*
$3^4 = 81$, *product of rate used as a factor.*
$\frac{1}{2} \div 81 = \frac{1}{162}$, *the first term.*
Or, $5 - 1 = 4$, *number of terms less 1.*
$\frac{1}{2} \div 3^4 = \frac{1}{2} \div 81 = \frac{1}{162}$, *first term.*

EXPLANATION. — Since the given extreme is the product of the required extreme multiplied by the product of 3 used four times as a factor, the required extreme, or first term, is the quotient of $\frac{1}{2}$ divided by 81, or $\frac{1}{2} \div 81$, which is $\frac{1}{162}$, the first term required.

462. *Example* 2. — The first term in a decreasing progression is $\frac{3}{4}$, the last is $\frac{3}{64}$, and the number of terms is 5. What is the rate?

SOLUTION.

$\frac{3}{4} \div \frac{3}{64} = 16$, *product of rate used as a factor.*
$5 - 1 = 4$, *number of times " " "*
$\sqrt[4]{16} = 2$, *rate.*
Or, $5 - 1 = 4$, *number of terms less 1.*
$\sqrt[4]{\frac{3}{64} \div \frac{3}{4}} = \sqrt[4]{\frac{1}{16}} = \frac{1}{2}$, *rate.*

EXPLANATION.—Since the last term equals the first divided by the product of all the divisions, 16 is the product of the rate used 4 times as a factor.

Hence, the 4th root of 16, or 2, is the rate required.

But as the rate in a decreasing series is usually expressed as a fraction, the rate may be found by taking the 4th root of the quotient of $\frac{3}{64}$ divided by $\frac{3}{4}$, giving $\frac{1}{2}$, the rate as a common multiplier.

463. *Example* 3. — The first term is 100, the last term is 6.25, and the rate is .5. Find the number of terms.

SOLUTION.

6.25 ÷ 100 = .0625, prod. of rate used as a factor.
.0625 = product of .5 used 4 times " "
4 + 1 = number of terms.
Or, 6.25 ÷ 100 = .0625 = .5⁴.
4 + 1 = 5, number of terms.

EXPLANATION. — Since the last term is the product of the first term multiplied by the rate used as a factor 1 time less than the number of terms, 6.25 ÷ 100, or .0625, is the product of .5 used 4 times as a factor. Hence, 4 + 1, or 5, is the number of terms required.

464. *Example* 4. — The extremes of a geometrical progression are 2 and 128, and the rate is 4. What is the sum of all the terms?

SOLUTION.

128 × 4 = 512, four times the last term.
512 − 2 = 510, three times the series.
510 ÷ 3 = 170, the sum of the series.
Or, 4 − 1 = 3, the rate less 1.
$\overline{(128 \times 4) - 2} \div (4 - 1) = 170$, *the sum.*

EXPLANATION.—If from the product of each term of the progression multiplied by the rate, 4, each term of the given progression be taken, there will remain 4 times the last term less the first term, and this equals 4 times the sum less 1 time the sum, or 3 times the sum. Hence, 510 divided by 3, gives *170*, the sum required.

465. From the preceding principles and explanations are deduced the following formulas, which the pupil should be required to prove, and to convert into rules:

1. $a \times r^{n-1} = l.$
2. $l \div r^{n-1} = a.$
3. $\sqrt[n-1]{l \div a} = r.$
   $\sqrt[n-1]{a \div l} = r.$
4. $l \div a = r^{n-1}.$
   $a \div l = r^{n-1}.$
5. $\overline{(l \times r) - a} \div (r - 1) = s.$
   $\overline{(a \times r) - l} \div (1 - r) = s.$

In a descending series of an infinite number of terms, the last term becomes so small that it is considered as 0, and formula 5 becomes $a \div (1 - r) = s.$

Problems.

Find the required extreme

1. If the first term is 5, the rate 2, and the number of terms 8. If the rate is $\frac{1}{2}$. If the last term is $\frac{1}{2}$ and the rate $\frac{1}{2}$.

2. If the last term is $30\frac{3}{8}$, the rate $1\frac{1}{2}$, and the number of terms 6. If the first term is 8.

3. If the first term is $\frac{1}{4}$, the rate .5, and the number of terms 5. If the last term is 31.25. If the last term is 6.25.

Find the rate

4. If the first term is 8, the last term 5000, and the number of terms 5. If the last term is $\frac{1}{2}$.

5. If the last term is .625, the first term 10, and the number of terms 5. If the last term is $390\frac{5}{8}$ and the first $\frac{5}{8}$.

6. If the first term is $\frac{1}{12}$, the last term $60\frac{3}{4}$, and the number of terms 7. If the first term is 3888. If the first is $44286\frac{3}{4}$.

Find the number of terms

7. If the first term is $2\frac{1}{2}$, the rate 3, and the last term $607\frac{1}{2}$. If the extremes are 81 and $\frac{1}{3}$.

8. If the first term is 320, the rate $\frac{1}{2}$, and the last term $2\frac{1}{2}$. If the extremes are $\frac{2}{6}\frac{9}{4}$ and $7\frac{1}{4}$.

9. If the first term is .1, the last term .00001, and the rate .1. If the extremes are 1000 and .001.

Find the sum of all the terms

10. If the extremes are $2\frac{1}{2}$ and $\frac{5}{4}\frac{6}{8}\frac{2}{6}$, and the rate $\frac{1}{3}$. If the first term is $\frac{2}{5}$ and the sixth term $97\frac{1}{5}$ and the rate 3.

11. If the first term is 1, the rate $\frac{1}{2}$, and the ninth term $\frac{1}{5\,1\,2}$. If the extremes are 512 and 1.

12. If the first term is 1, the rate $\frac{1}{3}$, and the last term 0. If the progression is 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., to infinity.

13. A clerk whose first year's salary was \$75, had his salary doubled every year for 5 years. What did he receive the last year? How much in all?

14. The first term of a geometrical progression is 4, and the seventh term is 2916. Find the rate and the sum.

15. A merchant doubled his capital of \$6000 every four years. How much was he worth at the end of 20 years?

16. How much can be saved in 10 years by placing in bank \$1 the first year, and doubling the amount each year?

17. If I commence business with $1000, and increase my capital by $\frac{1}{4}$ of itself each year, what will be my capital at the end of the sixth year?

18. The first term is 7, the rate 3, and the number of terms 6. What is the sum of the progression?

19. If $1 be doubled the first day of every month in the year, what will be the total amount of money?

20. If a body should move $\frac{1}{2}$ a mile the first minute, $\frac{1}{4}$ of a mile the second minute, and so on forever, how far would it move?

Section III.

ANNUITIES AND DEFERRED PAYMENTS.

466. An ***Annuity*** is a fixed sum of money to be paid annually or at the end of equal periods of time.

467. The ***Amount*** or ***Final Value*** of an annuity is the sum of all the payments each increased by the interest from the time it is due until the annuity ceases.

468. The ***Present Worth*** or ***Present Value*** of an annuity is such a sum as will, in the given time and at the given rate per cent., amount to the final value of the annuity.

CASE I.

Annuities and Deferred Payments at Simple Interest.

469. An annuity at simple interest forms an arithmetical progression in which

1. The ***Annuity*** is the *First Term.*
2. The ***Interest for one year*** is the *Common Difference.*
3. The ***Number of Intervals*** of time is the *Number* of *Terms.*
4. The ***Final Value*** is the *Sum of all the Terms.*

470. The combined principles of interest and arithmetical progression apply also to the process of finding the simple interest in which

1. *Principal* is a.
2. *Interest* for *1 year* is d.
3. *Number of years* + *1* is n.
4. *Amount* is l.

Note. — The same principles apply also to calculations on deferred payments by instalments, as in the case of rent, interest on mortgages, building association dues, etc.

The actual value of building association shares is, however, always affected by the *business* of the association; that is, by fines for non-payment, premiums on money loaned, losses, etc.

Written Exercises.

471. *Example* 1.—Find the amount and the interest of $200 for 5 years, at 6% simple interest.

SOLUTION.

$a + (n - 1)\,d = l.$
$\$200 + (\$12 \times 5) = \$260$, *Amt.*
$\$260 - \$200 = \$60$, *Int.*

EXPLANATION. — The principal is the first term. The number of years is 5, but the number of terms is 6, since the interest is added to the principal 5 times. Hence, $\$260$ is the amount required, and $\$60$ is the interest.

472. *Example* 2.—Find the amount of an annuity of $400 for 4 years at 7%.

SOLUTION.

$a + (n - 1)\,d = l.$
$\$400 + (\$28 \times 3) = \$484.$
$\overline{(a + l) \div 2} \times n = s.$
$\overline{(\$400 + 484) \div 2} \times 4 = \$1768$, *Amt.*

EXPLANATION.— If the annuity is left unpaid till the end of 4 years, the last payment will be the annuity without any interest, the third payment will bear one year's interest, the second two years' interest, and the first three years' interest. Hence, $\$484$ is the amount due on first payment, and $\$1768$ is the final amount required.

473. *Example* 3. — If a man rents a house at $10 a month in advance, and the rent remains unpaid for a year, what amount is then due, interest at 6%?

SOLUTION.

$\overline{(a + l) \div 2} \times n = s.$
$(\$.05 + \$.60) \div 2 \times 12 = \$3.90$, *Int.*
$\$120 + \$3.90 = \$123.90$, *Amt.*

EXPLANATION. — Since the number of terms is 12, the first term is the interest on the last instalment for one month, or 5

cents, and the last term is the interest on the first instalment for 12 months, or 60 cents. Hence, the interest due is *$3.90*, which added to the rent, $120, gives *$123.90*, the amount required.

474. *Example* 4.—Find the present value of an annuity of $300 for 15 years, at 6% simple interest.

SOLUTION.

$a + (n - 1)\ d = l.$

$\$300 + (\$18 \times 14) = \$552$, *Amt. of 1st.*

Amt. $\div$ $\overline{1 + (rate \times time)}$ = *pres. value.*

$\$6390 \div (\$1 + \$.90) = \$3363.15+$

EXPLANATION. — The final value of the annuity is $6390.

Since the present value of an annuity is the present worth of its final value, the present value is the quotient, of the final amount divided by the amount of $1 for the given rate and time, or *$3363.15* + the present value required.

Problems.

1. Find the amount of $800 for 8 years at 6%. At 7%.

2. If I save $150 each year, and put it out at 6%, simple interest, to how much will it amount in 10 years?

3. What is the final value of an annuity of $500 for 10 years at $4\frac{1}{2}$%? For 6 years at 5%?

4. What is the present worth of an annuity of $300 for 5 years at 6%? For 6 years at 5%?

5. What is the amount of an annuity of $200 for 4 years, payable quarterly, at 6% a year?

6. A widow has a dower of $300 at 6%, to continue until her child who is now 5 years old comes of age. What is its present value?

7. At what rate will an unpaid debt of $750 amount to $1170 in 8 years, simple interest?

8. A man rents a house at $300 a year, payable monthly in advance, but makes no payment till the end of the year. What does he then owe, interest 6%?

9. The monthly instalment on a building association share is $1. What is a share worth at the end of 4 years, considering interest only, at 6%?

10. If I buy a farm for $5000, payable in 10 semi-annual

instalments, and pay nothing till the end of the time, how much will be due, interest at 6%?

11. A merchant leased a store for 5 years at $600 a year, and paid the rent in advance less 6% discount. How much did he pay?

12. If after three years I shall come into possession of property that will yield an annual income of $500, how much cash can I borrow by "hypothecating" 5 years of this income, allowing 8% for the loan?

CASE II.

Annuities and Deferred Payments at Compound Interest.

475. An Annuity at compound interest forms a geometrical progression in which

1. The ***Annuity*** is the *First Term.*

2. The ***Amount of $1*** for one interval of time is the *Rate.*

3. The ***Number of Intervals*** of time is the *Number of Terms.*

4. The ***Final Value*** is the *Sum of all the Terms.*

476. The combined principles of compound interest and geometrical progression apply to the process of finding compound interest in which

1. *Principal* is a.
2. *1 plus rate* is r.
3. *Number of years plus 1* is n.
4. *Compound Amount* is l.

The same principles apply also to payments and to deferred payments by instalments at compound interest.

Written Exercises.

477. *Example* 1. — Find the final value of an annuity of $200 for 5 years, at 5% compound interest.

SOLUTION.

$a \times r^{n-1} = l.$

$\$200 \times 1.05^4 = \$243.1012$, *Comp. Amt.*

$\overline{(l \times r) - a} \div (r - 1) = s.$

$\overline{(\$243.1012 \times 1.05) - 200} \div .05 = \$1105.125.$

EXPLANATION. — If the annuity is left unpaid until the end of 5 years, the last payment is the annuity without interest, and henc

forms the first term; the fourth payment bears one year's comp. interest, and equals 1.05 times $200, etc.; and the first payment equals 1.05^4 times $200, or *$243.1012*.

Hence, the final value is *$1105.125*, the sum of all the payments required.

478. *Example* 2.—Find the present value of an annuity for $300 at 6% compound interest, if unpaid for 5 years.

SOLUTION.

$a \times r^{n-1} = l.$

$300 × 1.06^4 = $378.7431, Comp. Amt.

$(l \times r) - a \div (r - 1) = s.$

($378.7431 × 1.06) − 300 ÷ .06 = $1691.13.

$1691.13 ÷ $1.338226 = $1263.71+.

EXPLANATION. — The compound amount of $1 for 5 years at 6% is $1.338226.

The final value of $300 for 5 years at 6% comp. int. is $1691.13. The present value is as many dollars as the number of times the compound amount of $1 is contained in the final value, giving *$1263.71*, the present value required.

Problems.

1. What is the compound interest of $500 for 3 years at 6%?

2. What is the final value of $300 annually for 6 years at 4%? For 4 years at 6%? For 5 years at 5 %?

3. If I deposit $50 every six months in a savings-bank to draw interest semi-annually at 2%, what is it worth at the end of 3 years? At the end of 5 years?

4. If it costs me $75.25 a year to have my life insured, what ought a ten-year endowment policy to be worth at the end of that time?

5. What sum at 6% compound interest amounts to $89.25 in 4 years? To $11304.52 in 14 years?

6. What is the present value of an annuity of $200 at 6% compound interest for 4 years? For 3 years at 5%?

7. What sum of money deposited in a savings-bank at 5% compound interest will amount to as much as $100 annually for 5 years?

8. An annuity of $300 for 3 years at 6% is to commence 2 years hence. What is the present value of this annuity in reversion?

CHAPTER X.

CIRCULATING DECIMALS.

479. A ***Finite Decimal*** is a decimal that ends with the figures used to express it.

480. A ***Circulating Decimal*** is a decimal in which a figure, or a set of figures, is constantly repeated.

Thus, .25, .875, or 1.0625, is a *finite decimal.* And

.333 +, .2727 +, or .135135 + is a *circulating decimal.*

481. A ***Repetend*** is the figure, or set of figures, constantly repeated.

482. A repetend of one figure is expressed by writing the figure once and placing a dot or point over it.

483. A repetend of more than one figure is expressed by writing the set of figures once and placing a point over the first and over the last figure.

Thus, .333 + is written $.\dot{3}$; .135135 + is written $.\dot{1}3\dot{5}$.

484. A repetend of one figure may be called a *simple repetend;* and a repetend of more than one figure may be called a *compound repetend.*

485. A ***Pure Circulating Decimal*** is a decimal that commences with the repetend.

486. A ***Mixed Circulating Decimal*** is a decimal in which the repetend is preceded by one or more decimal figures, called the *finite part* of the decimal.

Thus, $.\dot{6}$, $.\dot{2}9\dot{7}$, is a *pure circulating decimal.* And

$.1\dot{2}$ or $.06\dot{2}\dot{7}$, is a *mixed circulating decimal,* .1 or .06 being the *finite part.*

487. The following illustrates the origin and the law of circulating decimals:

1. $\frac{1}{9} = .1111+ = .\dot{1}.$	$.\dot{1} = .1111+ = \frac{1}{9}.$
2. $\frac{1}{99} = .010101+ = .\dot{0}\dot{1}.$	$.\dot{0}\dot{1} = .010101+ = \frac{1}{99}.$
3. $\frac{1}{999} = .001001+ = .\dot{0}0\dot{1}.$	$.\dot{0}0\dot{1} = .001001+ = \frac{1}{999}.$

488. Principles of Circulating Decimals.

I. *Annexing a cipher to a number or multiplying it by* 10 *introduces into the result the prime factors* 2 *and* 5.

II. *A common fraction in its lowest terms having a denominator which contains no other prime factors than* 2 *or* 5 *is equivalent to a finite decimal.*

III. *A common fraction in its lowest terms having a denominator which contains other prime factors than* 2 *or* 5 *is equivalent to a circulating decimal.*

IV. *A common fraction in its lowest terms having a denominator which contains* 2 *or* 5 *with other prime factors is equivalent to a mixed circulating decimal.*

V. *A pure circulating decimal is equivalent to a common fraction whose numerator is the given repetend, and whose denominator is as many* 9*'s as there are figures in the repetend.*

CASE I.

To Change a Common Fraction to a Circulating Decimal.

Written Exercises.

489. *Example.* — Change $\frac{32}{33}$ to a decimal.

SOLUTION.

$\frac{32}{33} = .9696+ = .\dot{9}\dot{6}.$

$33)32.0000$

$.9696\frac{27}{33}.$

EXPLANATION.—By annexing decimal ciphers to the numerator and dividing by the denominator (362), the result is the complex decimal $.9696\frac{27}{33}$, in which the figures 96 are repeated. Hence, $.\dot{9}\dot{6}$ is the circulating decimal required.

RULE. — *Annex decimal ciphers to the numerator and divide by the denominator, until the quotient figures begin to repeat.*

Point off as many places as equal the number of decimal ciphers annexed, and place a point over the first and over the last figure of the repeating part.

Problems.

1. Change to finite decimals $\frac{1}{8}$; $\frac{1}{16}$; $\frac{7}{25}$; $\frac{15}{16}$; $\frac{9}{125}$.
2. Change to complex decimals of four places $\frac{2}{3}$; $\frac{5}{9}$; $\frac{8}{11}$.

3. Change to pure circulating decimals $\frac{1}{7}$; $\frac{8}{9}$; $\frac{12}{13}$; $\frac{141}{143}$.

4. Change to mixed circulating decimals $\frac{5}{7}$; $\frac{11}{15}$; $\frac{9}{44}$; $\frac{17}{150}$.

Change to finite or circulating decimals:

5. $\frac{1}{4}$; $\frac{10}{21}$; $\frac{19}{27}$; $\frac{9}{32}$; $\frac{11}{125}$.

6. $\frac{17}{64}$; $\frac{29}{33}$; $\frac{17}{56}$; $\frac{21}{22}$; $\frac{12}{66}$.

7. $\frac{11}{375}$; $\frac{9}{250}$; $\frac{9}{350}$; $\frac{19}{1050}$.

8. $8\frac{5}{7}$; $11\frac{9}{28}$; $21\frac{5}{180}$; $17\frac{1}{750}$.

CASE II.

To Change a Circulating Decimal to a Common Fraction.

Written Exercises.

490. *Example* 1.—Change $.\dot{0}2\dot{1}$ to a common fraction.

SOLUTION.

$.\dot{0}0\dot{1} = \frac{1}{999}$.

$.\dot{0}2\dot{1} = \frac{21}{999} = \frac{7}{333}$.

EXPLANATION.—Since $.\dot{0}0\dot{1}$ equals $\frac{1}{999}$, $.\dot{0}2\dot{1}$ equals 21 times $\frac{1}{999}$, or $\frac{21}{999}$, which changed to its lowest terms equals $\frac{7}{333}$, the common fraction required.

491. *Example* 2.—Change $.4\dot{0}\dot{9}$ to a common fraction.

SOLUTION.

$.4\dot{0}\dot{9} = \frac{4}{10} + \frac{9}{990} = \frac{405}{990} = \frac{9}{22}$.

Or, $.4\dot{0}\dot{9} = .4\frac{9}{99} = \frac{4\frac{9}{99}}{10} = \frac{9}{22}$.

EXPLANATION.—Since the repetend is not $\frac{9}{99}$, but $\frac{9}{99}$ of $\frac{1}{10}$, or $\frac{9}{990}$, write the finite part and the repetend each as a fraction, and find their sum, which is $\frac{9}{22}$. Or, Express the finite part and the repetend as a mixed decimal and change it to a common fraction, giving $\frac{9}{22}$, the fraction required.

To change a pure repetend to a common fraction:

RULE.—*Write the figures of the repetend as the numerator of the common fraction, and for the denominator write as many 9's as there are figures in the repetend, and change the fraction to its lowest terms.*

To change a mixed repetend to a common fraction:

RULE.—*Express the finite part and the repetend as a complex decimal, and change this decimal to a common fraction.*

Problems.

Change to common fractions in their lowest terms:

1. $.\dot{3}\dot{6}$; $.\dot{4}\dot{5}$; $.\dot{6}\dot{6}$; $.\dot{7}\dot{8}$.
2. $.\dot{1}2\dot{6}$; $.\dot{3}0\dot{3}$; $.\dot{4}5\dot{9}$; $.\dot{8}8\dot{8}$.
3. $.\dot{3}2\dot{4}$; $.\dot{9}2\dot{7}$; $.\dot{1}06\dot{2}$; $\dot{6}43\dot{5}$.
4. $.\dot{9}80\dot{1}$; $.\dot{9}512\dot{1}$; $.\dot{1}42857$.

5. Change to improper fractions $4.\dot{4}$; $11.\dot{6}\dot{6}$; $10.0\dot{0}3$; $3.0\dot{2}\dot{7}$.

Change to common fractions in their lowest terms:

6. $.3\dot{2}$; $.7\dot{9}$; $.2\dot{1}\dot{6}$; $.0\dot{1}\dot{8}$.
7. $.3\dot{2}\dot{7}$; $.00\dot{8}$; $.4\dot{7}\dot{2}$; $.0\dot{1}\dot{5}$.
8. $.08\dot{5}$; $.00\dot{6}\dot{6}$; $.100\dot{0}\dot{9}$.
9. $.02\dot{7}0\dot{2}$; $.0\dot{2}71\dot{8}$; $.027\dot{1}\dot{8}$.

10. Change $4.00\dot{3}$ and $2.00\dot{\frac{1}{3}}$ to improper fractions.
11. Change $3.\dot{6}\frac{1}{2}$, $4.0\frac{1}{2}$, $6.00\dot{3}\frac{1}{3}$ to improper fractions.

CASE III.

Similar and Dissimilar Repetends.

492. A finite decimal may be expressed as a circulating decimal by making the repetend to consist of ciphers.

Thus, .25 may be expressed $.25\dot{0}$ or $.25\dot{0}0\dot{0}$; for each changed to a common fraction equals $\frac{1}{4}$.

493. A simple repetend may be expressed as a compound repetend by repeating the figures forming the repetend.

Thus, $.\dot{7}$ may be expressed $.\dot{7}\dot{7}$, or $.\dot{7}7\dot{7}\dot{7}$; for each changed to a common fraction equals $\frac{7}{9}$.

494. A compound repetend may be expressed by moving the right hand point over an exact number of periods.

Thus, $.\dot{3}\dot{6}$ may be expanded to $.\dot{3}6\dot{3}\dot{6}$, or $.\dot{3}6\dot{3}6\dot{3}\dot{6}$; for each changed to a common fraction equals $\frac{4}{11}$.

495. Both points over a repetend may be moved to the same number of places to the right without changing the value.

Thus, $.0\dot{2}\dot{1}$ may be expressed $.0\dot{2}1\dot{0}$, or $.02\dot{1}0\dot{2}$; for each changed to a common fraction equals $\frac{7}{333}$.

Written Exercises.

496. *Example* 1. — Make $.\dot{6}$, $.0\dot{4}\dot{5}$, and $6.3\dot{9}$ similar.

SOLUTION.

$.\dot{6} = .6666666\ldots$

$.0\dot{4}\dot{5} = .0450450\ldots$

$6.3\dot{9} = 6.3963963\ldots$

EXPLANATION. — To be similar, the repetends must begin at the same place and end at the same place.

To begin at the same place the left-hand point must be over the second figure, since one of the decimals has a finite decimal figure.

To end at the same place the right-hand point must be moved to the right over an exact number of periods. Now the periods or sets of figures in the given repetends are composed of 1, 2, and 3 places, respectively. Hence, the number of places in the new repetend must be the common multiple of 1, 2, and 3, which is 6. The right-hand point must, therefore, be moved to the right so that each repetend shall contain six places.

497. *Example* 2. — Find the sum of $1.2\dot{7}$, $.4\dot{8}\dot{1}$, and $7.2\dot{9}$.

SOLUTION.

$1.2\dot{7} = 1.272727\ldots$

$.4\dot{8}\dot{1} = .481481\ldots$

$7.2\dot{9} = 7.297297\ldots$

$9.051506\ldots$

EXPLANATION. — Since only similar fractional units can be added, the repetends must be made similar.

Since, if the repetends were expanded, the sum of the next column to the right would be 25, add 2 to the sum of the right-hand column, giving $9.0\dot{5}1506\ldots$, the sum required.

498. *Example* 3. — From $3.0\dot{5}$ take $1.0\dot{9}\dot{6}$.

SOLUTION.

$3.0\dot{5} = 3.053053\ldots$

$1.0\dot{9}\dot{6} = 1.096969\ldots$

$1.956083\ldots$

EXPLANATION. — Since only similar fractional units can be subtracted, etc.

Since, if the repetends were expanded, there would be 1 borrowed from the right-hand figure of the minuend, take 1 from that order in the minuend, leaving $1.9\dot{5}6083\dot{3}$, the remainder required.

Note 1.—Operations involving circulating decimals are of little practical importance, except in the higher mathematics. In ordinary computations, they are carried to four or five places, and are then treated as other decimals.

2. Circulating decimals are most conveniently added, subtracted, multiplied, and divided by changing them to common fractions, and then performing the operations required.

3. A few problems are given for practice; those in addition and sub-

traction to be performed by either or both methods, and the remaining problems according to the following

499. GENERAL RULE. — *Change the circulating decimals to common fractions, perform the required operations, and change the result to an equivalent circulating decimal.*

Problems.

1. Add together $6.\dot{5}$, $3.0\dot{9}$, $4.\dot{0}7\dot{4}$, and $9.0\dot{5}\dot{5}$.
2. Find the sum of $18.29\dot{3}2\dot{4}$, $.\dot{2}1\dot{6}$, $10.00\dot{5}$, and $4.0\dot{2}\dot{2}$.
3. Subtract $9.0\dot{4}$ from $27.8\dot{3}$; $\dot{5}.6\dot{2}$ from 10.5.
4. From $24.32\dot{5}$ take $21.\dot{3}$; and from $\frac{4}{9}$ take $.\dot{2}3\dot{4}$.
5. Find the value of $(9.\dot{6} - 3.0\dot{3}) + 4.0\dot{7}2\dot{9}$.
6. Multiply $3.81\dot{8}$ by $2.2\dot{2}$; $5.0\dot{2}\dot{7}$ by $9.8\dot{8}$.
7. What is the product of $9\dot{2}7.\dot{9}$ multiplied by $7.0\dot{6}$?
8. Find the value of $7.25 \times (8.\dot{3}\dot{6} - 5.00\dot{8}) - 19$.
9. Divide $.05\dot{9}$ by .75; $8.\dot{2}\dot{7}$ by 25; $7.0\dot{8}$ by $\frac{4}{5}$.
10. What is the quotient of $17.\dot{2} \div .00\dot{6}$? Of $75 \div .\dot{7}\dot{5}$?
11. To $24.\dot{8}$ add $12\frac{4}{9}$ and multiply the sum by $9\frac{5}{6}$.
12. Find the value of $(\frac{5}{9}\frac{5}{9} \times .0\dot{1}\dot{8}) + \frac{1}{2}$ of $4.0\dot{4}\dot{4}$.
13. What is the result of $(8.\dot{2}85714 \div 8.1\dot{8}) + 6.02$?
14. Find the value of £$10.06\frac{2}{3}$ in lower denomination.
15. Change $4.8\dot{8}$ A. to its value in square rods.

CHAPTER XI.

THE METRIC SYSTEM.

500. The ***Metric System*** is a system of measures and weights based upon the decimal scale.

The *Primary Unit* of the Metric System is the *Meter.*

501. The ***Meter*** is one ten-millionth part of the distance from the equator to either pole, measured on the earth's surface at the level of the sea.

502. From the meter are formed the standard units of *square* and *land measures; cubic* and *wood measures; measures of capacity;* and *measures of weight.*

503. The *higher units* or denominations of any measure are named by prefixing to the name of the standard unit of that measure, the *Greek* numerals, ***Deka*** (*dek'-a*) *10;* ***Hecto*** (*hek'to*) *100;* ***Kilo*** (*kil'o*) *1000;* and ***Myria*** (*mir'ea*) *10000.*

504. The *lower units* or *denominations* of any measure are named by prefixing to the name of the standard unit of that measure, the *Latin* numerals ***Deci*** (*des'e*) *.1;* ***Centi*** (*sent'-e*) *.01;* and ***Milli*** (*mill'e*) *.001.*

Note. — The numeral prefixes furnish the key to the whole system, since the name of the unit shows whether it is greater or less than the standard unit, and also how many times greater or less than that unit.

MEASURES OF EXTENSION.

I. Linear Measures.

505. The ***Meter*** is the *Standard Unit* of *measures of length.*

The value of the meter in the United States is *39.37 in.* nearly.

Table.

10 meters	are	*1 dekameter*	= *393.7 in.*	= *32.81 ft.*
10 dekameters	"	*1 hectometer*	= *328.1 ft.*	= *19.93 rd.*
10 hectometers	"	*1 kilometer*	= *199.3 rd.*	= *.6214 mi.*
10 kilometers	"	*1 myriameter*	= *6.214 miles.*	

Also,

.1 meter	is	*1 decimeter*	=	*3.937 in.*
.1 decimeter	"	*1 centimeter*	=	*.3937 in.*
.1 centimeter	"	*1 millimeter*	=	*.03937 in.*

Note 1. — The *meter,* about $1\frac{1}{12}$ *yd.,* is used in measuring cloth, etc., and short distances. The *kilometer,* about $\frac{5}{8}$ *of a common mile,* in measuring long distances. The *centimeter* and the *millimeter* for very minute lengths.

2. The nickel 5-cent piece is two hundredths of a meter in diameter. Fifty of them, placed side by side in a straight line, measure 1 meter.

II. Surface Measures.

506. The ***Square Meter,*** or ***Centare,*** is the *Standard Unit* of ordinary *surface measures.*

The *Are* is the standard unit of *land measures.*

Note. — The *are* is a square dekameter, or a square each side of which is 10 meters, and is equal to *119.6 sq. yd.*

Table.

100 square meters	are	*1 are*	=	*119.6 sq. yd.*
100 ares	"	*1 hectare*	=	*2.471 acres.*

Also,

.01 square meter	is	*1 sq. decimeter*	=	*15.5 sq. in.*
.01 square decimeter	"	*1 sq. centimeter*	=	*.155 sq. in.*

A *square meter,* about $10\frac{3}{4}$ *sq. ft.,* or $1\frac{1}{5}$ *sq. yd.,* is used in measuring flooring, ceilings, etc. The *are,* nearly *4 sq. rd.,* and the *hectare,* about $2\frac{1}{2}$ *A.,* in measuring land.

III. Solid Measures.

507. The ***Cubic Meter*** is the *Standard Unit* of ordinary *solid measures.*

The *Stere* is the standard unit of *wood measures.*

Note. — The *stere* is the cubic meter, or a cube each edge of which is 1 meter, and is equal to *35.316 cu. ft.,* or *.2759 cd.*

Table.

10 decisteres	are	*1 stere*	=	*35.31 cu. ft.*
10 steres	"	*1 dekastere*	=	*2.759 cd.*

Also,

.001 cu. meter	is	*1 cu. decimeter*	=	*61.022 cu. in.*
.001 cu. decimeter	"	*1 cu. centimeter*	=	*.06102 cu. in.*
.001 cu. centimeter	"	*1 cu. millimeter*	=	*.00006 cu. in.*

A *cubic meter* is about $1\frac{1}{3}$ *cu. yd.;* and a *stere,* about $2\frac{1}{5}$ *cd. ft.*

MEASURES OF CAPACITY.

508. The ***Liter*** is the *Standard Unit* of both *dry* and *liquid measures.*

Note. — The *liter,* or cubic decimeter, is a vessel equal in volume to a cube each side of which is one-tenth of a meter, and equals 61.022 cu. in., or *.9081 qt.* dry measure, or *1.0567 qt.* liquid measure.

Table.

			Dry.		Liquid.
10 liters	are	*1 dekaliter*	= *9.08 qt.*	=	*2.6417 gal.*
10 dekaliters	"	*1 hectoliter*	= *2.837 bu.*	=	*26.417* "
10 hectoliters	"	*1 kiloliter*	= *28.37 bu.*	=	*264.17* "
10 kiloliters	"	*1 myrialiter*	= *283.7 bu.*	=	*2641.7* "

Also,

.1 liter	is	*1 deciliter*	= *6.1022 cu. in.*	=	*.845 gi.*
.1 deciliter	"	*1 centiliter*	= *.6102* "	=	*.338 fl. oz.*
.1 centiliter	"	*1 milliliter*	= *.0610* "	=	*.27 fl. dr.*

Note 1. — The *liter*, about $\frac{9}{10}$ *of a dry quart*, or $1\frac{1}{18}$ *qt. liquid*, is used for measuring milk, water, etc., in moderate quantities.

The *dekaliter*, about $2\frac{1}{2}$ *gal.*, for measuring liquids in large quantities, and the *hectoliter*, about $2\frac{5}{6}$ *bu.*, for grain, fruit, etc., in large quantities.

2. The *kiloliter* equals 1 cu. meter, or 1 stere, or *35.316 cu. ft.*

MEASURES OF WEIGHT.

509. The ***Gram*** is the *Standard Unit* of *weight.*

Note. — The *gram* is the weight of a volume of distilled water at 39.2° Fahrenheit, equal to a cubic centimeter, a cube each edge of which is *one-hundredth* of a meter, and equals *15.432 Troy grains.*

Table.

10 grams	are	*1 dekagram*	=	*.3527 oz. av.*
10 dekagrams	"	*1 hectogram*	=	*3.527* "
10 hectograms	"	*1 kilogram*	=	*2.2046 lbs. av.*
10 kilograms	"	*1 myriagram*	=	*22.0462* "
10 myriagrams *100 kilograms*	"	*1 quintal*	=	*220.4621* "
10 quintals *1000 kilograms*	"	*1 Tonneau,* *or Ton,*	=	*2204.621* "

Also,

.1 gram	is	*1 decigram*	=	*1.5432 Tr. gr.*
.1 decigram	"	*1 centigram*	=	*.15432* "
.1 centigram	"	*1 milligram*	=	*.01543* "

The *kilogram*, or *kilo*, about $2\frac{1}{5}$ *lb. avoir.*, is used for ordinary purposes. The *tonneau*, or *metric ton*, about *2205 lbs.*, is used in weighing very heavy articles; and the *gram*, about $15\frac{1}{2}$ *Troy gr.*, in weighing jewels, gold, etc.

The weight of the five-cent nickel piece is 5 grams.

510. Metric Numbers are written according to the decimal system of notation of integers and decimals.

511. Metric numbers are usually expressed by writing the number of primary units as an integer, and each lower order as a decimal of that primary unit.

Thus, 4 meters 6 decimeters 5 centimeters is written *4.65 M.*

Note. — The abbreviated name of the primary unit and of each higher unit is generally commenced with a capital letter; and of each lower unit with a small letter.

Thus, the abbreviation of *meter* is *M.*; of *dekameter*, *Dm.*; of decimeter, dm.; of are, A.; centiare, ca.; hectare, Ha.; etc.

512. Since the orders of *linear measures*, measures of *capacity*, and measures of *weight* are on the scale of *10*, the units of each order occupy *one place.*

Thus, *5 M. 2 dm. 4 cm.* is written *5.24 M.*; 3 Dm. 5 mm., 30.005 M.

513. Since the orders of *surface measures* are on the scale of *100*, each order lower than the primary unit occupies *two* decimal places.

Thus, *25 Sq. M. 36 sq. dm.* is written *25.36 Sq. M.*; 4 Sq. M. 6 sq. dm. 8 sq. cm., 4.0608 Sq. M.

514. Since the orders of *cubic measures* are on the scale of *1000*, each order lower than the primary unit occupies *three* decimal places.

Thus, *12 Cu. M. 357 cu. dm.* is written *12.357 Cu. M.*; 3 Cu. M. 25 cu. dm. 8 cu. cm. is written 3.025008 Cu. M.

515. If the decimal part of an expression in surface measures contains a number of figures less than a multiple of 2, a cipher is understood to be annexed before it is read in terms of the lower orders.

Thus, *3.5 Sq. M.* is understood to be *3.50 Sq. M.*, and is read *3 Sq. M. 50 sq. dm.*; 3.057 Sq. M. is read 3 Sq. M. 5 sq. dm. 70 sq. cm.

516. If the decimal part of an expression in cubic measures contains a number of decimal places less than a multiple of 3, one or two ciphers are understood to be annexed before it is read in terms of the lower orders.

Thus, *4.6 Cu. M.* is understood to be *4.600 Cu. M.*, and is read 4 Cu. M. 600 cu. dm.; 4.01575 Cu. M., 4 Cu. M. 15 cu. dm. 750 cm.

517. When metric numbers are expressed by figures, the integral part is usually read in the denomination of the primary unit; and the decimal part is read either as a decimal of the primary unit, or in the denomination of the right-hand figure.

Thus, *25.37 M.*, instead of being read 2 Dm. 5 M. 3 dm. 7 cm., is read *25* and *37* hundredths meters, or *25 M. 37 cm.*

Exercises.

1. Name the primary unit of measures of length; of measures of surface; land measures; measures of solids; wood measures; measures of capacity; measures of weight. Give the value of each primary unit in denominations of the common system.

2. Name the orders or denominations of measures of length. Repeat the table. Of surface measures. Of land measures. Repeat the tables. Of solid measures. Of wood measures. Repeat the tables. Of measures of capacity. Of measures of weight. Repeat the tables.

3. What decimal place is occupied by a number of decimeters? Centimeters? Millimeters? What decimal places by square decimeters? Square centimeters? Square millimeters? Centiares? What decimal places by cubic decimeters? Cubic centimeters? Cubic millimeters? Decisteres? What decimal place by deciliters? Centiliters? Milliliters? Decigrams? Centigrams? Milligrams?

4. In expressing measures of length, what order or denomination occupies the first decimal place? The third? The second? In expressing measures of capacity? Measures of weight? In measures of surface, what occupies the first and the second place? The fifth and the sixth? The third and the fourth? In measures of solids, what occupies the fourth, the fifth, and the sixth place? The first, the second, and the third?

5. Express as an integer, or a decimal, or a mixed decimal,

75 millimeters as the decimal of a meter; 41 Km. 12 M. as Km.

42 M. 25 cm. as meters; 3 m. 2 dm. 7 mm. as dekameters.

3 A. 75 ca. as hectares; 5 Sq. M. 5 sq. dm. as square meters.

3 St. 4 dst. as steres; 5 Cu. M. 25 cu. cm. as cubic meters.

5 Dl. 6 L. 7 cl. as liters; 125 Kl. 45 L. as kiloliters; as deciliters.

425 Kg. 35 G. as kilos; as tons; 95 tonneaux 456 kilos as tons; as kilos.

6. Read as a number of the several orders of metric units —

1. *31.675 M.*	5. *275.05 Hm.*	9. *104.037 L.*
2. *1765.307 Km.*	6. *1924.7 St.*	10. *345.104 kilos.*
3. *827.05 Cm.*	7. *14.0507 Sq. M.*	11. *32.7654 quintals.*
4. *5732.25 Sq. M.*	8. *1.00575 Cu. M.*	12. *123.0706 tons.*

Reduction and Computation.

518. Since the metric system is a decimal system, a number expressed in units of one order may be changed or reduced to units of another order by multiplying or dividing by 10, or some power of 10.

519. A number of any order expressed in figures is changed to units of higher or lower orders by removing the decimal point and placing it at the right of the figure denoting the order required, and then adding the proper name of the new order.

Thus, *24.75 meters* may be expressed as

Dekameters,	*2.475 Dm.*	*Decimeters,*	*247.5 dm.*
Hectometers,	*.2475 Hm.*	*Centimeters,*	*2475 cm.*
Kilometers,	*.02475 Km.*	*Millimeters,*	*24750 mm.*

520. Computations in metric numbers are similar to those in integers and decimals, or in United States money.

Units of the metric system may be changed to units of the common system by the aid of the preceding tables. And units of the common system may be changed to units of the metric system by the aid of the following comparison of values: —

LINEAR MEASURES.

1 inch = *2.54 centimeters.*
1 foot = *.3048 meter.*
1 yard = *.9144 meter.*
1 rod = *5.029 meters.*
1 mile = *1.6093 kilometers.*

SURFACE MEASURES.

1 sq. inch = *6.4528 sq. centimeters.*
1 sq. foot = *.0929 sq. meter.*
1 sq. yard = *.8361 sq. meter.*
1 sq. rod = *25.29 centiares.*
1 acre = *40.47 ares.*
1 sq. mile = *259 hectares.*

SOLID MEASURES.

1 cu. inch = *16.39 cu. centimeters.*
1 cu. foot = *.02832 cu. meter.*
1 cu. yard = *.7646 cu. meter.*
1 cord = *3.625 steres.*

MEASURES OF CAPACITY.

1 fl. ounce = *2.958 centiliters.*
1 liquid quart = *.9465 liter.*
1 gallon = *3.786 liters.*
1 dry quart = *1.101 liters.*
1 bushel = *.3524 hectoliter.*

MEASURES OF WEIGHT.

1 grain Troy = *64.8 milligrams.*
1 ounce Troy = *31.104 grams.*
1 pound Troy = *.3732 kilo.*
1 ounce av. = *28.359 grams.*
1 pound av. = *.4536 kilo.*
1 ton = *.907 tonneau.*

Problems.

1. Find the sum of 29 Hm., 846 M., 184 cm., 48 mm.

2. How many ares in 2 Ha., 2 A., and 4 sq. m.?

3. Express 25 Ds., 72 S., and 12 ds. as steres.

4. From 75 Ha. take 155 centares. Give result as ares.

5. At 5 francs per meter, what is the value of 4 meters of dress goods? Of 6.75 meters?

6. How much land will cost $8065.125 at $100.50 per hectare? How many ares? How many acres?

7. Change 3 yd. 2 ft. 6 in. to meters. To dekameters.

8. Change 4 Km. 8 M. 5 dm. to miles. To yards.

9. In 30 A. 80 sq. rd. how many ares? Hectares?

10. Express 10 Ha. 7 A. 25 sq. m. as acres. As sq. ch.

11. How many dekasteres in 28 cd. 96 cu. ft.? Steres?

12. How many cords in 27 Ds. 50 ds.? How many cu. ft.?

13. In 25 gal. 3 qt. of water, how many liters? Kiloliters?

14. Change 12 kiloliters to bushels. To bu. pk.

15. Express the value of 1 pound avoirdupois by the metric system. Express the value of 1 lb. Troy.

16. What is the value of 5000 pounds in kilos? What is the value in quintals? In tonneaux?

17. How many liters in 25 gallons? In 10 gal.?

18. In 36 gal. 2 qt. how many liters? In 5 bu. 2 pk.?

19. At $37\frac{1}{2}$ cents a kilo, how much will 15 kilograms of beef cost? What will 10 pounds cost?

20. If a gold ring was sold at $1 per grain, what was the price per gram?

21. If wine is worth $1.25 per quart, what is the price per liter? The value of 20 Dl. 5 L.?

22. At 25 cents per hectoliter, what is the cost of 150 bushels of potatoes? Of 175 bushels?

23. If a kilogram weighs 2.2046 lb., how many grams are there in an ounce? How many ounces in a dekagram?

24. If a meter is 3.2809 ft., how many millimeters are there in an inch? How many inches in a decimeter?

25. If 15 steres of wood cost $21.25, how much must be paid for 22.5 steres?

26. How much must be paid for excavating 18.5 cu. yd. of earth at $1.50 per cubic meter?

27. How many kilometers from Philadelphia to New York, the distance being about 100 miles?

28. At the rate of 4 kilometers an hour, how long will it take to walk from Lancaster to Philadelphia, the distance being about 70 miles?

29. A pile of wood is 4.5 meters long, 1.25 meters wide, and 2 meters high. Find the value at $6 a cord.

30. I bought 70.5 liters of wine in Paris at 4 francs per liter, paid $.50 a liter duty, and sold it at $2 a quart. How much did I gain?

31. If a merchant buys 100 meters of silk at 19 francs a meter, pays 10% *ad valorem* duty, and sells it at $4 a yard, what does he gain or lose?

32. Two cog-wheels work together, one having 360 teeth, and the other 100 teeth. If the first revolves 20 times in a minute, how many times will the second revolve in an hour? If the latter turns a drum 5 ft. in circumference, how many kilos of thread could it wind in a working day of ten hours?

Problems in Analysis.

Note. — The basis of every operation in arithmetical analysis is *the unit*, and in every process the reasoning should be directed first *to the unit*, and then *from the unit*.

Although in the more complicated problems, new groups or collections may seem to bear new relations to each other, or to form new bases of reasoning, yet the underlying principles and processes are similar to those which apply to the unit.

In the problems following, set forms of analysis should not be required, nor need there be given any other than the following

521. General Directions.

I. *From the number and the value of the things given, find the value of the unit of the thing required.*

II. *From the value of the unit of the thing required, find the value of the entire number of units of the thing required.*

1. Three men bought 60 barrels of apples, and as often as the first paid \$4, the second paid \$5, and the third paid \$6. How many barrels should each have had?

2. If the sum of three numbers is 46, and they are in the ratio of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, what are the numbers?

3. If 4 oxen or 6 cows can eat a certain quantity of hay in 6 days, in how many days can 6 oxen and 4 cows eat the same quantity?

4. If 5 boys or 3 men earn \$10 in a certain time, how much can 15 men and 6 boys earn in the same time?

5. How much will 5 apples cost if 3 apples are worth 9 pears and 4 pears are worth 8 cents?

6. What will 10 sheep cost if 5 sheep are worth as much as 2 cows, and 4 cows cost \$80?

7. If 8 men can do a piece of work in 5 hours, in what time will the work be completed, if 3 men leave when the work is half done?

8. Twenty men ordered a supper; but as 5 were absent from the supper, the expense of those present was increased 50 cents each. Find the cost of the supper.

9. Ten men hired a coach, and after going a short distance they took in two more men, thereby decreasing the expense of each 25 cents. What was paid for the coach?

10. A pipe will fill a cistern in 4 hours, and another will empty it in 6 hours. How soon will it be filled if both pipes run at the same time?

11. A, B, and C can do a piece of work in 6 days, A and B in 9 days, and A and C in 8 days. How soon can B and C together do it? Each alone?

12. If James can plow an acre of ground in $\frac{3}{5}$ of a day, and Thomas in $\frac{3}{4}$ of a day, how long will it take both together to mow 10 acres?

13. Henry and Jacob can cut a quantity of wood in 5 days, and Henry alone can cut it in 9 days. How long will it take Jacob to cut what Henry leaves after working alone for 5 days?

14. A man received \$2 a day for his work, and paid \$6 a week for his board. If at the end of 20 weeks he had saved \$100, how many days was he idle?

15. A mason agreed to work for \$2.50 a day, and to forfeit 50 cents every day he was idle. If at the end of 40 days he had saved \$70, how many days did he work?

16. If a laborer engaged for a year at \$440 and a suit of clothes, left at the end of 5 months, receiving \$160 and the suit of clothes, what was the suit worth?

17. The amount of a certain principal for 7 years at a certain rate % was \$540, and for 10 years at the same rate it was \$600. Find the principal and the rate.

18. A man left \$8800 to his two children, whose ages were 11 years and 16 years, so that their respective shares at 5% simple interest should amount to the same sum on coming of age. Find the share of each.

19. Divide an estate of \$5800 between two children aged 12 years and 15 years so that their respective shares at 6% shall amount to the same sum when they become 21 years old.

20. The cost of a house and farm is \$9400. What is the cost of each if $\frac{3}{4}$ of the cost of the house increased by \$600 equals $\frac{2}{3}$ of the cost of the farm?

21. The head of a fish is 4 inches long; the tail is as long as the head plus $\frac{1}{4}$ of the body; and the body is as long as the head and the tail. How long is the fish?

22. The tail of a fish weighs 3 ounces; the head weighs as much as the tail plus $\frac{1}{4}$ of the body; and the body weighs as much as the head and the tail both. Find the weight of the fish.

23. An apple-woman sold apples at 3 cents each, and gained 10 cents; but if she had sold them at 5 cents each she would have gained 30 cents. How many apples did she sell?

24. A drover bought some cattle for $300; but if he had bought 5 more at $6 less each, all would have cost $420. What did he pay for each?

25. A butcher bought some sheep for $600; but 20 of them died, and he sold $\frac{3}{5}$ of the remainder at cost, receiving for them $300. How many did he buy?

26. A dealer having lost 10 horses, sold $\frac{3}{4}$ of the remainder at cost price, and received for them $6000, which was $4000 less than all cost. How many did he sell?

27. If an acre of land furnishes pasturage for 3 cows, and an acre of roots enough for 5 cows, how many cows can be kept on 32 acres?

28. A stock raiser allows an acre of grass to 4 cows, and an acre of roots to 6 cows. How much of 40 acres must he have in grass and how much in roots?

29. If a man travels by rail at the rate of 20 miles an hour, and returns by boat at the rate of 12 miles an hour and is gone 16 hours, how far does he go?

30. Henry rode 24 miles at a certain rate and walked back at the rate of 3 miles an hour. If he was gone 11 hours, at what rate did he ride?

31. A steamboat moving at the rate of 12 miles an hour in still water goes down a river whose current is 3 miles an hour. How far down may it go that it may be back in 16 hours?

32. Henry is 20 years old and Edgar is 4 years old. In how many years will Henry be just three times as old as Edgar?

33. Sarah is 18 years old, and this is $\frac{3}{5}$ of her aunt's age. How long since the aunt was twice as old as Sarah?

34. Helen is 3 times as old as Owen, but in 4 years she will be only twice as old. How old is each now?

35. A is 20 years old; the sum of B's and C's is 4 times A's; and B's age is $\frac{1}{7}$ of A's and B's together. Find the age of each.

36. John is 40 yards ahead of Harry, but he runs only 3 yards while Harry runs 5 yards. How far must Harry run to catch John?

37. A and B travel in the same direction, and A is 18 miles ahead of B at starting. If B travels 7 miles while A travels 4 miles, where will B overtake A?

38. A hare is 80 leaps before a hound and takes 3 leaps while the hound takes 2 leaps; but 2 of the hound's equal 5 of the hare's. How many will each make before they are together?

39. What is the time of day if $\frac{1}{2}$ of the time past noon equals the time to midnight? If $\frac{1}{4}$ of the time to midnight equals the time past noon.

40. What is the hour if $\frac{1}{3}$ of the time past midnight equals the time past noon? If $\frac{1}{2}$ of the time past midnight equals the time till midnight again?

41. Find the time of day when $\frac{1}{3}$ of the time past midnight equals $\frac{1}{2}$ of the time till noon. When $\frac{1}{5}$ of the time past noon equals $\frac{1}{3}$ of the time till noon again.

42. A man being asked the hour said, "It is between 2 and 3 o'clock, and the hands on the dial of my watch are exactly together." What was the time of day?

43. A boy has 24 apples and pears, and he has twice as many apples as pears. How many more apples must he get to have three times as many apples as pears?

44. In a school of 90 pupils there are three girls to every 2 boys. How many more girls than boys are there?

45. In a school of 90 pupils there are 2 girls to every 3 boys. How many girls must be admitted that there may be 3 girls to 2 boys? How many of each will there be?

46. An alloy of 60 pounds contains 3 pounds of silver to 2 pounds of copper. How much silver must be added that there may be 6 pounds of silver to 2 pounds of copper?

47. If 100 lb. of sea-water contain 3 lb. of salt, how many pounds of fresh water must be added to these 100 lb. so that 20 lb. of the new mixture shall contain $\frac{1}{2}$ lb. of salt?

General Review Problems.

1. A has \$384, B \$576, and C \$768, with which they are to buy horses at the highest price per head that will allow each to invest all his money. How many can each buy?

2. A certain number multiplied by $\frac{3}{5}$ of $1.2\frac{1}{2}$, and divided by $\frac{2}{3}$ of .075, gives 1. What is the number?

3. A man bought 140 acres of land at \$87.50 per acre. He sold at one time 50 acres at \$75 per acre, and at another time 40 acres at $\$92.37\frac{1}{2}$ per acre. What must be the selling price of the remainder to make a gain of \$1053.50 on the cost?

4. Divide $\frac{\frac{4}{5} \text{ of } 6\frac{2}{3} \times 7\frac{1}{2}}{\frac{5}{6} - 4\frac{2}{3} \div 8\frac{2}{5}}$ by $\frac{10 + \frac{5}{7} \text{ of } 9\frac{4}{5}}{\frac{5}{9} \text{ of } 5\frac{2}{5} - 2\frac{9}{10}}$.

5. If $\frac{3}{4}$ of a bushel of corn be worth $\frac{3}{7}$ of a bushel of wheat, and wheat be worth \$1.75 a bushel, how many bushels of corn can be bought for $\$37\frac{1}{2}$?

6. If a certain number be increased by $3\frac{2}{3}$, then diminished by $\frac{9}{10}$, this remainder multiplied by $5\frac{3}{5}$, and this product be divided by $3\frac{4}{7}$, the quotient will be $9\frac{4}{5}$. Find the number.

7. Find the sum of $2\frac{1}{8}$ decimal units of the first order, $3\frac{3}{5}$ of the second order, $4\frac{1}{2}\frac{1}{0}$ of the third order, and $5\frac{3}{3}\frac{3}{2}$ of the fourth order.

8. Simplify $\frac{.00\frac{3}{4} + .0\frac{1}{2} - \frac{1}{4}\frac{1}{0}}{\frac{3}{5} \text{ of } 6.6\frac{2}{3} \div .0\frac{5}{8}}$; $\frac{5\frac{1}{2} + \frac{.7\frac{2}{5}}{5} - .7\frac{1}{4}}{(4 + 3.4\frac{1}{2}) \div .0\frac{1}{4}}$.

9. A barrel holding $31\frac{1}{2}$ gallons is $\frac{1}{2}$ of $\frac{8}{9}$ full. If $10\frac{1}{4}$ gallons be drawn off, what part of the contents will remain?

10. A Western farmer sold $213\frac{3}{8}$ bu. of wheat at $\$2\frac{1}{8}$, $300\frac{3}{4}$ bu. at $\$2\frac{1}{10}$, and 275.5 bu. at $\$2\frac{3}{16}$. Find the entire selling price, and the average price.

11. Change $\frac{8\frac{3}{4} - 6\frac{7}{8}}{\frac{5}{6} \text{ of } 4\frac{2}{7}} \div \frac{(\frac{1}{6} \div \frac{1}{5}) - \frac{1}{4}}{(\frac{1}{2} + \frac{1}{3}) \times \frac{1}{4}}$ to a decimal.

12. A builder bought 8760 feet of scantling at \$1.50 per M, 20340 feet of lumber at \$2.75 per C, 1025 feet of pine boards at \$20 per M, and paid $\$.37\frac{1}{2}$ per thousand for planing 17875 feet of boards. Find the amount of the bill.

13. How many English books, at 4 s. $6\frac{1}{2}$ d. each, can be purchased for £11 7 s. 1 d.? What are they worth in U. S. money?

14. What must be the length of a bin 6 ft. 8 in. wide, and 4 ft. 6 in. deep, to hold 125 bu. of wheat? 125 bu. of corn in the ear?

15. How many rolls of paper 8 yd. long, 18 in. wide, will paper the walls of a room 18 ft. long, $12\frac{1}{2}$ ft. wide, and 12 ft. high, allowing for 2 windows and a door, each 7 ft. by 3 ft. 3 in.?

16. What % must be added to the weight of a brass ounce weight that it may be as heavy as an ounce of gold?

17. A coal-dealer bought 150 T. 10 cwt. of coal at $\$4.87\frac{1}{2}$ a long T., and sold it at $\$5.37\frac{1}{2}$ per short T. What % did he gain?

18. What depth of rain-fall upon a roof 30 ft. long, and 30 ft. from ridge to eaves, will fill a tank 10 ft. long, 6 ft. wide, 4 ft. deep?

19. My cellar was 30 ft. long, 20 ft. wide, and 6 ft. deep. I had it made a foot deeper, a foot longer at each end, and a foot wider each way. How many cu. yd. of earth were removed?

20. A pillar is 10 ft. 8 in. long, 3 ft. 2 in. wide, 2 ft. 4 in. thick. Find the area of its surface, and its solidity.

21. A pond covers $\frac{3}{4}$ of an acre. How many hhd. of water would fall into it during a rain storm in which $\frac{1}{4}$ in. of rain fell?

22. I bought 57 A. 2 sq. ch. 6 sq. yd. of land at \$75 an acre. I sold $\frac{2}{5}$ of it at \$90 per acre, $\frac{3}{4}$ of the remainder at \$1 per sq. rd., and the rest at \$.005 per sq. ft. Find the gain.

23. What fractional part of a solar year is a common year? What decimal part of a leap year is a solar year?

24. A owns $\frac{7}{10}$ of a tract of land, and B the remainder. $\frac{4}{5}$ of the difference between their shares is 10 A. 16 sq. rd. 10 sq. yd. What is the share of each?

25. What is $\frac{3}{4}$ of the difference between $\frac{4}{5}$ of 3 mi. 40 rd. 3 yd. 2 feet and $4\frac{2}{3}$ times 25 mi. 100 yd. $2\frac{1}{2}$ feet?

26. Required the sum in avoirdupois weight of $3\frac{2}{3}$ lb. Troy, and $16\frac{1}{3}$ lb. avoirdupois.

27. Add together $.0\frac{7}{8}$ sq. mi., $4.0035\frac{2}{3}$ sq. rd., $3.0\frac{1}{5}$ sq. yd., and $.0006\frac{3}{8}$ sq. ft. Result in all lower denominations.

28. A can do a piece of work in 5 days, B can do it in 6 days, and C in 8 days. In what time can all do it working together?

29. A and B can do a piece of work in 3 days, A and C in 4 days, and B and C in 6 days. In what time can each man do it?

30. If A can mow an acre of grass in $5\frac{2}{3}$ hr., and B can mow $1\frac{3}{4}$ A. in $9\frac{1}{3}$ hr., in what time can they jointly mow $8\frac{1}{4}$ A.?

31. If 4 cows or 7 sheep eat $11\frac{1}{4}$ T. of hay in $1\frac{1}{8}$ years, how long will $29\frac{1}{7}$ T. last 13 cows and 113 sheep?

32. If 12 horses in 45 days eat $3\frac{1}{2}$ T. of hay, how long will $4\frac{1}{5}$ T. last 10 horses, 15 cows, and 7 sheep, each cow eating $\frac{3}{4}$ as much as a horse, and each sheep $\frac{1}{3}$ as much as a cow?

33. Three persons engaged in a speculation, and cleared \$7650, which they divided in the ratio of 2, 3, 4, according to their capital. What did each receive?

34. A, B, and C engaged in business for one year with a capital of \$5000. A put in his capital for 7 mo., B for 8 mo., and C for the year. Of its profits, A received \$105, B \$200, and C \$120. What did each put in?

35. Brown and Smith formed a partnership. Brown put in \$6000, and Smith \$4000. Brown valued his services at \$2000, and Smith valued his at \$3000. If the year's profits were \$8000, what was the share of each?

36. A wholesale dealer sells goods at a profit of $12\frac{1}{2}\%$, and the retailer gains $37\frac{1}{2}\%$. What $\%$ of the price paid by the consumer is profit on the first cost of the goods?

37. The selling price of a carriage bought for \$225 is marked at 25% advance. What $\%$ may the marked price be reduced and still leave a profit of 20%?

38. A merchant retails his goods at a gain of $33\frac{1}{3}\%$. If he sells them at wholesale for 10% less than the retail price, what $\%$ does he gain at wholesale?

39. What $\%$ is gained or lost by buying goods for \$1000 cash, and selling immediately for \$1200 on 9 mo. credit, getting the note discounted at bank at 7%?

40. I sold a bill of goods at 8% gain, invested the proceeds, and sold at an advance of $12\frac{1}{2}\%$; invested the proceeds again, and sold at a loss of 4%, when I had \$1749.60. With what capital did I commence?

41. For value received, Boston, May 20, 1900, I promise to pay to the order of John Hood, $750, with interest, Jan. 1, 1901. Find the amount of this note when due.

42. A man who has been paying $500 a year rent, borrows $6000 at 6%, and buys the house. He pays yearly $75 taxes, $12 water-rent, and $50 for repairs. What does he gain or lose?

43. A merchant bought goods on 6 mo. credit, and sold them at an advance of 20%, one-half on 3 mo. credit, and the remainder on 9 mo. What was his gain, money being worth 6%?

44. A merchant sold a bill of goods for $1500 on 4 mo., and made a discount of 5% off for 30 days, and a further discount of 10% for cash. Find the cash proceeds.

45. A jobber bought goods for $5000 cash, sold them on 4 mo. at $12\frac{1}{2}$% advance, and had the note discounted at bank at 7% to pay the bill. What % did he gain?

46. A note for $500, dated New York, Sept. 5, 1901, was due in 9 mo., with interest at 7%. In 60 days it was discounted at 8%. What was realized from it? If discounted at bank?

47. A father invested $1666\frac{2}{3}$ at 6% for his son at a certain age, so that, on becoming 21 years of age, the sum should amount to $2500. Find the son's age at the time of investment.

48. If the annual interest of A's money at 7% is $15.40, and B's at 6% is $3.40 less, which has the greater sum of money, and how much?

49. A owes $3000, with interest at 7%. He pays at the end of each year for interest, and in part payment of the principal, $500. Find the amount of his debt at the end of five years?

50. A tradesman marks his goods with two prices, one for cash, and the other for credit of 6 months, money at 7%. Find the cash price of goods when the credit price is $37.50.

51. If I pay 85 cents on a dollar for bonds which pay 4% semi-annual dividends on their face, what % per annum does this give me, money being worth 10%?

52. The premium paid for insuring a steam-mill at $4\frac{1}{2}$%, to cover $\frac{3}{4}$ of its value, was $631.80. Find the value of the mill.

53. A bought 6% bonds for 112, kept them a year, and sold them for 118¾. What rate of interest did the investment pay him?

54. What difference of income is made by transferring $4500 from 4% stock at 85 to a 6% stock at 101?

55. I have $3375 which I wish to invest in 5% bonds. When they are quoted at 112¼, and brokerage is ¼%, how many $100-bonds can I purchase with my money?

56. The capital of an insurance company was $250000. Its receipts for one year were $51170, and its expenses and losses were $34920. What rate of dividend could it declare?

57. Philadelphia, Jan. 25, 1901, due Edward Pierson, for value received, 44\frac{75}{100}$, with interest at 6%.

THOMAS PHILLIPS.

What was due on this due bill June 10, 1901?

58. Date of note Feb. 8, 1897; face, $750; rate %, 6; endorsement, Dec. 23, 1897, $225; Oct. 10, 1898, $20; Aug. 15, 1899, $15; Jan. 20, 1900, $31. What was due April 26, 1900?

59. I wish to get $100 at bank for 60–63 days at 6%. For what amount must my note be drawn?

60. Find the difference between $\sqrt{.9}$ and $\sqrt[3]{.8}$ to three places.

61. If A owes B $1000 due in 6 mo., how large a payment must he make in 3 mo. to extend the payment of the balance 6 mo.?

62. To-day I owe $250 due in 30 days, $300 in 60 days, and $500 in 90 days. If I give my note for the whole amount made payable at the average time, when will the note be due?

63. What is the area of the bottom of a cubical box whose capacity is 46656 cubic inches?

64. A liquor-dealer buys 24 gal. of liquor for $36. How much water may he add to sell it at $1.40 and clear $4.50?

65. A laborer engaged to work for $1.25 a day on condition that he should forfeit 50 cents for every day he was idle. If at the end of 100 days he received $55, how many days did he work?

66. John and Harry together have 226\frac{25}{100}$, and John has $70¾ more than Harry. How much does John lack of having $150?

67. If my watch is set to Philadelphia time, 75° 9′ 23″ W., how much faster or slower is Chicago local time, 87° 38′ 1″ W.?

68. A merchant bought goods at 25% below their nominal value, and sold them at 20% above, thereby making $1920. What sum of money did he invest in the goods?

69. The amount of A's money for 8 years at 10% is $392 more than its amount for 6 years at 8%. How much has he?

70. Gold being the standard of value, which is the better price for wheat, $1.45 per bushel in gold when it is quoted at $115\frac{1}{2}$, or $1.60 in currency at par?

71. A merchant sold some goods at such rate that $\frac{3}{4}$ of the selling price was 20% less than the cost. Find his gain %.

72. What % is made by buying stocks at 15% below par and selling them at 10% above par?

73. A company of men paid $750.76 for a supper, each man paying as many cents as there were men in the company. How much money did each pay?

74. What sum placed at simple interest for 3 yr. 6 mo. 20 da. at 6% will amount to as much as $1500 placed at compound interest for the same time, at the same rate, payable semi-annually?

75. A father at his death left $10000 to his son aged 15 yr. 4 mo. 15 da., the money to be paid when he became 21 years of age, with 6% interest compounded annually. What did he receive?

76. A, B, and C gave $75 for a reaper, agreeing that A should pay $\frac{1}{2}$, B $\frac{1}{3}$, and C $\frac{1}{4}$. In that ratio, what amount did each really pay? What % of the cost did each pay?

77. A merchant rented an office for $300 a year, payable quarterly in advance; but he made no settlement till the end of 3 years. What did he then owe, interest at 6%?

78. Simplify $1.\dot{3}\ (2.\dot{4} + 7.\dot{5}) + 2.3\dot{6}\dot{4} - 1.69\dot{7}$.

79. Find the exact square of the result of $\sqrt{3 + \sqrt{3}}$.

80. If I rent a house for a year for $480, payable monthly in advance, how much money at the beginning of the year would be required to pay the rent?

81. How many shares of stock at 50 must be added to 10 shares at par, and 50 shares at 80, to make all the shares average the market value of 75?

82. Add $.01\dot{5}$ and $.\dot{0}\dot{9}$, and prove the result by fractions.

83. A father deposited \$150 annually at 6% simple interest for his son, beginning on his twelfth birthday. What was the amount due the son on his 21st birthday?

84. A manufacturer hired 10 mill hands, paying to the men \$10 a week and to the boys \$5. How many boys were there, provided he paid them all \$480 in 6 weeks?

85. Multiply $.5\dot{7}14285$ by $.6\dot{3}$ and prove that the result is $.\dot{3}\dot{6}$.

86. If £1 is worth 25.2 francs, $9\frac{1}{2}$ thalers are worth 35 francs, and 60 thalers are worth 107 Austrian paper florins, how many florins are £10 worth?

87. A merchant offered to settle with his creditors at 60 cents on the dollar, or to pay them in full at the end of five years if permitted to continue business. Which was the better for them?

88. Find the value of 10 building association shares at the end of 6 yr. 6 mo. at 6% simple interest.

89. Express decimally the result of $\frac{1}{3}\frac{5}{7} - .1\dot{3}5\dot{1}$.

90. What amount of cash deposited in a savings-bank at 5% compound interest would amount to as much as \$500 annually for 6 years?

91. An importer bought silk at \$3.20 a yard, and after keeping it a year he sold it at \$3.50 a yard, but did not collect the account for 6 months. Money at 6%, what did he gain or lose?

92. A merchant imported 20 barrels of liquor, each 32 gallons, invoiced at $\$.87\frac{1}{2}$ per gallon. After paying \$75 for freight and a duty of 25% *ad valorem*, what per cent. will he gain by selling it for \$930?

93. A dealer bought 2000 bushels of wheat at \$1.50 per bushel. How much per bushel must he ask that he may fall 20% on the asking price and still make $16\frac{2}{3}$%, after allowing 10% of the sales for bad debts?

MENSURATION.

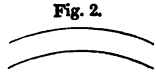
LINES.

1. ***Mensuration*** is the process of finding the lengths of lines, the areas of surfaces, and the volumes of solids.

Fig. 1.

2. A ***Line*** is that which has length only.

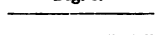
Fig. 2.



3. A ***Right Line*** is a line whose direction does not change at any point; as, Fig. 1.

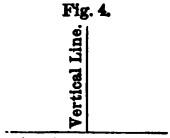
4. A ***Curved Line*** is a line whose direction changes at every point; as, Fig. 2.

Fig. 3.



5. ***Parallel Lines*** are lines which have the same direction; as, Fig. 2 and Fig. 3.

Fig. 4.



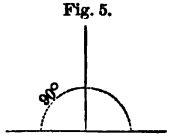
6. A ***Perpendicular Line*** is a right line which meets another so as to incline no more towards one side than towards the other.

7. A ***Horizontal Line*** is a right line parallel to the horizon or water-level. And

A ***Vertical Line*** is a right line perpendicular to a horizontal line.

ANGLES.

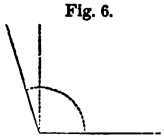
Fig. 5.



8. An ***Angle*** is the opening between two right lines meeting in a common point, called the *Vertex.*

9. Angles are of three kinds — *Right Angle, Obtuse Angle,* and *Acute Angle.*

Fig. 6.

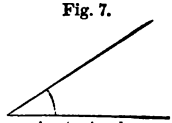


10. A ***Right Angle*** is an angle formed by two right lines that are perpendicular to each other.

11. An ***Obtuse Angle*** is greater than a right angle.

12. An ***Acute Angle*** is less than a right angle.

Fig. 7.



PLANE FIGURES.

13. A ***Plane,*** or ***Plane Surface,*** is a surface which is everywhere perfectly flat or even.

14. A ***Plane Figure*** is a plane surface bounded by right or curved lines.

15. The ***Base*** of a figure is the side on which it is supposed to stand.

16. The ***Altitude*** of a figure is the perpendicular distance from the base to the opposite side or opposite angle.

17. The ***Diagonal*** of a figure is a right line which joins the vertices of any two opposite angles.

18. The ***Perimeter*** of a figure is the sum of all the sides that bound the figure.

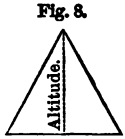
19. The ***Area*** of a plane figure is the extent of surface within the lines which bound the figure.

20. A ***Polygon*** is a plane figure bounded by right lines.

21. A ***Regular Polygon*** is one that has all of its sides equal and all of its angles equal.

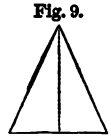
THE TRIANGLE.

Fig. 8.



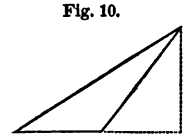
Equilateral.

Fig. 9.



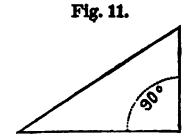
Isosceles.

Fig. 10.



Scalene.

Fig. 11.



Right-Angled.

22. A ***Triangle*** is a plane figure bounded by three right lines, and having three angles.

23. According to the nature of their *sides*, triangles are divided into three classes: *Equilateral*, *Isosceles*, and *Scalene*.

24. An ***Equilateral Triangle*** has all of its sides equal.

25. An ***Isosceles Triangle*** has but two of its sides equal.

26. A ***Scalene Triangle*** has all of its sides unequal.

27. According to the nature of their *angles*, triangles are divided into three classes: *Right-angled*, *Obtuse-angled*, and *Acute-angled*.

28. A ***Right-angled Triangle*** has one right angle; as, in Fig. 11.

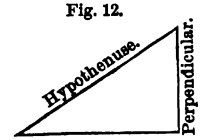
29. An ***Obtuse-angled Triangle*** has one obtuse angle; as, in Fig. 10.

30. An ***Acute-angled Triangle*** has all of its angles acute; as, in Figs. 8 and 9.

31. The ***Hypothenuse*** of a right-angled triangle is the side opposite to the right angle.

The two sides forming the right angle are called the *Base* and the *Perpendicular*.

Fig. 12.



Note.—The rules of Mensuration depend upon principles of Geometry; hence, no attempt is made to explain or demonstrate them. They are briefly stated, with such general principles as seem necessary to guide the pupil in performing the required operations.

Problems.

32. *To Find the Area of a Triangle when the Base and the Altitude are given.*

RULE. — *Multiply the base by the altitude, and divide the product by* 2.

Find the area of a triangle

1. Whose base is 16 ft. and altitude 10 ft. 8 in.

2. Whose altitude is 10 rd. and base 12 rd. $8\frac{1}{4}$ ft. Base 5 ch. 2 rd.

3. Find the area of an isosceles triangle whose base is 35 rd. and altitude $\frac{4}{5}$ as much. If the base is 4 ch. $16\frac{1}{2}$ ft,

4. A board $14\frac{2}{3}$ ft. long is $14\frac{1}{2}$ in. wide at one end, and tapers to a point. Find the value at $4\frac{1}{2}$ cents per sq. ft.

33. *To Find the Base or the Altitude of a Triangle when the Area and one Dimension are given.*

RULE. —*Multiply the area by* 2, *and divide by the given dimension.*

Find the required dimension of a triangle

5. Whose area is 150 sq. ch. and base 14 rd. Base 4 ch. $2\frac{4}{5}$ yd.

6. Whose area is 8 A. 150 sq. rd. and altitude 30 rd. Area 17 A. $8\frac{3}{4}$ sq. ch. and base 6 ch. 1 rd.

7. Whose area is $3\frac{15}{16}$ A. and altitude $31\frac{1}{4}$ rd. Area 20 sq. ch. 9 sq. ft.

8. Find the perimeter of an equilateral triangle whose area is 17 A. $51\frac{1}{3}$ sq. rd. and its altitude 17.32 ch.

9. At $\$6\frac{1}{4}$ per square rd., a triangular lot cost \$1281.25. The base being 40 rd., what was the length?

34. *To Find the Area of a Triangle when the Three Sides are given.*

RULE.—*From half the sum of the side subtract each side separately. Multiply the half sum and the three remainders together, and extract the square root of the product.*

Find the area of a triangle

10. Whose sides are 13 ft., 14 ft., and 15 ft. The sides each $11\frac{1}{2}$ rd.

11. Whose sides are 15 yd., $3\frac{7}{11}$ rd., $1\frac{3}{22}$ ch. Each side 5 ft. 6 in.

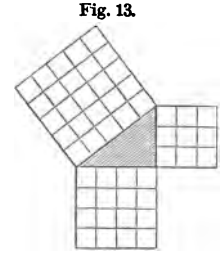
12. Whose base is $12\frac{2}{3}$ rd., and each of its other sides $10\frac{1}{2}$ rd.

13. How many square feet of boards in the gables of a barn 40 ft. wide, the rafters being 25 ft. long?

14. At \$75 an acre, what is the cost of a piece of ground in the form of an equilateral triangle whose perimeter is 150 ch.?

35. To Find the Hypothenuse of a Right-angled Triangle.

Fig. 13.



PRIN. I.—*The square on the hypothenuse of a right-angled triangle equals the sum of the square on the other two sides.*

PRIN. II.—*The square on either side of a right-angled triangle equals the square on the hypothenuse less the square on the other side.*

RULE.—*To the square of the base add the square of the perpendicular, and extract the square root of the sum.*

Find the hypothenuse of a right-angled triangle

15. Whose base is 15 ft. and perpendicular 20 ft.

16. Whose perpendicular is 4 ft. 6 in. and base 6 ft. Each 8.5 ft.

17. Find the diagonal of a square whose side is 4 rd. $2\frac{3}{4}$ yd. Of a rectangle whose length is 18 ch. 3 rd. and breadth $14\frac{1}{16}$ ch.

18. The height of a gable is 16 ft., and the width of the barn 60 ft. How long are the rafters, which project 1 ft. 6 in.?

19. A room is 32 ft. long, 24 ft. wide, and 9 ft. high. How far from a lower corner to the opposite upper corner?

36. To Find the Base or the Perpendicular of a Right-angled Triangle.

RULE.—*From the square of the hypothenuse subtract the square of the given side, and extract the square root of the remainder.*

Find the base or the perpendicular of a right triangle

20. Whose hypothenuse is 119 rd. and perpendicular 14 ch.

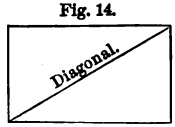
21. Whose base is 3 rd. $3\frac{1}{2}$ yd. and hypothenuse 7 rd. 4 yd.

22. The diagonal of the ceiling of a room 16 ft. wide is 34 ft. How long is the border of paper around the walls?

23. A vessel from a certain port sails east 12 mi. an hour, and another northeast 15 mi. an hour. At the end of 3 days how far apart will they be?

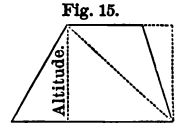
THE QUADRILATERAL.

Fig. 14.



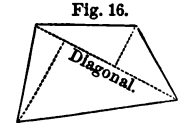
Parallelogram.

Fig. 15.



Trapezoid.

Fig. 16.

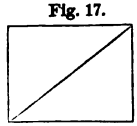


Trapezium.

37. A ***Quadrilateral*** is a plane figure bounded by four right lines, and having four angles.

38. According to the nature of their sides and angles, quadrilaterals are divided into three classes; the *Parallelogram*, the *Trapezoid*, and the *Trapezium*.

Fig. 17.

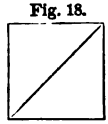


Rectangle.

39. A ***Parallelogram*** is a quadrilateral which has its opposite sides parallel.

40. There are four kinds of parallelograms: the *Rectangle*, the *Square*, the *Rhomboid*, and the *Rhombus*.

Fig. 18.



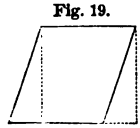
Square.

41. A ***Rectangle*** is a parallelogram whose angles are all right angles.

42. A ***Square*** is a parallelogram whose sides are all equal and whose angles are all right angles.

43. A ***Rhombus*** is a parallelogram whose sides are all equal, but whose angles are not right angles.

Fig. 19.

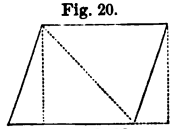


Rhombus.

44. A ***Rhomboid*** is a parallelogram whose opposite sides are equal, but whose angles are not right angles.

45. A ***Trapezoid*** is a quadrilateral which has but two of its opposite sides parallel; as, Fig. 15.

Fig. 20.



Rhomboid.

46. A ***Trapezium*** is a quadrilateral which has none of its opposite sides parallel; as, Fig. 16.

Problems.

47. To Find the Area of a Parallelogram.

RULE. — *Multiply the base by the altitude.* (See 528.)

Find the area

1. Of a square whose side is 4 ch. 1 rd. Whose perimeter is 137 ch.

2. Of a rhombus whose width is 10 rd. 5 yd. and perimeter 74.5 ch.

3. Of a rhomboid whose length is 20.5 ch. and altitude 30 rd. 5 yd.

4. Find the difference in area between a square and a rhombus, each 136 rd. in perimeter, the rhombus being 20 rd. wide.

48. To Find the Required Dimension of a Parallelogram.

RULE. — *Divide the area by the given dimension.* (See 528.)

Find the required dimension

5. Of a square whose area is 39 sq. ch. 1 sq. rd.

6. Of a rhombus whose area is 10 A. $6\frac{1}{4}$ sq. ch. and width 8.5 ch.

7. Of a rhomboid whose altitude is 40 rd. and area 15 A. 10 sq. ch.

8. The side of a square field is 6 ch. 2 rd. Find the perimeter of a field of equal area, shaped like a rhombus, the width being 16 rd.

49. To Find the Area of a Trapezoid.

RULE. — *Multiply half the sum of the parallel sides by the altitude.*

Find the area of a trapezoid

9. Whose parallel sides are 12 ft. 6 in. and 19 ft. 4 in., and width 10 ft.

10. Whose altitude is $20\frac{1}{4}$ rd. and parallel sides 10 ch. and $37\frac{1}{2}$ rd.

11. How many square feet in 3 boards, each 18 ft. long, 1 ft. 6 in. wide at one end, and 9 in. wide at the other?

12. How wide is a farm containing 67 A. 96 sq. rd., having two parallel sides 20 ch. and 30 ch. in length?

50. To Find the Area of a Trapezium.

RULE. — *Multiply the diagonal by the sum of the perpendiculars, and divide the product by* 2.

Find the area of a trapezium

13. Whose diagonal is 30 yd. and perpendiculars 15 ft. and 10 ft.

14. Whose diagonal is 10 rd. $2\frac{3}{4}$ yd. and perpendiculars 8 ch. and 8 rd.

15. How many acres in a quadrilateral field whose diagonal is 100 rd. and the perpendiculars 36.5 rd. and 22.5 rd.?

16. The area of a trapezium is $\frac{1}{2}$ A., and the perpendiculars to the diagonal 28 yd. and 22 yd. Find the diagonal.

THE CIRCLE.

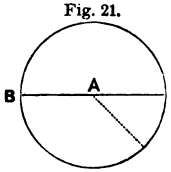
51. A ***Circle*** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the *centre.*

52. The ***Circumference*** of a circle is the curved line which bounds the circle.

53. The ***Diameter*** of a circle is a right line which passes through the centre and terminates at both ends in the circumference.

54. The ***Radius*** of a circle is a right line which extends from the centre to the circumference.

Fig. 21.



Circle.

The radius of a circle is one-half of the diameter.

Problems.

55. To Find the Circumference or the Diameter of a Circle.

RULES. — *Multiply the diameter by* 3.1416, *for the circumference.*

Divide the circumference by 3.1416, *for the diameter.*

Find the circumference or the diameter of a circle

1. Whose diameter is $30\frac{1}{4}$ ch. Whose radius is $\frac{1}{4}$ mi.

2. Whose circumference is 18 ft. 3.912 in. Circumference $\frac{1}{2}$ mi.

3. How many hoops 20 in. in diameter can be made from a piece of wire whose length is 104 ft. 8.64 in.?

4. Find the diameter of a wheel which makes 200 revolutions in running a mile. 150 revolutions per mile.

56. *To Find the Area of a Circle.*

RULES. — *Multiply the square of the diameter by* .7854. Or,

Multiply the square of the circumference by .07958.

Find the area of a circle

5. Whose diameter is 20 rd. Whose radius is 10 ft. 6 in.

6. Whose circumference is 50 ch. Circumference 12 rd. $2\frac{2}{4}$ yd.

7. At $187.50 per acre, what is the value of a circular lawn enclosed by 100 rd. of fence? Of a tract whose radius is 50 ch.?

8. Find the area of a ring, the diameter of the inner circle being 50 ft., and the circumference of the outer 200 ft.

57. *To Find the Diameter or the Circumference of a Circle when the Area is given.*

RULES.—*Divide the area by* .7854, *and extract the square root of the quotient for the diameter.* Or,

Divide by .07958, *and extract the square root for the circumference.*

Find the

9. Diameter of a circle whose area is $314\frac{4}{25}$ sq. ft. Area $\frac{1}{2}$ A.

10. Circumference of a circle containing $795\frac{1}{5}$ sq. ch. Area 1 A.

11. At $5 a rod, what will a fence encircling 100 sq. rd. of land cost?

12. Find the width of a circular walk whose area is 235.62 sq. yd., the inner circular plat being 10 yd. across. How many bricks $8\frac{1}{4}$ in. by $4\frac{1}{8}$ in. are required to pave it?

SOLIDS.

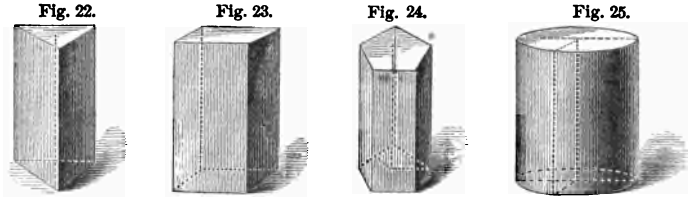
58. A ***Solid*** is that which has length, breadth, and thickness.

59. The ***Solidity*** or ***Volume*** of a solid is the space bounded by the surfaces of the solid.

60. A ***Prism*** is a solid whose ends are two equal parallel plane figures, and whose faces are parallelograms.

THE PRISM AND THE CYLINDER.

Fig. 22. Fig. 23. Fig. 24. Fig. 25.

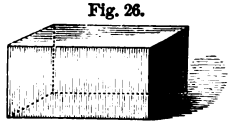


Triangular Prism. Quadrangular Prism. Pentagonal Prism. Cylinder.

61. According to the number of sides in their equal ends or bases, prisms are divided into *triangular, square, pentagonal,* etc.

62. A ***Parallelopipedon*** is a solid bounded by six plane parallelograms, every opposite two being equal and parallel; as, Figs. 23 and 26.

Fig. 26.



Parallelopipedon.

63. A ***Cube*** is a parallelopipedon having six equal square faces (421).

64. A ***Cylinder*** is a solid whose ends are two equal parallel circles, and whose surface is uniformly curved.

65. The ***Altitude*** of a prism or cylinder is the perpendicular distance between its two ends or bases.

66. A ***Convex Surface*** is the outer curved surface of a solid.

67. A ***Concave Surface*** is the inner curved surface of a hollow body.

Problems.

68. *To Find the Convex Surface of a Prism or Cylinder.*

RULE. — *Multiply the perimeter of the base by the altitude.*

To find the entire surface, *Add the area of the base to the convex surface.*

Find the convex surface and the entire surface

1. Of a rectangular solid 9 ft. 6 in. long, the end being $4\frac{3}{4}$ ft. by 3 ft. 8 in.

2. Of a triangular prism 10 ft. long, the sides of the base being 6 ft., 8 ft., and 10 ft. If the sides are 8 ft., 15 ft., 17 ft.

3. The cost of polishing the whole surface of a block of granite whose ends are $1\frac{1}{2}$ ft. square, at 30 cents per sq. ft., was \$14.55. Find the length.

4. Find the entire surface of a cylinder formed by the revolution about its longest side of a rectangle 7 ft. 6 in. by 5 ft. 4 in. 8 ft. 2 in. by 10 ft. 3 in.

69. *To Find the Volume of a Prism or Cylinder.*

RULE. — *Multiply the area of the base by the altitude.*

Find the solidity

5. Of a cylinder 15 ft. 6 in. long and 10 in. in diameter.

6. Of a cube whose entire surface is 73 sq. ft. 72 sq. in.

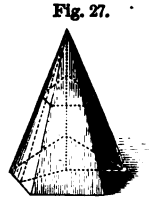
7. How many 4-in. cubes can be cut from a piece of timber 3 ft. long, 16 in. wide, 1 ft. thick, allowing no waste?

8. Find the depth of a well 10 ft. in diameter from which 235.62 cu. yd. of earth have been removed.

9. At the rate of 1000 gal. per hour, how long would it take to fill a cylindrical water-tank 10 ft. deep and 20 ft. in diameter?

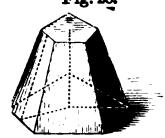
THE PYRAMID AND THE CONE.

Fig. 27.



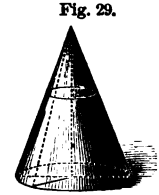
Pyramid.

Fig. 28.



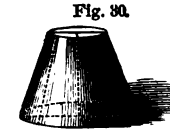
Frustum of a Pyramid.

Fig. 29.



Cone.

Fig. 30.



Frustum of a Cone.

70. A ***Pyramid*** is a solid whose base is any plane figure, and whose faces are triangles which meet in a common point, called the *Vertex.*

71. A ***Cone*** is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the vertex.

72. The ***Altitude*** of a pyramid or a cone is the perpendicular distance from the base to the vertex.

73. The ***Slant Height*** of a *pyramid* is the distance from the middle of any side of the base to the vertex.

74. The ***Slant Height*** of a *cone* is the distance from the circumference of the base to the vertex.

75. The ***Frustum*** of a pyramid or cone is the part which remains after cutting off the top by a plane parallel to the base.

Problems.

76. To Find the Convex Surface of a Pyramid or Cone.

RULE. — *Multiply the perimeter of the base by the slant height, and divide the product by* 2.

To find the entire surface *Add the area of the base to the convex surface.*

Find the convex surface and the entire surface

1. Of a cone whose base is 10 in. in diameter and slant height 2 ft. 6 in.

2. Of a pyramid, base 3 ft. 6 in. square, slant height 8 ft. 6 in.

3. At $.60 a sq. yd., what will it cost to paint a church-spire whose base is an octagon 6 ft. on each side, and slant height 75 ft.?

4. The convex surface of a square pyramid is 960 sq. in. and the slant height is 40 in. Find the entire surface.

5. The entire surface of a cone is 1256.64 sq. ft. and the diameter of the base is 20 ft. What is the slant height?

77. To Find the Solidity of a Pyramid or a Cone.

RULE.—*Multiply the area of the base by the altitude, and divide by* 3.

Find the solidity of a

6. Triangular pyramid, each side of the base 2 ft. 6 in. and altitude 30 ft.

7. Cone whose slant height is $31\frac{1}{4}$ in. and radius of base 25 in.

8. Find the value of a conical piece of metal $6\frac{2}{3}$ in. high, and $3\frac{1}{2}$ in. in diameter at the base, at $12\frac{1}{2}$ cents per cubic inch.

9. A square pyramid 81 ft. high is to contain 360 perches of stone. How many square feet will its base cover?

10. Find the altitude of a conical tower whose solidity is 628.32 cu. ft. and the diameter of the base 10 ft.

78. To Find the Convex Surface of a Frustum of a Pyramid or a Cone.

RULE.—*Multiply the sum of the perimeters or the circumferences of the two bases by the slant height, and divide the product by* 2.

To find the entire surface, *Add the areas of the bases to the convex surface.*

Find the convex surface and the entire surface

11. Of the frustum of a cone whose slant height is 10 ft. 6 in., the circumference of the lower base 15 ft. 6 in., and of the upper 10 ft. 8 in.

12. At $.25 per sq. yd., what will it cost to paint the faces of a pedestal in the shape of a frustum of a pyramid, the sides of the lower base being 15 ft. by 12 ft., and of the upper 10 ft. by 8 ft., the slant height 12 ft.?

13. The convex surface of a frustum of a cone is 376.992 sq. ft., the slant height 20 ft., and the diameter of the greater end 8 ft. Find the diameter of the smaller end.

79. To Find the Volume of the Frustum of a Pyramid or a Cone.

RULE.—*To the sum of the areas of both bases, add the square root of their product, multiply the sum by the altitude, and divide by* 3.

Find the volume of the frustum

14. Of a square pyramid, the altitude being 12 ft. 6 in., each side of the upper base 3 ft. 4 in., and of the lower base $6\frac{1}{3}$ ft.

15. Of a cone, the altitude being 20 ft., the diameter of the upper base 10 ft., and the circumference of the lower 50 ft.

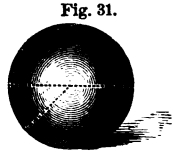
16. At $5 per cu. ft., what is the value of a block of marble 18 ft. high, the upper base being 16 in. square and the lower 40 in. square?

17. How high is the frustum of a cone containing 65.9736 cu. ft., the diameter of one end being 2 ft. and of the other 4 ft.?

THE SPHERE.

80. A ***Sphere*** is a solid bounded by a curved surface, every point of which is equally distant from a point within, called the *centre.*

Fig. 31.



Sphere.

81. The ***Diameter*** of a sphere is a right line which passes through the centre and terminates at both ends in the surface.

82. The ***Radius*** of a sphere is a right line which extends from the centre to the surface.

Problems.

83. To Find the Convex Surface of a Sphere.

RULE.—*Multiply the circumference of the sphere by its diameter.*

Find the convex surface of a sphere

1. Whose diameter is 6 ft. 6 in. Whose radius is 5 ft. 8 in.

2. Whose circumference is 10 in. If it is $314\frac{4}{25}$ ft.

3. The cost of polishing a marble globe 3 ft. 6 in. in circumference was $\$1.949\frac{1}{2}$. Find the cost per sq. ft.

4. The expense of gilding a ball at $4.80 per sq. ft. was $\$94.24\frac{4}{5}$. What was the diameter of the ball?

84. To Find the Volume of a Sphere.

RULE.—*Multiply the convex surface by the radius, and divide by* 3.

Find the volume of a sphere

5. Whose radius is 5 ft. Whose convex surface is $1256\frac{8}{25}$ square feet.

6. If the volume of a sphere 18 in. in diameter is 1 cu. ft. $1325.62\frac{1}{2}\frac{3}{5}$ cu. in., what would be the cost of gilding its surface @ $\$4.12\frac{1}{2}$ per sq. ft.?

7. At 8 cents per cubic inch, a ball of metal cost $\$41.88\frac{4}{5}$. What was the diameter? The convex surface?

8. Find the difference in feet between the diameter and the circumference of a globe whose volume is $1436.75\frac{2}{2}\frac{1}{5}$ cu. in.

THE END.